Name:

Quiz 1

In the following the integral

$$\int_0^\infty du\, u^n\, e^{-u} = n! \qquad (1)$$

will be useful, where $0! = 1$ and $n! = n(n-1)\ldots 1$ for integer $n > 0$.

For the wave function $\psi(x) = C\, x\, e^{-\alpha x}$ on the interval $x \geq 0$:

1. What is the constant, C, so that the wave function is normalized?

$$\int_0^\infty dx\, |\psi(x)|^2 = \frac{C^2}{(2\alpha)^3} \int_0^\infty dx\; x^2 e^{-2\alpha x} (2\alpha)^3 = \frac{2C^2}{(2\alpha)^3} = 1$$

$$\boxed{C = 2\alpha^{3/2}}$$

2. What is the expectation value of p for this wave function?

0 because for real ψ

$$\int_0^\infty dx\; \psi^*(x) \frac{\hbar}{i} \frac{d\psi}{dx} = \frac{\hbar}{i} \int_0^\infty dx\, \frac{1}{2} \frac{d\psi^2}{dx} = \frac{\hbar}{2i} \psi^2(x) \Big|_0^\infty = 0.$$

3. What is the expectation value of p^2 for this wave function?

$$p^2 = -\hbar^2 \frac{d^2}{dx^2} \rightarrow p^2 C x e^{-\alpha x} = -\hbar^2 C (\alpha^2 x - 2\alpha) e^{-\alpha x}$$

$$\langle p^2 \rangle = -\hbar^2 C^2 \int_0^\infty dx\; (\alpha^2 x^2 - 2\alpha x) e^{-2\alpha x}$$

$$= -\hbar^2 (4\alpha^3) \left(\frac{2\alpha^2}{(2\alpha)^3} - \frac{1}{2\alpha} \right) = \frac{\hbar^2 \alpha^3}{\alpha} = \boxed{\hbar^2 \alpha^2}$$