

Formulas to Memorize for Exam 1

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The following is a list of formulas that you need to know for Exam 1. I suspect that you already know most of them from working the problems. The first part of the exam will be short answer questions involving these formulas.

1. General

(a) Time dependent Schrodinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

(b) Expectations values:

$$\begin{aligned}\langle x \rangle &= \int dx \psi^*(x, t) x \psi(x, t) \\ \langle p \rangle &= \int dx \psi^*(x, t) p \psi(x, t),\end{aligned}$$

where $p = -i\hbar\partial/\partial x$. Other expectation values, e.g. $\langle x^2 \rangle$ and $\langle p^2 \rangle$, may be computed by substituting the appropriate expectation values instead of x and p above.

(c) Uncertainty principle:

$$\begin{aligned}\sigma_x &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \\ \sigma_p &= \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \\ \sigma_x \sigma_p &\geq \hbar/2.\end{aligned}$$

(d) Time independent Schrodinger equation:

$$E\varphi(x) = -\frac{\hbar^2}{2m} \frac{d^2 \varphi}{dx^2} + V\varphi = H\varphi$$

(e) Completeness:

If $H\varphi_n = E_n\varphi_n$ are the solutions to the time independent Schrodinger equation, then the solution to the time dependent Schrodinger equation have the form

$$\psi(x, t) = \sum_n c_n \varphi_n(x) e^{-iE_n t/\hbar},$$

where

$$c_n = \int dx \varphi_n^*(x) \psi(x, 0).$$

(f) Orthonormality:

For different energies the wave functions are orthogonal. The wave functions are also normalized.

$$\int dx \varphi_m^*(x) \varphi_n(x) = \delta_{m,n}$$

2. Infinite square well

$$\begin{aligned}\varphi_n(x) &= \sqrt{\frac{2}{a}} \sin(kx) \\ k &= \frac{n\pi}{a} \\ E_n &= \frac{\hbar^2 k^2}{2m}, \text{ for } n = 1, 2, 3, \dots\end{aligned}$$

3. Harmonic oscillator

$$\begin{aligned}
 H &= \hbar\omega \left(a_+ a_- + \frac{1}{2} \right) \\
 E_n &= \hbar\omega \left(n + \frac{1}{2} \right), \text{ for } n = 0, 1, 2, \dots \\
 [a, a_+] &= 1 \\
 [x, p] &= i\hbar \\
 a_+ \psi_n &= \sqrt{n+1} \psi_{n+1} \\
 a_- \psi_n &= \sqrt{n} \psi_{n-1} \\
 a_+ a_- \psi_n &= n \psi_n
 \end{aligned}$$

4. Free particle

$$\begin{aligned}
 \psi(x, t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \frac{\hbar k^2}{2m} t)} dk \\
 \phi(k) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x, 0) e^{-ikx} dx
 \end{aligned}$$

5. Piecewise constant potentials

The solution to the time independent Schrodinger equation for piecewise constant potentials (constant V_o),

$$E\varphi(x) = -\frac{\hbar^2}{2m} \frac{d^2\varphi}{dx^2} + V_o\varphi,$$

is

$$\varphi(x) = Ae^{ikx} + A'e^{-ikx} \text{ with } k = \sqrt{\frac{2m(E - V_o)}{\hbar^2}} \text{ for } E > V_o,$$

and

$$\varphi(x) = Be^{\rho x} + B'e^{-\rho x} \text{ with } \rho = \sqrt{\frac{2m(V_o - E)}{\hbar^2}} \text{ for } E < V_o.$$

The wave function and its first derivative are continuous at the boundaries (except for delta function potentials). The probability current is

$$j = \frac{\hbar}{2mi} \left(\psi^* \frac{\partial\psi}{\partial x} - \psi \frac{\partial\psi^*}{\partial x} \right).$$

It satisfies the continuity equation for the probability

$$\frac{\partial|\psi(x, t)|^2}{\partial t} + \frac{\partial j}{\partial x} = 0,$$

and for stationary states (solutions to the time independent Schrodinger equation) satisfies $\partial j/\partial x = 0$. The transmission and reflection probabilities are

$$\begin{aligned}
 T &= \frac{\dot{j}_{\text{transmitted}}}{\dot{j}_{\text{incoming}}} \\
 R &= \frac{\dot{j}_{\text{reflected}}}{\dot{j}_{\text{incoming}}} \\
 T + R &= 1.
 \end{aligned}$$

Formulas Printed on Exam 1

These formulas will be printed on the exam. You do not need to memorize them.
Harmonic oscillator:

$$\begin{aligned}a_+ &= \frac{1}{\sqrt{2\hbar m\omega}}(-ip + m\omega x) \\a_- &= \frac{1}{\sqrt{2\hbar m\omega}}(+ip + m\omega x) \\x &= \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-) \\p &= i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-) \\\psi_0(x) &= \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right) \\\psi_n &= \frac{1}{\sqrt{n!}}(a_+)^n \psi_0\end{aligned}$$

Delta function potential $V(x) = \alpha \delta(x)$:

$$\frac{d\varphi(0^+)}{dx} - \frac{d\varphi(0^-)}{dx} = \frac{2m\alpha}{\hbar^2} \varphi(0).$$