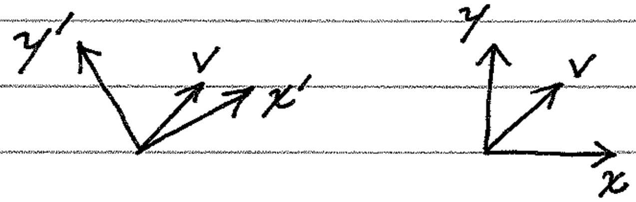
Representations:



$\vec{v} = \alpha'\hat{x}' + \beta'\hat{y}'$ $\quad$ $\vec{v} = \alpha\hat{x} + \beta\hat{y}$

Vector same, but coefficients different.

$\alpha' = \hat{x}'\cdot\vec{v}$ $\quad \begin{pmatrix}\alpha' \\ \beta'\end{pmatrix}$ vs. $\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ $\quad \alpha = \hat{x}\cdot\vec{v}$

$\beta' = \hat{y}'\cdot\vec{v}$ $\quad \beta = \hat{y}\cdot\vec{v}$

Always need to specify basis.

In quantum mechanics we have

basis

$$|\psi\rangle = \sum_n c_n |\psi_n\rangle$$

$$c_n = \langle\psi_n|\psi\rangle.$$

Momentum eigenstates:

$$\frac{\hbar}{i}\frac{d}{dx} f_p(x) = p\, f_p(x)$$

$$\rightarrow \quad \frac{df_p}{dx} = \frac{ip}{\hbar} f_p \rightarrow f_p \propto e^{ipx/\hbar}$$

Book's Convention: $f_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$

$$\rightarrow \langle f_{p'} | f_p \rangle = \int dx\, \frac{e^{-ip'x/\hbar}}{\sqrt{2\pi\hbar}} \frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}}$$

$$= \delta(p-p')$$

Expansion in momentum basis:

$$f(x) = \int c(p)\, f_p(x)\, dp$$

$c(p) = \langle f_p | f \rangle.$ (compare to discrete case)

This is often written as:

$$|f\rangle = \int c_p\, |p\rangle\, dp$$

$$c_p = \langle p | f \rangle.$$

Position eigenstates:

$$x\, g_y(x) = \underset{\substack{\uparrow \\ \text{eigenvalue}}}{y}\, g_y(x) \leftarrow \text{eigenvector}$$

$$g_y(x) = \delta(x-y)$$

$$\langle g_{y'} | g_y \rangle = \int dx\, \delta(x-y')\,\delta(x-y) = \delta(y-y')$$

Expansion in position basis:

$$f(x) = \int c(y)\, g_y(x)\, dy$$

$$c(y) = \langle g_y | f \rangle = f(y)$$

This is often written as:

$$|f\rangle = \int c_y\, |y\rangle\, dy$$

$$c_y = \langle y | f \rangle.$$

Energy-time uncertainty relation:

Let Q be any Hermitian operator.

$$\frac{d}{dt}\langle Q\rangle = \frac{d}{dt}\langle\psi|Q|\psi\rangle$$

$$= \langle\frac{d\psi}{dt}|Q|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \langle\psi|Q|\frac{d\psi}{dt}\rangle$$

From the Schrodinger equation

$$i\hbar\frac{d|\psi\rangle}{dt} = H|\psi\rangle \rightarrow \frac{d|\psi\rangle}{dt} = \frac{1}{i\hbar}H|\psi\rangle$$

$$\rightarrow \frac{d\langle\psi|}{dt} = \frac{-1}{i\hbar}\langle\psi|H$$

$$\Rightarrow \frac{d\langle Q\rangle}{dt} = \frac{-1}{i\hbar}\langle\psi|HQ|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle + \frac{1}{i\hbar}\langle\psi|QH|\psi\rangle$$

$$= \langle\psi|\frac{1}{i\hbar}[Q,H]|\psi\rangle + \langle\psi|\frac{\partial Q}{\partial t}|\psi\rangle$$

$$\Rightarrow \boxed{\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle[H,Q]\rangle + \langle\frac{\partial Q}{\partial t}\rangle}$$

Let $Q = x$. $\frac{\partial x}{\partial t} = 0$ since no time dependence in operator itself.

$$\frac{d\langle x\rangle}{dt} = \frac{i}{\hbar}\langle [H, x]\rangle$$

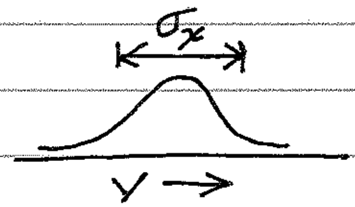
From th uncertainty relation with $A = H$, $B = x$

$$\sigma_A \sigma_B = \sigma_H \sigma_x \geq \left|\frac{1}{2i}\langle [A, B]\rangle\right|$$

$$= \left|\frac{1}{2i}\langle [H, x]\rangle\right|$$

$$= \left|\frac{1}{2i}\frac{\hbar}{i}\frac{d\langle x\rangle}{dt}\right| = \frac{\hbar}{2}\left|\frac{d\langle x\rangle}{dt}\right|$$

For a wave packet moving with average velocity $v = \frac{d\langle x\rangle}{dt}$, the time it takes to pass is approximately



$$\Delta t = \frac{\sigma_x}{v} = \frac{\sigma_x}{d\langle x\rangle/dt}.$$

$$\Delta E = \sigma_H$$

$$\Rightarrow \boxed{\Delta E \, \Delta t \geq \frac{\hbar}{2}}$$