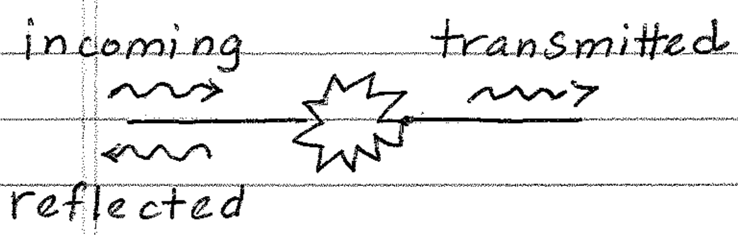
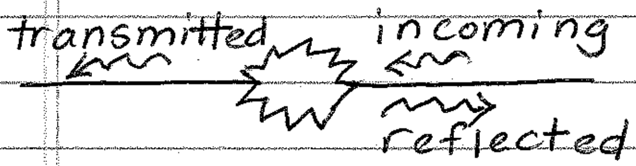
Last time:





e^{ikx} right moving

e^{-ikx} left moving

Can combine different k's to get a wave-packet as we did for the Gaussian wavepacket.

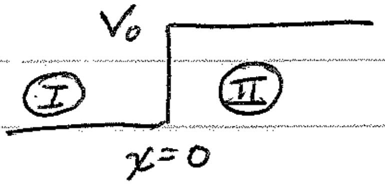
Boundary conditions: ψ continuous

$d\psi/dx$ continuous

Probability current: $j = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$

For $Ae^{ikx} + A'e^{-ikx}$, $j = \frac{\hbar k}{m}\left(|A|^2 - |A'|^2\right)$

Step Barrier:



$E < V_0$

(I) $x<0$: $\dfrac{-\hbar^2}{2m}\dfrac{d^2\varphi}{dx^2} = E\varphi \longrightarrow \dfrac{d^2\varphi}{dx^2} = -k^2\varphi$

with $k = \sqrt{\dfrac{2mE}{\hbar^2}}$

(II) $x>L$: $\dfrac{-\hbar^2}{2m}\dfrac{d^2\varphi}{dx^2} = (E-V_0)\varphi \longrightarrow \dfrac{d^2\varphi}{dx^2} = \rho^2\varphi$

with $\rho = \sqrt{\dfrac{2m(V_0-E)}{\hbar^2}}$

(I) $x<0$ $\quad \varphi(x) = Ae^{ikx} + A'e^{-ikx}$

(II) $x>0$ $\quad \varphi(x) = \underbrace{Be^{\rho x}}_{\text{unphysical}} + B'e^{-\rho x}$

$\longrightarrow B = 0$

Boundary conditions:

φ continuous: $\quad A + A' = B'$

φ' continuous: $ik(A - A') = -\rho B'$

$$\rightarrow A - A' = \frac{i\rho}{k} B'$$

$$\rightarrow \quad 2A = \left(1 + \frac{i\rho}{k}\right) B$$

$$2A' = \left(1 - \frac{i\rho}{k}\right) B$$

$$\rightarrow \frac{A'}{A} = \frac{k - i\rho}{k + i\rho} \quad \text{and} \quad \frac{B}{A} = \frac{2k}{k + i\rho}$$

$$R = \frac{j_{refl}}{j_{inc}} = \frac{\cancel{k}|A'|^2}{\cancel{k}|A|^2} = \frac{|k - i\rho|^2}{|k + i\rho|^2} = \frac{k^2 + \rho^2}{k^2 + \rho^2} = 1$$

$T = \frac{j_{trans}}{j_{inc}} = 0$ because $j_{trans} = 0$ for $e^{-\rho x}$.

Example: $\psi(x) = Be^{\rho x} + B'e^{-\rho x}$

$$\frac{\partial \psi}{\partial x} = \rho B e^{\rho x} - \rho B' e^{-\rho x}$$

$$\psi^* = B^* e^{\rho x} + B'^* e^{-\rho x}$$

$$\frac{\partial \psi^*}{\partial x} = \rho B^* e^{\rho x} - \rho B'^* e^{-\rho x}$$

$$\psi^* \frac{\partial \psi}{\partial x} = \rho |B|^2 e^{2\rho x} - \rho |B'|^2 e^{-2\rho x} + \rho B B'^* - \rho B' B^*$$

$$\psi \frac{\partial \psi^*}{\partial x} = \rho |B|^2 e^{2\rho x} - \rho |B'|^2 e^{-2\rho x} + \rho B^* B' - \rho B'^* B$$

$$\Rightarrow \psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x} = 2\rho B B'^* - 2\rho B' B^*$$

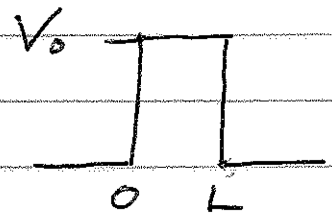
$$\Rightarrow j = \frac{\hbar}{mi} (\rho B B'^* - \rho B' B^*)$$

$$= \frac{\hbar}{mi} \rho \, 2i \, \mathrm{Im}\{BB'^*\}$$

$$= \frac{\hbar \rho}{m} \, 2 \, \mathrm{Im}\{BB'^*\}$$

If one of B or B' is zero, then j is zero.

Potential Barrier:



$E < V_0$

$$\phi(x<0) = A_1 e^{ikx} + A_1' e^{-ikx}$$
$$\phi(0<x<L) = B_2 e^{\rho x} + B_2' e^{-\rho x}$$
$$\phi(x>L) = A_3 e^{ikx} + A_3' e^{-ikx}$$

with $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $\rho = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$.