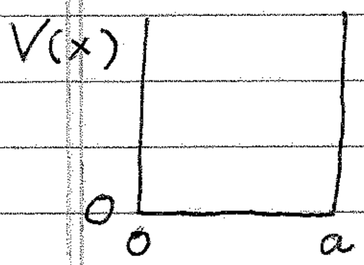
<u>Review last time:</u>



Solutions of infinite square well:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{\pi n x}{a}\right)$$

$$E_n = \frac{\hbar^2}{2m}\left(\frac{\pi n}{a}\right)^2$$

<u>Orthonormality:</u> $\int_0^a dx\; \psi_n^*(x)\,\psi_m(x)\,dx = \delta_{mn}$

<u>Completeness:</u> $f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$, where

$$c_n = \int_0^a \psi_n^*(x)\,f(x)\,dx.$$

or equivalently: $\sum_{n=1}^{\infty} \psi_n^*(x)\,\psi_n(x') = \delta(x - x')$

Normalization: $f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$

$$\int_0^a f^*(x) f(x)\, dx =$$

$$= \int_0^a \left(\sum_{m=1}^{\infty} c_m^* \psi_m^*(x) \right) \left(\sum_{n=1}^{\infty} c_n \psi_n(x) \right) dx$$

$$= \sum_{m,n=1}^{\infty} c_m^* c_n \int_0^a \psi_m^*(x) \psi_n(x)\, dx$$

$$= \sum_{m,n=1}^{\infty} c_m^* c_n \delta_{m,n} = \boxed{\sum_{n=1}^{\infty} |c_n|^2 = 1}$$

Time evolution:

$$\boxed{\begin{array}{l} \text{If } \quad \Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x), \\ \text{then } \quad \Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-\frac{iE_n}{\hbar} t} . \end{array}}$$

Expectation values:

$$\langle H \rangle = \int \psi^*(x,t)\, H\, \psi(x,t)\, dx$$

$$= \int \left(\sum_{n=1}^{\infty} c_n^* e^{\frac{iE_n t}{\hbar}} \psi_n^*(x) \right)$$

$$H \left(\sum_{m=1}^{\infty} c_m e^{-\frac{iE_m t}{\hbar}} \psi_m(x) \right) dx$$

$$= \int \left(\sum_{n=1}^{\infty} c_n^* e^{\frac{iE_n t}{\hbar}} \psi_n^*(x) \right)$$

$$\left(\sum_{m=1}^{\infty} c_m e^{-\frac{iE_m t}{\hbar}} E_m \psi_m(x) \right) dx$$

$$= \sum_{m,n=1}^{\infty} c_n^* c_m e^{-i\frac{(E_m - E_n)t}{\hbar}} E_m \underbrace{\int \psi_n^*(x)\, \psi_m(x)\, dx}_{\delta_{m,n}}$$

$$\boxed{\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n}$$

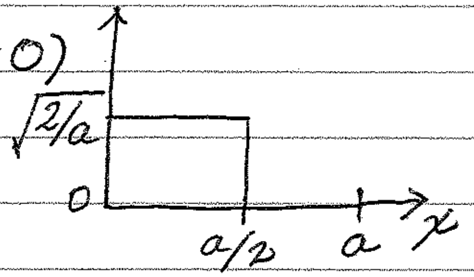
Had this been x we would have gotten:

$$\langle x \rangle = \sum_{m,n=1}^{\infty} c_n^* c_m e^{-i\frac{(E_m - E_n)t}{\hbar}} \int \psi_n^*(x)\, x\, \psi_m(x)\, dx$$

An example:

Take:

$\psi(x, t=0)$ (normalized to 1)



$a/2 \leftarrow$ because 0 for $\frac{a}{2} \leq x \leq a$

$$C_n = \int_0^{a/2} \sqrt{\frac{2}{a}} \sin\left(\frac{\pi n x}{a}\right) \sqrt{\frac{2}{a}}\, dx$$

$$= \frac{2}{a} \frac{a}{\pi n} \left. -\cos\left(\frac{\pi n x}{a}\right)\right|_0^{a/2}$$

$$C_n = \frac{2}{\pi n}\left(-\cos\left(\frac{\pi n}{2}\right) + 1\right)$$

See matlab codes:

leftside.m ... plots $\psi(x,t)$ at one time keeping $n \leq nmax$ terms

movie.m ... plots $\psi(x,t)$ at different times with a 1/2 second pause to create a movie