

# Solution

Name: \_\_\_\_\_

## Quiz 3

$$H = E_0 \begin{pmatrix} 4 & -2i \\ 2i & 1 \end{pmatrix}$$

1. What are the eigenvalues of the above Hamiltonian,  $H$ ?

$$\begin{aligned} 0 = \det(H - E\mathbb{1}) &= \det \begin{pmatrix} 4E_0 - E & -2iE_0 \\ 2iE_0 & E_0 - E \end{pmatrix} \\ &= 4E_0^2 - 5EE_0 + E^2 - 4E_0^2 \\ &\rightarrow 0 = E(E - 5E_0) \rightarrow E = 0, 5E_0 \end{aligned}$$

2. What are the eigenvectors of the Hamiltonian?

$$E = 0$$

$$\begin{pmatrix} 4 & -2i \\ 2i & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

$$\rightarrow 2ic_1 + c_2 = 0$$

$$\rightarrow |E=0\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ -2i \end{pmatrix}$$

$$E = 5E_0$$

$$\begin{pmatrix} 4 & -2i \\ 2i & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 5 \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\rightarrow 4c_1 - 2ic_2 = 5c_1 \rightarrow c_1 = -2ic_2$$

$$|E=5E_0\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} -2i \\ 1 \end{pmatrix}$$

3. For the eigenvector,  $|\psi\rangle$ , with the largest eigenvalue, what is  $|\psi\rangle\langle\psi|$ ?

$$|5E_0\rangle\langle 5E_0| = \frac{1}{5} \begin{pmatrix} -2i \cdot 2i & -2i \cdot 1 \\ 1 \cdot 2i & 1 \cdot 1 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 4 & -2i \\ 2i & 1 \end{pmatrix}$$