

## Homework 7

(due Friday, October, 25)

In the last assignment we practiced finding eigenvalues and eigenvectors and using the Dirac notation. This assignment applies those skills to finding the possible outcomes of measurements and the probabilities of those outcomes.

1. A two state system,  $|1\rangle$  and  $|2\rangle$ , has Hamiltonian

$$H = E_0 \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} = 4E_0(|1\rangle\langle 1| + |2\rangle\langle 2|) + 3E_0(|2\rangle\langle 1| + |1\rangle\langle 2|). \quad (1)$$

- (a) Compute the eigenvalues and eigenvectors of  $H$ .  
(b) The position operator is

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad (2)$$

What are the eigenvalues and eigenvectors of  $X$ ?

- (c) At  $t = 0$  a position measurement is made, and the result is 1. What is the state of the system immediately after the measurement?  
(d) What is the state of the system at time  $t = \tau$  after the position measurement in (c)?  
(e) At time  $\tau$  a second position measurement is made. What are the possible outcomes of the measurement and the probabilities of those outcomes? Based on the probability of the outcomes, what is the average value of the position if many identical measurements were made?  
(f) Compare your average value to the expectation value computed as  $\langle \psi(\tau) | X | \psi(\tau) \rangle$ .

2. The Hamiltonian of a three state system is

$$H = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \quad (3)$$

Two other observables,  $A$  and  $B$ , are represented by the matrices

$$A = \alpha \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \text{ and } B = \beta \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (4)$$

The coefficients  $\omega$ ,  $\alpha$ , and  $\beta$  are all real.

- (a) Compute the commutators  $[H, A]$ ,  $[H, B]$ , and  $[A, B]$ . For which pairs of operators is it possible to find a complete set of simultaneous eigenvectors?  
(b) Find the eigenvalues and eigenvectors of  $H$ ,  $A$ , and  $B$ . You do not have to solve the characteristic equation if you can find the eigenvectors and eigenvalues by inspection. How do your results support your conclusions for (a)?

(c) Suppose at  $t = 0$  the system is in the state

$$|\psi(0)\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \quad (5)$$

where  $|c_1|^2 + |c_2|^2 + |c_3|^2 = 1$ . If an energy ( $H$  operator) measurement is made, what are the possible outcomes and their associated probabilities? If many measurements were made on identically prepared states, what would the average value of the energy be? Compare this result to  $\langle\psi(0)|H|\psi(0)\rangle$ .

(d) Repeat (c) for the operator  $A$ .

(e) Repeat (c) for the operator  $B$ .

(f) For the  $|\psi(0)\rangle$  in part (c), what is  $|\psi(t)\rangle$  with no intervening measurements?

(g) If an energy measurement is made at time  $t$ , what are the possible outcomes and their associated probabilities? Compare your results to those at  $t = 0$  (part (c)) and explain why they should or should not be the same.

(h) Repeat (g) for the operator  $A$ .

(i) Repeat (g) for the operator  $B$ .