**Bonus Homework 9**
(due Friday, Nov. 8)

Because of Veteran's Day on Monday Nov. 11 and Exam 2 on Nov. 13, I am treating this homework as a bonus assignment. Your score on this will add to your overall homework grade. To get the bonus points you must hand it in on the due date: Friday, Nov. 8. No points will be given for late assignments. I will post the solution on Saturday, Nov. 9 so that you can study for the exam on the following Wednesday. Note: you can not get more than 100% on the homework part of your grade. Also, the references to the book below are for the previous edition of the textbook.

1. **Hydrogen atom:**
   Solve for the radial wave functions of the hydrogen atom for $n = 2$. There are two possible $l$ values for $n = 2$: $l = 0$ and $l = 1$.

   (a) The first step is to solve for the coefficients of the power series using the recursion relation

   $$c_{j+1} = \frac{2(j+l+1-n)}{(j+1)(j+2l+2)}c_j \tag{1}$$

   For $n = 2$ and $l = 1$ show that $c_1 = 0$. For $n = 2$ and $l = 0$ show that $c_2 = 0$. Note: I am using the book's notation, which is slightly different from mine. Please stick with their conventions, which are listed here, to avoid confusion.

   (b) Using the book's notation, the function $u(\rho)$ is then

   $$u(\rho) = \rho^{l+1}e^{-\rho}\sum_{j=0}^{\infty} c_j\rho^j. \tag{2}$$

   Express $u$ in terms of $r$ instead of $\rho$ using $\rho = r/(a_o n)$, where $n = 2$ here.

   (c) Apply the normalization condition

   $$\int_0^{\infty}(u(r))^2 dr = 1 \tag{3}$$

   to determine the coefficient $c_0$.

   (d) Finally compute $R(r) = u(r)/r$ and compare your results to those in Table 4.7 in the book.

2. **3D Particle in a box:** (Problem 4.9 in book)
   In three dimensions a particle in a finite box has the potential $V(r) = -V_0$ for $r < a$ and $V(r) = 0$ for $r > a$ with $V_0 > 0$. We are going to look for solutions to the radial Schrodinger equation for $-V_o < E < 0$ and for $l = 0$. By using $l = 0$ we will readily be able to solve the radial equation for $u$.

   Hint: For this problem if you define

   $$k = \sqrt{\frac{2m((E+V_0)}{\hbar^2}} = \sqrt{\frac{2m((V_0-|E|)}{\hbar^2}} \tag{4}$$

   $$\kappa = \sqrt{\frac{2m|E|}{\hbar^2}}, \tag{5}$$

then the relation

$$k^2 + \kappa^2 = \frac{2mV_0}{\hbar^2} \tag{6}$$

will allow you to eliminate $\kappa$ in part (d).

(a) Write down the equation for $u(r)$ for $r < a$ with $l = 0$ and solve it. (See Eq. 4.37 in the book.) It is a second order differential equation so there will be two independent solutions. Given that $R(r) = u(r)/r$ which of the solutions is the physical one?

(b) Write down the equation for $u(r)$ for $r > a$ with $l = 0$ and solve it. Again there will be two independent solutions, but only one of them will be physical (normalizable). Indicate which solution is physical.

(c) As in one dimensional problems, match the solutions for $u(r)$ at $r = a$ by assuming that both $u$ and its first derivative are continuous at $r = a$. Derive an equation to determine the bound state energies.

(d) Show that there is no bound state if $V_0 a^2 < \pi^2\hbar^2/8m$.

(e) For a general $V_0$ how many bound states are there?