

# solution

Name: \_\_\_\_\_

## Quiz 4

A Hamiltonian,  $H$ , has energy eigenvectors which satisfy:  $H|1\rangle = \hbar\omega|1\rangle$ ,  $H|2\rangle = 0|2\rangle$ , and  $H|3\rangle = -\hbar\omega|3\rangle$ . Another operator,  $A$ , has eigenvectors and eigenvalues of

$$|a_1\rangle = \frac{1}{\sqrt{3}}(|1\rangle - |2\rangle + |3\rangle) \text{ with eigenvalue } 1 \quad (1)$$

$$|a_2\rangle = \frac{1}{\sqrt{6}}(2|1\rangle + |2\rangle - |3\rangle) \text{ with eigenvalue } 2 \quad (2)$$

$$|a_3\rangle = \frac{1}{\sqrt{2}}(|2\rangle + |3\rangle) \text{ with eigenvalue } 3. \quad (3)$$

1. A measurement of  $A$  made at  $t = 0$  finds a value of 2. What is the state of the system ~~write~~ after the measurement is made?

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$$|\psi(0^+)\rangle = \frac{1}{\sqrt{6}}(2|1\rangle + |2\rangle - |3\rangle)$$

2. What is the state of the system a time  $t$  after the measurement?

$$|\psi(t)\rangle = \frac{1}{\sqrt{6}}(2e^{-i\omega t}|1\rangle + |2\rangle - e^{+i\omega t}|3\rangle)$$

3. At time  $t$  another measurement of  $A$  is made. What is probability that the outcome is 3? Plot the probability as a function of  $\omega t$ .

$$\begin{aligned} |\langle a_3 | \psi(t) \rangle|^2 &= \left| \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{6}} (1 - e^{i\omega t}) \right|^2 \\ &= \frac{1}{6} \frac{1}{2} (2 - \cos(\omega t)) = \frac{1}{6} (1 - \cos(\omega t)) \end{aligned}$$

