Time Dependence of $\vec{S}$ in $\vec{B}$:

$$S_x = \frac{\hbar}{2}\sigma_x \ , \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\sigma_y \ , \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_z = \frac{\hbar}{2}\sigma_z \ , \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Eigenvalues $\pm \hbar/2$

Eigenvectors for general case: $\hat{n}\cdot\vec{S}$,
where $\hat{n} = \cos\theta\,\hat{z} + \sin\theta\cos\varphi\,\hat{x} + \sin\theta\sin\varphi\,\hat{y}$,

$+\hbar/2$ $\quad \begin{pmatrix} \cos\theta/2\ e^{-i\varphi/2} \\ \sin\theta/2\ e^{i\varphi/2} \end{pmatrix}$

$-\hbar/2$ $\quad \begin{pmatrix} -\sin\theta/2\ e^{-i\varphi/2} \\ \cos\theta/2\ e^{i\varphi/2} \end{pmatrix}$.

Time dependence: $H = -\vec{M}\cdot\vec{B}$

$$\vec{M} = \underset{\substack{\uparrow \\ \text{gyromagnetic} \\ \text{ratio}}}{\gamma}\vec{L} = \gamma\vec{S} = g\left(\frac{-e}{2m}\right)\vec{S} \quad , g \approx 2 \text{ for electron}$$

$$= -g\underbrace{\left(\frac{e\hbar}{2m}\right)}_{\mu_B}\frac{\vec{\sigma}}{2} \approx -\mu_B\vec{\sigma}$$

Take B in $\hat{z}$ direction:

$$H = \mu_B B\sigma_z \quad , \qquad \text{... electron}$$

$$= \mu_B B\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

If in $+\hbar/2$ $\vec{S}\cdot\hat{n}$ eigenstate: $\omega = \mu_B B/\hbar$

$$\begin{pmatrix} \cos\theta/2 \; e^{-i\varphi/2} \; e^{-i\omega t} \\ \sin\theta/2 \; e^{i\varphi/2} \; e^{i\omega t} \end{pmatrix}$$

Seems to rotate about $\hat{z}$ axis:

$$\varphi \rightarrow \varphi + 2\omega t .$$

Classically we have

$$\frac{d\vec{L}}{dt} = \gamma \vec{L} \times \vec{B}$$

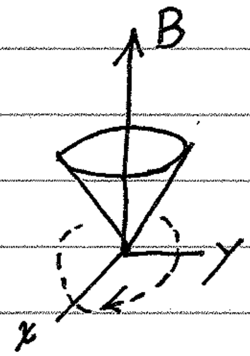
$$\frac{dL_x}{dt} = \gamma L_y B$$

$$\frac{dL_y}{dt} = -\gamma L_x B$$

$$\frac{dL_z}{dt} = 0$$

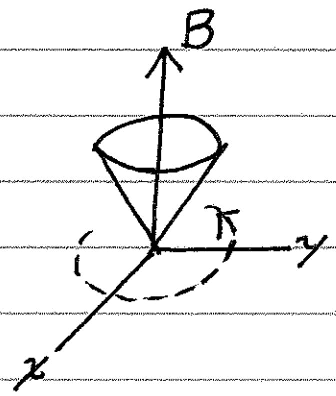
This has solution

$$L_x = -L_\perp \cos(\gamma B t + \varphi)$$
$$L_y = L_\perp \sin(\gamma B t + \varphi)$$
$$L_z = L_\parallel$$



For $\gamma < 0$

$$L_x = -L_\perp \cos(|\gamma| B t - \varphi)$$
$$L_y = -L_\perp \sin(|\gamma| B t - \varphi)$$
$$L_z = L_\parallel$$



Remember for electrons $\gamma \approx \frac{-e}{m} \rightarrow |\gamma| B = \frac{2\mu_B B}{\hbar} = 2\omega$ ✓

$$\langle \sigma_x \rangle = \left(\cos\frac{\theta}{2} e^{i\varphi/2} \;\; \sin\frac{\theta}{2} e^{-i\varphi/2} \right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi/2} \\ \sin\frac{\theta}{2} e^{i\varphi/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} \sin\frac{\theta}{2} \left(e^{i\varphi} + e^{-i\varphi} \right)$$

$$= \sin\theta \cos\varphi$$

$$\langle \sigma_y \rangle = \left(\cos\frac{\theta}{2} e^{i\varphi/2} \;\; \sin\frac{\theta}{2} e^{-i\varphi/2} \right) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi/2} \\ \sin\frac{\theta}{2} e^{i\varphi/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} \sin\frac{\theta}{2}\, i \left(e^{-i\varphi} - e^{i\varphi} \right)$$

$$= \sin\theta \sin\varphi$$

$$\langle \sigma_z \rangle = \left(\cos\frac{\theta}{2} e^{i\varphi/2} \;\; \sin\frac{\theta}{2} e^{-i\varphi/2} \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi/2} \\ \sin\frac{\theta}{2} e^{i\varphi/2} \end{pmatrix}$$

$$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$= \cos\theta$$

$$\Rightarrow \langle \vec{\sigma} \rangle = \hat{n}$$

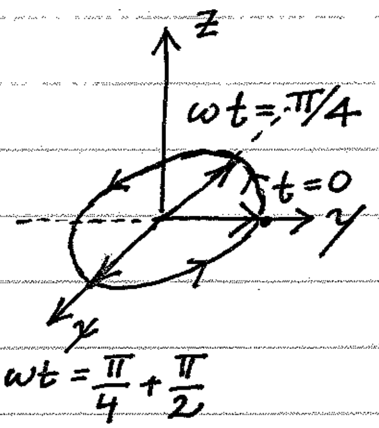
Example: $\quad H = \hbar\omega\sigma_z \quad$ (same as before)

At $t=0$ $\frac{+\hbar}{2}$ eigenvector of S_y:

$$\begin{pmatrix} \cos\pi/4 \; e^{-i\pi/4} \\ \sin\pi/4 \; e^{i\pi/4} \end{pmatrix} = e^{-i\pi/4}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

At $t>0$ an S_x measurement is made

outcome	probability
$+\hbar/2$	$\left\lvert \frac{1}{\sqrt{2}}(1 \;\; 1)\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\pi/4}e^{-i\omega t} \\ e^{i\pi/4}e^{i\omega t} \end{pmatrix}\right\rvert^2 = \cos^2(\omega t + \frac{\pi}{4})$
$-\hbar/2$	$\left\lvert \frac{1}{\sqrt{2}}(1 \; -1)\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\pi/4}e^{-i\omega t} \\ e^{i\pi/4}e^{i\omega t} \end{pmatrix}\right\rvert^2 = \sin^2(\omega t + \frac{\pi}{4})$



period: $\omega\tau = \pi$

frequency: $\frac{1}{\tau} = \frac{\omega}{\pi}$

ang. freq: $\frac{2\pi}{\tau} = 2\omega$ ✓

100% probability when in an eigenstate of measuring operator.