Momentum:

We have $\langle x\rangle = \int_{-\infty}^{+\infty} x\,|\psi(x,t)|^2\,dx$.

What is $\frac{d\langle x\rangle}{dt}$?

$$\frac{d\langle x\rangle}{dt} = \int_{-\infty}^{+\infty} x\,\frac{\partial|\psi(x,t)|^2}{\partial t}\,dx$$

$$\frac{\partial|\psi(x,t)|^2}{\partial t} + \frac{\partial j}{\partial x} = 0, \text{ where}$$

$$j = \frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)$$

$$\rightarrow \frac{d\langle x\rangle}{dt} = \int_{-\infty}^{+\infty} x\,\frac{\partial}{\partial x}\,\frac{-\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx \leftarrow \int u\,dv$$

integration by parts

$$= \int_{-\infty}^{+\infty}\frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx \leftarrow -\int v\,du$$

$$+ \underbrace{x}_{u}\underbrace{\left(\frac{-\hbar}{2mi}\right)\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)}_{v}\Bigg|_{-\infty}^{+\infty} \leftarrow uv$$

Assuming $|\psi|^2$ is normalized, it vanishes at $\pm\infty$ faster than $1/x$. The second term vanishes.

$$\boxed{\frac{d\langle x\rangle}{dt} = \int_{-\infty}^{+\infty}\frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)dx}$$

Again assuming $|\psi|^2$ vanishes at $\pm\infty$, this can be integrated by parts to get

$$\frac{d\langle x\rangle}{dt} = \int_{-\infty}^{+\infty} \frac{\hbar}{mi}\psi^* \frac{\partial \psi}{\partial x}\, dx.$$

Introduce the momentum operator

$$\boxed{p = \frac{\hbar}{i}\frac{\partial}{\partial x}}$$

so that

$$\boxed{\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} = \frac{1}{m}\int_{-\infty}^{+\infty}\psi^* p\, \psi\, dx}.$$

The expectation value of the kinetic energy is

$$\left\langle \frac{p^2}{2m}\right\rangle = \int_{-\infty}^{+\infty}\psi^* \frac{p^2}{2m}\psi\, dx$$

$$= \int_{-\infty}^{+\infty}\psi^* \frac{1}{2m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2\psi\, dx$$

$$= \int_{-\infty}^{+\infty}\psi^* \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}\, dx$$