Homework 10

(due Wednesday, Dec. 4)

This is the last homework assignment! There is one problem on spin and one problem on addition of angular momentum.

1. **Spin:**
   Any two state system may be thought of as a spin 1/2 particle. This analogy is mathematically accurate and gives us valuable physical intuition. Consider a two state system with a Hamiltonian written in matrix form as

$$\begin{pmatrix} \epsilon & v \\ v & -\epsilon \end{pmatrix}. \tag{1}$$

   The Hamiltonian of an electron in a magnetic field of magnitude B and direction $\hat{n}$ is

$$\mu_B B \hat{n} \cdot \vec{\sigma}. \tag{2}$$

   (a) Comparing Eqs. (1) and (2), what is the effective B and $\hat{n}$ for the Hamiltonian in Eq. (1)? What is the angle, θ, that $\hat{n}$ makes with the z-axis?

   (b) Suppose the spin is initially in pointing in the $+z$ direction. Sketch how the spin will precess about the effect magnetic field for the following three cases.

      i. $\epsilon = v$

      ii. $\epsilon \gg v$

      iii. $\epsilon \ll v$

   (c) What is the angular frequency and period of the precession of the spin?

   (d) Now we are going to solve this same problem without making analogy to a spin in a magnetic field.

      i. Find the eigenvalues and eigenvectors of the Hamiltonian matrix of Eq. (1).

      ii. Suppose that the state of the system at $t = 0$ is

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{3}$$

      What is the state of the system at time $t > 0$?

      iii. What is the probability of again being in the state of Eq. (3) at time $t > 0$.

   (e) Compare the results of (c) with those of (d)iii.

2. **Addition of angular momentum:**
In this problem we will compute the Clebsch-Gordon coefficients for adding spin 1 and spin $\frac{1}{2}$ angular momentum. The results will be compared to those in Table 4.8 of the book.

   (a) List the (six) possible pairs of m_1 and m_2 for $j_1 = 1$ and $j_2 = \frac{1}{2}$. Give the $m = m_1 + m_2$ values for these two states.

   (b) The state with $m_1 = 1$ and $m_2 = \frac{1}{2}$ has total angular momentum $\frac{3}{2}$. To avoid confusion use the notation of the book and call this state $|1, 1\rangle|\frac{1}{2}, \frac{1}{2}\rangle$. Apply the lowering operator to this state to get the the three other states with $j = 3/2$. Again following the notation of the book denote these states at $|\frac{3}{2}, m\rangle$ for $m = \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$.

   (c) Find a state which is orthogonal to $|\frac{3}{2}, \frac{1}{2}\rangle$, but still has an m value of $\frac{1}{2}$. Normalize it and make the coefficient of $|1, 1\rangle|\frac{1}{2}, -\frac{1}{2}\rangle$ positive. Verify that this state has total angular momentum $\frac{1}{2}$ by acting with S^2 on it.

   (d) Apply the lowering operator, S_-, to the state of (c) to obtain the $j = \frac{1}{2}$, $m = -\frac{1}{2}$ state.

   (e) Compare your results in (a)-(d) to those in Table 4.8.