

# $\gamma$ - $\gamma$ Coincidence in $^{22}\text{Na}$

## Experiment GGC

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PHY4803L — Advanced Physics Laboratory

### Objective

You will learn how to use two NaI scintillation detectors with specialized electronic modules to measure and analyze the coincident emission of gamma ray pairs created in the positron-electron annihilation process. You will also learn about “square root statistics” for counting experiments, and about the attenuation of gamma rays as they pass through lead.

### References

Jerome L. Duggan, *Laboratory Investigations in Nuclear Science*, (The Nucleus, Oak Ridge, TN 1988).

A. C. Melissinos, *Experiments in Modern Physics*, (Academic Press, New York, NY 1966).

Positron Emission Tomography article on Wikipedia

### Annihilation Radiation

$^{22}\text{Na}$  radioactively decays to an excited state of  $^{22}\text{Ne}$  either by emission of a positron (90% probability) or by electron capture (10% probability). The excited  $^{22}\text{Ne}$  nucleus decays with a mean life  $3 \times 10^{-12}$  s to the ground state with the emission of a 1.274 MeV gamma.

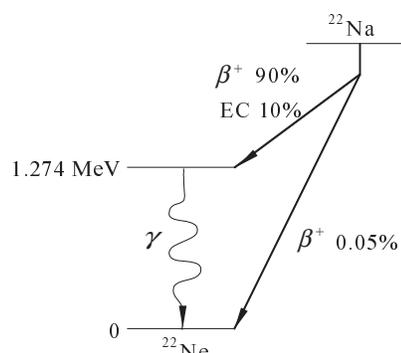


Figure 1: The decay of  $^{22}\text{Na}$  proceeds by positron  $\beta^+$  emission (90%) or electron capture (10%) to produce an excited state of  $^{22}\text{Ne}$  which decays by emission of a 1.274 MeV gamma.

The positrons are emitted with a range of kinetic energies up to about 0.5 MeV. They lose this energy quickly ( $10^{-9}$  s) in the material surrounding the source and, when they reach atomic (eV) energies, capture an electron to form positronium—a hydrogen-like “atom.” The positronium decays (with a lifetime on the order of  $10^{-10}$  s) by annihilation of the  $e^+$  and  $e^-$  into two gammas. By energy conservation, the energy of the gammas must equal the energy (including the rest mass energy) of the positronium and, by momentum conservation, the net momentum of the two gammas must equal the initial momentum of the positron-

ium.

The magnitude of the photon momentum is given by

$$p_\gamma = \frac{E_\gamma}{c} \quad (1)$$

In the rest frame of the positronium, there is no initial momentum and thus the two annihilation gammas must be of opposite momentum in that frame. Consequently, the two photons must be equal in energy and propagate in opposite directions. Since the initial energy of the positronium (neglecting the binding energy of a few eV) is simply the rest mass energy of an electron and positron (0.511 MeV each), each gamma will have an energy  $E_\gamma = 0.511$  MeV.

In the laboratory frame, the positronium will be moving with a range of kinetic energies up to a few eV (typical energy of electrons with which it forms). Then, depending on the direction of the initial positronium momentum relative to the gamma emission direction, the transformation to the lab frame gives gamma energies that might differ from 0.511 MeV and/or produce gammas that are not emitted exactly  $180^\circ$  apart.

**Exercise 1** *Assume the positronium is moving with a kinetic energy of 10 eV in the lab frame. Determine the lab-frame energy of each gamma if they are emitted in the rest frame parallel to the direction of motion. Determine the lab frame angle between the gammas if they are emitted in the rest frame perpendicular to the direction of motion.*

The previous exercise demonstrates that in the lab frame the two annihilation gammas are emitted within a degree or less of  $180^\circ$  from one another. The main objective of this experiment is to verify this behavior.

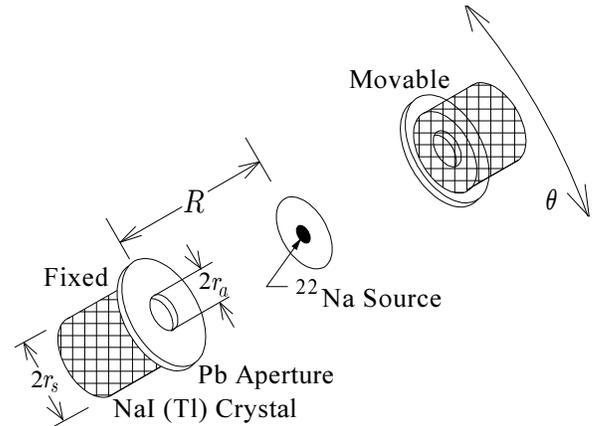


Figure 2: The two detectors for coincidence measurements have lead (Pb) shields with apertures in front of scintillation crystals.

## Apparatus

Read up in Experiment GA on the principles of *Gamma Ray Spectroscopy*. There are two scintillation detectors for this experiment, one to detect each of the annihilation gammas. Each detector consists of an integral NaI scintillation crystal, photomultiplier tube (PMT), and a preamplifier. Each preamplifier output is connected to a linear amplifier with variable gain. The amplifier output consists of analog pulses—voltage pulses having a duration of a few microseconds and having a height or amplitude proportional to the gamma ray energy deposited in the scintillator. Read up about the pulse height analyzer (PHA)—an electronic module inside the computer that sorts, records, and histograms the analog pulses according to their pulse heights. Learn about the spectrum of analog pulse heights expected from a monoenergetic gamma source; the photopeak, Compton plateau, and backscatter peak are features you will need to be able to recognize in the pulse height spectrum.

Figure 2 shows how the detectors and source will be arranged for the experiment. Sim-

ply stated, the experiment will detect *coincidences*. Our definition of a coincidence will be the near simultaneous detection of a gamma *in the 0.511 MeV photopeak* in each detector. The rate of these coincidences will be high when the detectors are on opposite sides of the  $^{22}\text{Na}$  source and will decrease as the detectors are moved away from this orientation. The theory section will describe a model predicting the coincidence rate, but before presenting this model, we next describe how a coincidence will be detected.

The electronics for the coincidence measurements is shown in Fig. 6.

Of fundamental importance for this experiment is the PHA's gate input. The gate is a logic input (0, 5 V) used with the analog pulse input. When the gate input is used, the PHA only processes those pulses that occur while the gate is high (at a 5 V level); it ignores those pulses occurring while the gate is low (at a 0 V level). To detect coincidences, the PHA will be set up to process analog pulses from the movable detector with its gate input derived from the analog pulses of the fixed detector. As described next, the Single Channel Analyzer and the Gate and Delay Generator will be used to create the gating signal and with the Delay Amplifier will reliably overlap it with the analog pulses.

The analog pulses from the fixed detector will be used to make the gating signal. To do so they are first processed by a **Single Channel Analyzer** (SCA), which is used to pick out those pulses that are in the 0.511 MeV photopeak. The SCA has a "window" from  $E$  to  $E + \Delta E$  with both the lower limit  $E$  and the window width  $\Delta E$  user adjustable. The SCA window is actually a voltage window but is labeled as an energy in anticipation of its use for gamma ray energy analysis. The SCA "looks at" its analog pulse input and "decides" whether the pulse height is within the window.

If it is, the SCA puts out a short 5 V logic pulse about  $0.5 \mu\text{s}$  long.  $E$  and  $\Delta E$  will be set so that the logic pulse is produced only for analog pulses in the 0.511 MeV photopeak.

The logic pulses from the SCA are routed to the **Gate and Delay Generator**. This module puts out a 5 V pulse that lasts a few microseconds after the arrival of the SCA logic pulse which initiated it. This rectangular pulse is called the *gating signal* and will be routed to the gate input of the PHA.

Before going to the PHA, the analog pulses from the movable detector are sent through the **Delay Amplifier**, a module which outputs these analog pulses with little or no change in their shape or height, but delayed from the input pulses by some fixed amount. This delay is needed because the SCA and the Gate and Delay Generator take some minimum time to create the gating signal. Without the delay, the gating signal would be too late to overlap with the analog pulses.

Examine Fig. 3 to see the timing relationships.

The name *singles* will be used to describe either those gammas which result in the production of an SCA logic pulse (fixed detector or SCA singles), or those gammas that result in a count in the 0.511 MeV photopeak of the PHA spectrum with gating off (movable detector or PHA singles). With the SCA properly set, both types of singles are gammas detected in the 0.511 MeV photopeak. We can now define more precisely that *coincidences* will refer to those gammas in 0.511 MeV photopeak of the PHA spectrum with gating on, i.e., in coincidence with 0.511 MeV gammas from the fixed detector. (The *Procedure* section will describe exactly how the modules are set up.)

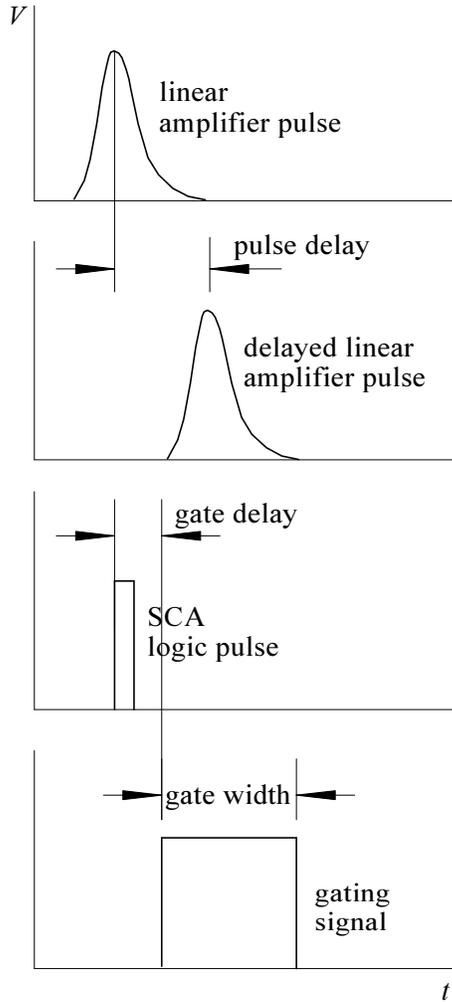


Figure 3: Relative timing of the various pulses.

## Theory

The nuclear decay rate (number of nuclear decays per second) is  $\Gamma_n = \alpha \cdot 3.7 \times 10^{10}$  decays/sec/curie, where  $\alpha$  is the source activity in curies. Ninety percent of the time, the nuclear decay proceeds by  $\beta^+$  emission, and these always annihilate with an electron to produce two 0.511 MeV gammas. Thus the rate of emission of 0.511 MeV gammas is  $0.9 \cdot 2 \cdot \Gamma_n = 1.8\Gamma_n$ . These gammas are emitted uniformly (in oppositely directed pairs) from the source. Thus at a distance  $R$  from the

source they are spread out over an area  $4\pi R^2$  and the flux  $\Phi$  (number per unit area per second) will be

$$\Phi = \frac{1.8\Gamma_n}{4\pi R^2} \quad (2)$$

The geometry of the source and detectors is shown in Fig. 2. The source is about 3 mm in size located near the middle of the disk and the annihilation events occur within a few millimeters of the source. One detector will be fixed and the other will be rotated about the source. Both use  $1'' \times 1''$  cylindrical NaI scintillators. In front of each detector there is a lead shield with a small aperture of radius  $r_a$  drilled through its center. The source-aperture distance will be measured and is given the symbol  $R$ . The apertures, the scintillators and their relationship to the source and each other are an important factor in determining the rates at which gammas are detected.

To calculate the true rate at which gammas are striking the face of the scintillator, the effect of the lead aperture needs to be taken into account. We call the effective aperture radius  $r_a$  and the effective scintillator radius  $r_s$ . The term “effective” is used because we will not take into account the full three dimensional nature of the shielding aperture or of the scintillator. We will determine the rate of passage of gammas as the rate passing through circles of the effective radii assuming both are a distance  $R$  from the source.

The fraction of 0.511 MeV gammas which get through the lead shielding will also need to be specified. It is expected to be on the order of 20% and will be expressed by the symbol  $\kappa$ .

With these considerations, the rate  $Q$  of 0.511 MeV gammas striking the face of the scintillator can be calculated.

$$Q = \Phi[A_a + \kappa(A_s - A_a)] \quad (3)$$

where the area of the aperture is  $A_a = \pi r_a^2$ , and the area of the scintillator is  $A_s = \pi r_s^2$ .

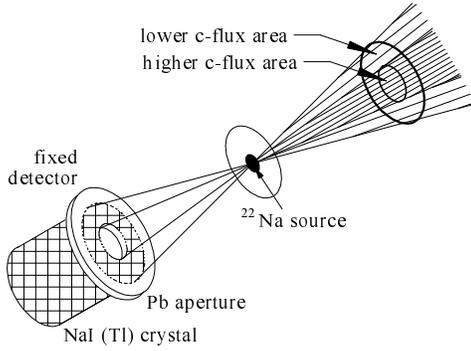


Figure 4: With the assumption of  $180^\circ$  gamma pairs, real coincidences only occur when the movable scintillator detects a gamma in the *c-flux*—outgoing gammas for which the opposite member of the pair has been detected in the fixed scintillator.

Thus  $A_s - A_a$  is the annular area of the detector shielded by lead.

For a variety of reasons, SCA or PHA singles will occur for only a fraction  $f$  of the 0.511 MeV gammas entering the corresponding scintillation crystal.  $f$  is called the photopeak efficiency and depends on crystal size, SCA or PHA settings and source/detector geometry. The singles rates are thus suppressed (from the value  $Q$ ) by the factor  $f$ , and the singles rates  $S$  will be given by

$$S_{f,m} = f_{f,m} * Q \quad (4)$$

where we have allowed for the possibility that the photopeak efficiency may be different for the fixed (subscript  $f$ ) and movable (subscript  $m$ ) detectors.

### Coincidence rate

With the assumption that 0.511 MeV gammas are always produced in pairs emitted  $180^\circ$  apart, we may model the detection of singles in the fixed detector as creating a *flux of possible coincidence gammas* moving in the opposite

direction. We will call this *c-flux* (for coincidence possible flux) and use the symbol  $\Phi'$  for it. The situation is as shown in Fig. 4.

The c-flux is not the same as the raw flux of 0.511 MeV gammas from the source. It is not uniform; it extends only into the region opposite the fixed detector from the source. With both detectors at the same distance  $R$  from the source, the c-flux will be the product of the raw flux at that distance and the probability that the fixed detector will register a count in the photopeak. In the region opposite the aperture, it is reduced in intensity from the raw flux by the photopeak efficiency.

$$\Phi'_{\text{center}} = \Phi \cdot f_f \quad (5)$$

In the annular region it is further reduced by the attenuation due to the Pb shielding.

$$\Phi'_{\text{annulus}} = \Phi \cdot \kappa f_f \quad (6)$$

It will help in the analysis to remodel these c-flux regions as the sum of two complete circles of c-flux: a higher c-flux extending over a circular area of radius  $r_a$

$$\Phi'_a = \Phi(1 - \kappa)f_f \quad (7)$$

and a lower c-flux extending over a circular area of radius  $r_s$

$$\Phi'_s = \Phi\kappa f_f \quad (8)$$

The sum of these c-fluxes give the same result as the two c-fluxes  $\Phi'_{\text{center}}$  and  $\Phi'_{\text{annulus}}$ .

Keep in mind the c-flux is a real flux of gammas radiating out from the source opposite the fixed detector. They are modeled based on the assumption that the 0.511 MeV gammas are always created in  $180^\circ$  pairs. The c-flux is simply a selected fraction of the total flux, for which the opposite gamma of the pair has been detected in the fixed detector. Consequently,

any of the c-flux gammas detected in the movable detector's photopeak will be counted as coincidences.

Because of the movable detector's photopeak efficiency and shielding aperture, it will only count a fraction of the c-flux gammas. Over its aperture area, the detection probability is  $f_m$ ; in its annular (shielded) area, the probability is lower— $\kappa f_m$ .

Again, this situation is more easily modeled as the sum of two full circular detection regions: one of (scintillator) radius  $r_s$  with a lower detection probability  $\kappa f_m$  and another of (aperture) radius  $r_a$  with a higher detection probability  $(1 - \kappa)f_m$ . The efficiency of the sum of these two detection regions will be the same as that of circular and annular regions discussed in the previous paragraph.

Real coincidences should only occur when there is some overlap of the movable detector into the region of the c-flux. If the fixed and movable detectors are very far from being opposite one another, there will be no overlap and no chance of real coincidences. As the movable detector is brought in to the c-flux, real coincidences just start to occur when its shielded area starts touching the circular area containing the lower c-flux. As the overlap area grows, the coincidence rate increases in proportion to that area. In fact, the coincidence rate would be the product of the overlap area, the magnitude of the c-flux in that area and the movable scintillator's detection probability for that area. Of course, as there begins to be overlap with the higher c-flux areas and higher detection probability regions, these will also need to be taken into account. The way the c-flux and movable detector efficiency were each modeled as a sum of two complete circular regions, there will be four possible overlapping areas, and the coincidence rate will be the sum of four such products.

The overlap areas between the various cir-

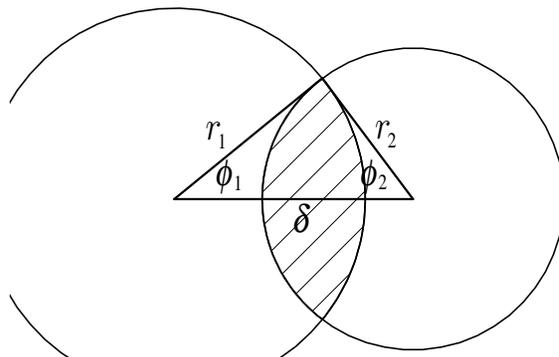


Figure 5: The overlap area  $A'$  (hatched) depends on the radii  $r_1$  and  $r_2$  of the circles and the separation  $\delta$  between their centers. The half-sector angles  $\phi_1$  and  $\phi_2$ , found from the law of cosines, is useful in expressing the area formula.

cles involved can be calculated from the geometry of the apparatus. They will depend on the radii of the two circles involved and on the separation  $\delta$  between the centers of the circles. We will assume the aperture and scintillator circles for each detector have common centers so that all overlap areas for a given position of the movable detector will have the same value for  $\delta$ .

**Exercise 2** With reference to Fig. 5, show that the hatched overlap area  $A'$  of two circles of radii  $r_1$  and  $r_2$  (where  $r_1 > r_2$ ) is given by

$$\text{for } \delta < r_1 - r_2 :$$

$$A' = \pi r_2^2$$

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$$\text{for } r_1 - r_2 < \delta < r_1 + r_2 :$$

$$A' = r_1^2 \left( \phi_1 - \frac{1}{2} \sin 2\phi_1 \right) + r_2^2 \left( \phi_2 - \frac{1}{2} \sin 2\phi_2 \right) \quad (9)$$

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$$\text{for } \delta > r_1 + r_2 :$$

$$A' = 0$$

where the half-sector angles  $\phi_1, \phi_2$  can be obtained from

$$\begin{aligned} r_1^2 &= r_2^2 + \delta^2 - 2r_2\delta \cos \phi_2 \\ r_2^2 &= r_1^2 + \delta^2 - 2r_1\delta \cos \phi_1 \end{aligned} \quad (10)$$

Components of the coincidence rate  $C$  can now be calculated as the product of the c-flux, the overlap area, and the detection probability.

Lower c-flux area with lower detection probability area

$$\begin{aligned} C_{ss} &= \Phi'_s \kappa f_m A'_{ss} \\ &= f_m f_f \Phi \kappa^2 A'_{ss} \end{aligned} \quad (11)$$

Lower c-flux area with higher detection probability area

$$\begin{aligned} C_{sa} &= \Phi'_s (1 - \kappa) f_m A'_{sa} \\ &= f_m f_f \Phi \kappa (1 - \kappa) A'_{sa} \end{aligned} \quad (12)$$

Higher c-flux area with lower detection probability area

$$\begin{aligned} C_{as} &= \Phi'_a \kappa f_m A'_{as} \\ &= f_m f_f \Phi (1 - \kappa) \kappa A'_{as} \end{aligned} \quad (13)$$

Higher c-flux area with higher detection probability area

$$\begin{aligned} C_{aa} &= \Phi'_a (1 - \kappa) f_m A'_{aa} \\ &= f_m f_f \Phi (1 - \kappa)^2 A'_{aa} \end{aligned} \quad (14)$$

In each case, the doubly subscripted area represents the overlap area of two circles of the corresponding two radii when their centers are offset by the amount  $\delta$ . Of course, the net coincidence rate will be the sum of the four components above.

$$C = C_{ss} + 2C_{sa} + C_{aa} \quad (15)$$

where use has been made of the fact that  $A'_{as} = A'_{sa}$ .

**Exercise 3** The coincidence rate would be greatest when the movable detector is centered over the c-flux pattern, i.e., for  $\delta = 0$ . Show that the maximum coincidence rate reduces to the expected result

$$C_{\max} = \Phi f_m f_f [A_a + \kappa^2(A_s - A_a)] \quad (16)$$

Also explain why this is the expected result.

Equations 4 and 16, respectively, predict the singles rates  $S_m$  and  $S_f$  and the maximum coincidence rate  $C_{\max}$  based on the (presumed unknown) flux  $\Phi$  and photopeak efficiencies  $f_m$  and  $f_f$  and the (presumed known) geometry factors  $A_a$  and  $A_s$  and attenuation factor  $\kappa$ . The next exercise asks you to invert these equations to predict the flux  $\Phi$  and the geometric mean of the photopeak efficiencies from the known geometry and attenuation factor and your actual measured rates for  $S_m, S_f$  and  $C_{\max}$ . Of course, you can then use the determination of  $\Phi$  with Eq. 2 and another known geometry factor ( $4\pi R^2$ ) and a theory factor (1.8) to determine the actual source activity  $\Gamma_n$ .

**Exercise 4** Show that the raw flux  $\Phi$  can be expressed

$$\Phi = \frac{S_m S_f}{C_{\max}} \left[ \frac{A_a + \kappa^2(A_s - A_a)}{(A_a + \kappa(A_s - A_a))^2} \right] \quad (17)$$

and that the geometric mean of the photopeak efficiencies can be expressed

$$\sqrt{f_m f_f} = \frac{C_{\max}}{\sqrt{S_m S_f}} \frac{A_a + \kappa(A_s - A_a)}{A_a + \kappa^2(A_s - A_a)} \quad (18)$$

### Random Coincidences

Even when SCA and PHA singles arrive randomly, there is a finite probability that one will arrive in each detector near enough in time

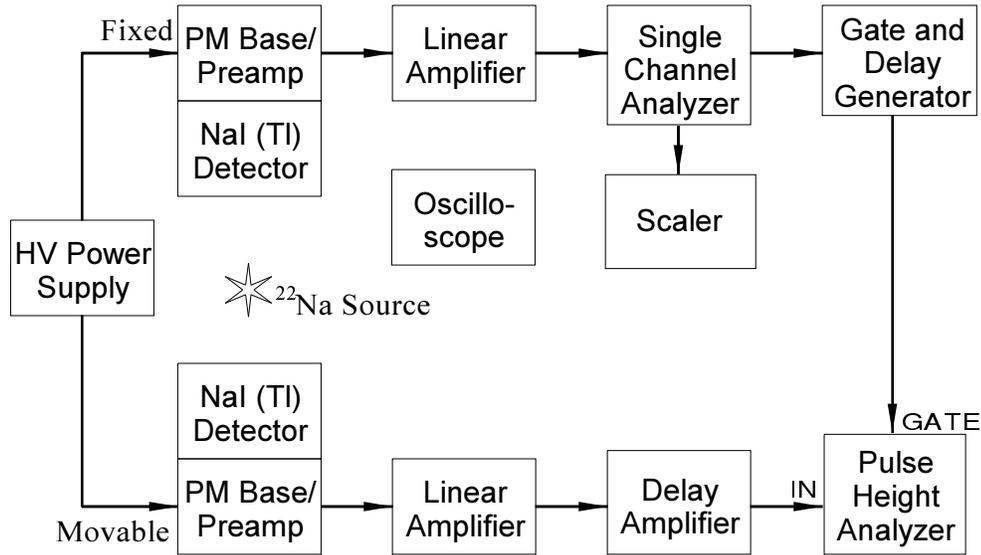


Figure 6: Schematic of the apparatus for the experiment.

to be counted as a coincidence. The average rate at which these *random coincidences* can be expected to occur is easily calculated and depends on the singles rates and the width  $\tau$  of the gating signal.

If SCA pulses arrive at a rate  $R_S$ , the product  $\tau R_S$  is the fraction of the time  $f$  that the PHA gate will be open. For example, consider the case where  $\tau$  is  $4 \mu\text{s}$  and  $R_S = 500/\text{s}$ . On average, every second, there will be 500 SCA pulses and thus, during that second, the PHA gate will be open for  $2000 \mu\text{s}$ . The fraction of the time the gating window is open is thus  $f = 2000 \mu\text{s}/\text{s} = 0.002$ .

If PHA-singles are occurring randomly at a rate  $R_P$ , the rate  $R_{rc}$  at which they should be detected arriving in the gating window (and thus count as coincidences) is  $R_{rc} = fR_P$ . Thus random coincidences can be expected to occur at the rate

$$R_{rc} = \tau R_S R_P \quad (19)$$

## Procedures

### Safety

The  $^{22}\text{Na}$  source used in this experiment is sealed in a mounting disk and can be handled without special equipment. As a precaution, minimize handling of the source, handle it by the edges, keep it away from your body, and wash your hands after handling it.

Always handle the lead sheets and lead apertures using gloves or a paper towel and wash your hands afterward.

Do not eat or drink in the laboratory.

There are many electronic modules used in this experiment. If you are unsure how to use them or just want to get more information about them, please consult the manuals. Reread the *Apparatus* section and refer to Fig. 6 for an overview of the connections.

### Setup

1. Measure and record the relevant geometric quantities such as  $R$  (the source-

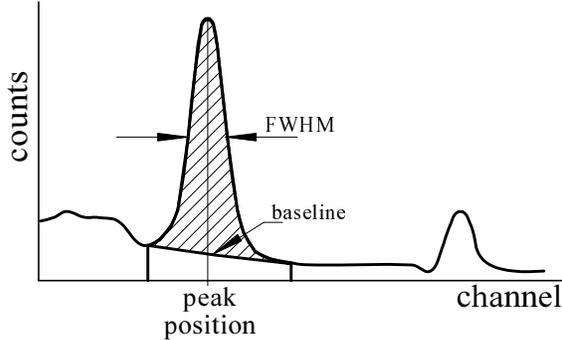


Figure 7: A region of interest set over a peak. The software provides the FWHM, the peak position, the gross area and the net area (hatched).

detector distance),  $r_a$  (the aperture radius), and  $d$  (the lead shielding thickness).

2. Place the  $^{22}\text{Na}$  source on its holder at the center of the table. To provide additional free electrons, place a thin steel disk on each side of the source disk. The combination should fit without wobbling in the mount located at the center of the table.
3. Place the fixed detector at the  $180^\circ$  setting on the table and the movable detector opposite it, at  $0^\circ$ . As its name implies, the fixed detector will not be moved during this experiment.
4. The HV supply has dual outputs and supplies voltage to both detector PM tubes. Make sure you use the special coax cable between the HV supply and the detector HV inputs. The connectors on these components are called MHV (on the detectors) and SHV (on the HV supply) and the special coax cables have corresponding mixed connectors. The BNC connectors found on most coax cables in the laboratory and usable for all other connections in this experiment are not designed to handle the high voltages. The high voltage should be set for positive polarity and 1000 V. The voltage reading on the HV supply may not indicate the set voltage. Trust the rotary dials used to set the voltage.
5. Connect the detector preamp outputs to the Linear Amplifier inputs. Set the Linear Amplifiers to direct, unipolar output. Set the coarse gain to x4 and the fine to about x5.
6. Put a BNC tee on the SCA input and on the oscilloscope channel 1. Connect the fixed detector's Linear Amplifier output to the SCA input, then to the oscilloscope channel 1, and then to the PHA analog input. (The PHA inputs are on the back of the computer; the analog pulse input is labeled IN.) With the oscilloscope triggering properly, pulses of various sizes, but with similar shape should be visible. A large fraction of the pulses should arise from the 0.511 MeV annihilation gammas; on the oscilloscope they should have nearly the same size and be quite a bit more intense than pulses of other sizes. Adjust the amplifier coarse and fine gain controls so that the 0.511 MeV annihilation gammas (the more intense group of traces) produce pulse heights around 3 V.
7. Start the Maestro software. Select **Acquire|ADC Setup** and then set the **Conversion Gain** to 1024 and the **Gating** to **Off**. Select **Acquire|Preset Limits** and clear any entries in the dialog box that appears. Select **Calculate|Calibration and Destroy Calibration**. Select **Acquire|Clear** and then **Acquire|Start**. The conversion gain of 1024 implies that a 10 V analog pulse will be sorted into channel 1024, with smaller

pulse heights sorted into correspondingly lower channels in the spectrum. Thus, the spectrum now coming in should show the 0.511 MeV photopeak at about 3 V (near channel 300 in the spectrum), and a weaker 1.274 MeV photopeak around 7.5 V (near channel 750). Adjust the Linear Amplifier gain if the peaks are too far off the suggested channel locations.

8. Acquire the PHA spectrum for the fixed detector with good statistics (until it looks smooth) and save it.
  9. Check the detector resolution for the 0.511 MeV and 1.274 MeV peaks. To do so, determine the **F**ull **W**idth at **H**alf the **M**aximum height of the peak (abbreviated FWHM). The ratio of this width to the center position of the photopeak is the detector resolution. The FWHM and the peak position must be measured in the same units, e.g., MeV, volts, or channels. Thus, measure the number of channels across the peak at half the peak height and divide by the position of the center of the peak in channels. This is easily done with the Maestro software. Set a **R**egion **O**f **I**nterest (ROI) using the Maestro software to cover the 0.511 MeV photopeak; include the whole peak with a few channels out in the shoulders to either side of the peak. Click in the region to get the FWHM and the center channel of the peak. For the 0.511 MeV peak, the resolution should be around 10-15%, and somewhat better for the 1.274 MeV peak.
  10. The ROI information also gives the *gross* and *net* counts in the peak. The gross count is simply the sum of all counts in each channel of the ROI. The net count is the count above a baseline at a height equal to the average of the first and last three points of the ROI. See Fig. 7. The true counts in the 0.511 MeV photopeak would be the net count; the gross count would include, for example, counts in the Compton plateau for the 1.274 MeV gammas. Determine the singles rate  $S_f$  from the net count and the live time.
- In the next few steps the electronics will be set up to make coincidence measurements of the fixed detector's analog pulses with a gating signal also generated from the fixed detector, i.e., coincidences of the fixed detector's pulses with themselves. As you will see, this will make it easier to set the timing properly and it will make it easy to set the SCA window. When finished, all that will be necessary to get true coincidences is to switch the PHA input to the movable detector's analog pulses.
11. Put a tee on the input of the Delay Amplifier. Disconnect the Linear Amplifier output/SCA input at the scope channel 1 tee and reconnect it to the Delay Amplifier input. Connect the Delay Amplifier output to the scope channel 1 (and then on to the PHA analog input). Connect the scope channel 2 to the Delay Amplifier input. Put in a total of 4  $\mu$ sec of delay on the Delay Amplifier and note how the output is delayed relative to the input with virtually no change in the pulse shape or height. If you do not see an output, jiggle the time delay switches on the Delay Amplifier. There seems to be an intermittency problem with these switches.
  12. Take and save a PHA spectrum with good statistics for these delayed pulses. Compare it with the one for the direct (undelayed) pulses taken in Step 8 to see if the Delay Amplifier has any effect.
  13. Set the SCA Mode switch to the middle position ( $\Delta E$ , 0 to 10 V), set  $E = 1.5$  V

(middle dial) and  $\Delta E = 3$  V (top dial). Connect the SCA output logic pulses to the Gate and Delay Generator.

14. On the Gate and Delay Generator, set the **Amplitude** dial to about 5 V (mid-way on the dial). Set the **Delay Range** switch to 0.1-1.1  $\mu\text{s}$  and set the **Delay** dial to the minimum (fully counterclockwise). Set the **Width** dial fully clockwise (about 5  $\mu\text{s}$ ). Put a BNC tee on the scope channel 2 and connect the Gate and Delay Generator output to it and then on to the PHA gate input (labeled **GATE** on the back of the PC). Set the scope trigger for channel 2 (the rising edge of the gate signal).
15. Next, you will overlap the (delayed) analog pulses and the gating signal. The 4  $\mu\text{s}$  delay on the Delay Amplifier should work well and need not be changed. For proper gating, the gate must occur before the peak of the analog pulses and extend for 0.5  $\mu\text{s}$  beyond the peak. Because of timing jitter, and because making the width of the gate bigger than is absolutely necessary is not very detrimental to the experiment, a gate width of around 5  $\mu\text{s}$  is recommended. Measure and record this width. Set the **Delay** to give a good overlap of the gate and the analog pulses.

The fixed detector's (delayed) analog pulses are now being processed by the PHA with the gate generated from these same (undelayed) pulses when they are within the SCA window. Next, the SCA will be adjusted while a spectrum is acquired with the PHA.

16. Select **Acquire|ADC Setup** and then set the **Gating** to **Coincidence**. The software often fails to use the gating signal (even when it is properly set to **Coincidence** mode) and

processes all pulses as if gating is set to **Off**. Turning the gating to **Off**, and then back to **Coincidence** seems to fix the problem. If you are unsure whether gating is working, it is wise to go through this procedure just to be sure. It seems to only happen when you start the Maestro software, so you should only have to do it once.

Clear and restart the PHA. The spectrum should now show only peak heights from 1.5 to 4.5 V, i.e., only channels from around 150 to 450 should now be receiving counts. Adjust the SCA  $E$  and  $\Delta E$ , clearing and restarting the PHA spectrum each time, until only the pulses in the 0.511 MeV photopeak are processed. Include all channels over the entire peak and even some on the flat shoulders to either side of the peak; it is better to include a few channels outside the peak than to miss any in the peak.

17. Acquire this windowed spectrum with good statistics (until it looks smooth) and save it. Set a **Region Of Interest (ROI)** using the Maestro software to cover all SCA windowed channels. Record the ROI gross counts. Connect a scalar to the SCA output, measure the SCA singles rate, and check that the ROI gross count rate is within a percent or so of the SCA singles rate.
18. Switch the Delay Amplifier input from the Linear Amplifier for the fixed detector to the Linear Amplifier for the movable detector. Now the movable detector's analog pulses go to the Delay Amplifier and on to the scope channel 1 and the PHA analog input. All connections ending at the PHA gate input should be the same; the gate should still be generated from the fixed detector.

19. Turn the PHA gating off and start acquiring a new spectrum. Adjust the Linear Amplifier gain to get the 0.511 MeV photopeak around channel 300 and the 1.274 MeV photopeak around channel 750. Set an ROI over the 0.511 MeV photopeak; again get into the shoulders.
20. Turn gating on and start a new PHA spectrum. If the detectors are  $180^\circ$  opposite one another relative to the  $^{22}\text{Na}$  source, the spectrum should come in quite a bit more slowly because these counts are mostly all real coincidences arising from the annihilation gamma pairs.
21. Move the movable detector to  $90^\circ$  and restart a PHA spectrum. Note how the spectrum now comes in really slowly. The counts coming in now are not true coincidences; they are random coincidences.

Before taking coincidence measurements there is one more adjustment which should help increase the rate at which coincidences will occur. We have found that the *active area* (the small spot where the  $^{22}\text{Na}$  nuclei are located on our source disk) may be significantly off-center on the disk. Thus, as the disk is rotated on its holder at the center of the apparatus, the active spot will likewise rotate about the center of the disk. Keep in mind that the center of the c-flux pattern is located on the line from the center of the fixed detector through the center of the source. Consequently, the position of the active area will affect the location where the movable detector would need to be positioned to get maximum area overlap, i.e., to get the maximum theoretical coincidence rate. Further keep in mind that the apparatus is designed so that the centers of the fixed detector, the disk, and all positions of the movable detector are constrained to lie in a horizontal plane.

Besides being able to rotate the disk about its center, note how the disk holder can be rotated about a vertical axis so that either the disk face or its edge can be oriented toward the fixed detector (or any position in between). For the following discussion, imagine the disk face (i.e., its normal) is pointed toward the fixed detector.

Now consider the effects if the active area is directly above the disk center (at the 12:00 o'clock position). The c-flux pattern would then be centered at the marked  $0^\circ$  position but above the plane in which the movable detector rotates. (How far above the plane would depend on how far the active area was off-center.) Consequently, the maximum overlap area and coincidence rate would occur when the movable detector is at its  $0^\circ$  position. However, the center of the detector would then be directly below the center of the c-flux pattern and depending on the distance between these centers, the maximum experimental coincidence rate could be significantly less than the maximum theoretically possible. A similar argument holds if the active area is below the disk center at the 6:00 o'clock position.

However, the situation is different if the active area is directly left or right of the disk center (at the 3:00 or 9:00 o'clock positions). For these two cases, the c-flux pattern would be centered left or right of the marked  $0^\circ$  position, but it would be in the plane in which the movable detector rotates. The center of the movable detector could then be positioned to coincide with the center of the c-flux pattern (somewhere left or right of the marked  $0^\circ$  position) and the maximum experimental coincidence rate would be very close to the maximum theoretically possible.

The following procedure is one way to approximately locate the active area. **Do not put any markings on the disk.**

22. Position the movable detector at the  $0^\circ$

position. Orient the disk holder so that the disk edge is in line with the detectors (the disk normal perpendicular to line connecting detectors). If the active area is off-center, rotating the disk will cause the center of the c-flux pattern to rise and fall and go through the center of the movable detector.

23. Select **Acquire|Preset Limits** and set the **Live Time** to around 200 s. Start a PHA spectrum with gating on. Record coincidence counts as you rotate the disk (not the holder) keeping track of the angle (via the notch filed on the disk mount) relative to vertical. You can use clock positions, e.g., 12:00, 1:00, 2:00, etc. Plot the coincidence rate vs. the angle. Explain whether or not the variations with angle behave as expected based on the model.
24. If the data seem reasonable, do a rough hand fit to find the two angles at which maximum coincidences are obtained, and rotate the source disk to one of these angles. The active area should now be in the horizontal plane.
25. Keeping the disk angle as set, rotate the disk holder so that the disk faces one of the detectors (disk normal parallel to the detector line). The active area should now be directly left or right of the disk center (at the 3:00 or 9:00 o'clock position). While measuring the coincidence rate, do a rough scan of the movable detector's angular position trying to locate the position left or right of  $0^\circ$  where the coincidence rate maximizes. Do as best you can, but within about  $0.5^\circ$  should be good enough. This locates the center of the c-flux pattern, and from the angle of the maximum left or right of  $0^\circ$ , determine approximately how far the active area is off-center on the disk. You should leave the disk and disk holder at these angles for the final coincidence measurements described next.
26. Turn gating off and take a non-coincidence spectrum for the movable detector with good statistics and save it. Check the resolution for this detector. Record the gross and net counts in the 0.511 MeV photopeak. Determine the singles rate  $S_m$  from the net count and the live time.
27. With the source and detector set for maximum coincidences, take a spectrum with good statistics and save it. (An hour or more may be necessary.) Record the gross and net counts in the 0.511 MeV photopeak. Determine the maximum experimental coincidence rate. (This is not  $C_{\max}$ .)
28. Position the movable detector near the  $90^\circ$  mark and restart a PHA spectrum. Take a random coincidence spectrum with good statistics and save it. (An overnight run is definitely necessary.) Determine the random coincidence rate  $R_{rc}$ .
29. Answer C.Q. 3 and 4.
30. Position the movable detector for maximum coincidences. Set the preset live time to get at least 3000 coincidences at the maximum coincidence rate. This should take no longer than 5 min per angle.
31. Make a data table for coincidences vs. the angle reading for the movable detector. Measure to both sides of the maximum

Now the apparatus is ready to make coincidence measurements.

until you get into the region where there are only random coincidences. Measurements every one degree seem to work well. Graph the coincidence count rate vs. detector angle.

**CHECKPOINT: The procedure should be completed through the previous step, including the graph of coincidence count rate vs. angle.**

## Mini-Experiments

### “Square root statistics”

If the number of detected gammas  $N$  (coincidences or singles) is measured over some time interval and the measurement is repeated over and over again under unchanging experimental conditions, the result is a sample of  $N$ -values. Because of the random nature of nuclear decay and gamma detection, the values of  $N$  will not all be the same. For an infinite number of samples, the values for  $N$  are predicted to occur with probabilities governed by the Poisson distribution, several features of which are discussed next.

A Poisson distribution is characterized by a single number—its mean  $\mu$  (which would be the mean of an infinite sample of  $N$ -values). The standard deviation  $\sigma$  of a Poisson distribution (or of an infinite sample) is given by  $\sigma = \sqrt{\mu}$ . One goal of this mini-experiment is to verify that the standard deviation of a sample of  $N$ -values is reasonably well-described by the square root of the sample average. Lastly, the shape of the Poisson distribution (for  $\mu$  greater than 30 or so), is very close to that of the Gaussian of the same mean and standard deviation.

When only a single measurement is made and a count value  $N$  is obtained, that value becomes the best estimate of the true expected average  $\mu$ . Furthermore, knowing that the

measured  $N$  is a sample from a Poisson distribution, one may then estimate its standard deviation, i.e., the uncertainty of the measured  $N$ , as the  $\sqrt{N}$ . This kind of measurement is often said to obey “square root statistics.” In calculations or fits using measured  $N$  values (greater than 30 or so), one can reliably apply techniques relying on the Gaussian distribution, using  $\sqrt{N}$  for the uncertainty in any measured  $N$ .

32. With gating off, adjust the preset live time so that the PHA singles count will be between 1000 and 2000. Take twenty separate singles counts without changing the experimental conditions.

33. Answer Comprehension Question 1

### Gamma ray attenuation

The transmission of monoenergetic gammas through shielding of thickness  $z$  follows an exponential law

$$S(z) = S_0 e^{-\mu z} \quad (20)$$

where the decay constant  $\mu$  depends on the shielding material and the energy of the gamma. Thus the value of  $\kappa$  discussed in the theory section is expected to be  $\kappa = e^{-\mu d}$  where  $d$  is the thickness of the lead apertures.

To determine the value of  $\mu$  (and thus  $\kappa$ ) experimentally, lead sheets of known thickness will be placed in front of one of the detectors and the count rate in the photopeak will be measured.

34. Set two ROI's; one over the 0.511 MeV photopeak and one over the 1.274 MeV photopeak. Place various thickness of lead sheets in front of the movable detector (leaving the original aperture in place) and record the photopeak count rates  $S$  (net counts in each ROI divided by the

live time) vs. the thickness of the lead sheets.

35. Fit the data for each photopeak to Eq. 20. (Add a constant to the equation to take into account background counts.) The different energy gammas may have different fitting parameters. Remember to use square root statistics for the data point uncertainties.
36. Answer Comprehension Question 2.

## Analysis of the Coincidence Data

37. Fit your coincidence counts vs. angle to Eq. 15 plus a constant  $B$  to represent random coincidences. Including as parameters in your fit  $R$ ,  $r_a$ ,  $r_s$ ,  $b$ ,  $\theta_0$ ,  $B$ ,  $\kappa$ , and an overall multiplicative constant representing the quantity  $f_m f_f \Phi$ . Remember to use square root statistics for the data point uncertainties. The next few paragraphs describe how to construct the prediction column for measured coincidence rate as a function of the angle setting,  $\theta$ .

Despite your best efforts, the c-flux pattern may still be somewhat above or below the plane of the movable detector. In addition, because of the finite size of the active area, the overlap cannot be perfect for all source points. The average overlap area might be reasonably well modeled by Eq. 15 with  $\delta$  given by

$$\delta = \sqrt{b^2 + [R(\theta - \theta_0)]^2} \quad (21)$$

where  $\theta_0$  describes the horizontal angle where the best case overlap is achieved, and  $b$  is an effective offset describing the imperfect overlap at  $\theta = \theta_0$ . Remember to convert all angular measures to radians before using them in any formulae.

Set aside a spreadsheet area for the fitting parameters. Populate those cells with reasonable guesses for their values. For  $b$ , start with the value 0.1 cm.

Set up columns for  $\theta$ , the measured coincidence counts  $N$ , and the live time  $t$ . Create a column for the coincidence count **rate**  $C'_{\text{meas}}$  ( $=N/t$ ) and another for its uncertainty  $\sigma_{C'_{\text{meas}}}$  ( $=\text{SQRT}(N)/t$ ). Enter your data in a consistent (monotonic) order for  $\theta$ .

Use your values of  $\theta$  with the cells for  $R$ ,  $b$  and  $\theta_0$  to construct a column for  $\delta$  according to Eq. 21.

To make columns for the three overlap areas, the IF(LOGICAL EXPRESSION, TRUE VALUE EXPRESSION, FALSE VALUE EXPRESSION) Excel function will be used to take care of the various cases given in Eq. 9. The IF function returns values based on one or the other of the two latter expressions based on the value of the first expression (which must evaluate to a true or false value).

What follows are equivalent spreadsheet translations of Eq. 9 with intermediate columns for the half angles as obtained from Eq. 11. For  $A'_{sa}$  the circle radii  $r_1$  and  $r_2$  and the half angles  $\phi_1$  and  $\phi_2$  in those equations become  $r_s$  and  $r_a$  and  $\phi_s$  and  $\phi_a$  below. For  $A'_{aa}$ , the radii  $r_1$  and  $r_2$  are both equal to  $r_a$  and the half angles  $\phi_1$  and  $\phi_2$  are both equal and taken as  $\phi_a$  below. For  $A'_{ss}$ , the radii  $r_1$  and  $r_2$  are both equal to  $r_s$  and the half angles  $\phi_1$  and  $\phi_2$  are both equal and taken as  $\phi_s$  below. Non-spreadsheet notations used below are (a) variable names are used while the spreadsheet needs cell addresses for the variables, (b) implied multiplication is shown while the spreadsheet needs the specific \* symbol, (c) squaring requires the ^ symbol.

A column for the overlap area  $A'_{sa}$  is most easily constructed using two intermediary columns. Create a column for  $\phi_a$  using

$$= \text{IF}(\delta > r_s - r_a, \text{IF}(\delta < r_s + r_a, \quad (22)$$

$$\text{ACOS}((r_a^2 + \delta^2 - r_s^2)/(2r_a\delta)), 0), \text{PI}())$$

and create a column for  $\phi_s$  using

$$= \text{IF}(\delta > r_s - r_a, \text{IF}(\delta < r_s + r_a, \quad (23) \\ \text{ACOS}((r_s^2 + \delta^2 - r_a^2)/(2r_s\delta)), 0), 0)$$

Then, create a column for  $A'_{sa}$  using

$$= r_a^2(\phi_a - 0.5 \text{ SIN}(2\phi_a)) + \quad (24) \\ r_s^2(\phi_s - 0.5 \text{ SIN}(2\phi_s))$$

The conditional  $\phi$ -values will give the proper area for all cases of Eq. 9.

Columns for the two overlap areas  $A'_{aa}$  and  $A'_{ss}$  are most easily constructed using one intermediary column for each area. The calculation is illustrated for  $A'_{aa}$  with the translation to  $A'_{ss}$  obtained by changing subscripts from  $a$  to  $s$ . Create a column for  $\phi_a$  using

$$= \text{IF}(\delta < 2r_a, \text{ACOS}(\delta/(2r_a)), 0) \quad (25)$$

Then, create a column for  $A'_{aa}$  using

$$A'_{aa} = 2r_a^2(\phi_a - 0.5 \text{ SIN}(2\phi_a)) \quad (26)$$

Construct a column for each of the terms  $C_{ss}$ ,  $2C_{sa}$ , and  $C_{aa}$  from Eqs. 12-15 and one more for their sum plus the random coincidence rate — the predicted coincidence rate  $C_{\text{pred}} = C_{ss} + 2C_{sa} + C_{aa} + B$ .

Construct a column for the contribution of each point to the total  $\chi^2$

$$= \frac{(C_{\text{meas}} - C_{\text{pred}})^2}{\sigma_{C_{\text{meas}}}^2} \quad (27)$$

Then construct a cell for the sum of all these contributions using the spreadsheet **SUM()** function. Finally, perform the fit by having the Excel Solver vary the fitting parameters to minimize this sum. You can eliminate bad data points from consideration in the fit by typing a zero in the cell where its  $\chi^2$  contribution would have been calculated. Remember to exclude  $R$  from the Solver variable list.

The fitting parameters, (in particular,  $b$  and  $r_a$ ) may interact in the fit. Both may simultaneously change with only a modest change in the shape of the overlap function. Consequently, relatively good data (high coincidence counts) are needed to fit both parameters. Watch out for this and, if necessary, fix (instead of fitting) the value of  $r_a$  to its measured value.

## Comprehension Questions

- For the mini-experiment on square root statistics: (a) Calculate the sample mean and standard deviation of the twenty trial values of  $N$  and compare with the prediction based on square root statistics. (b) Show that square root statistics are theoretically self consistent. That is, if  $N_1, N_2, \dots$  have  $\sigma_{N_i} = \sqrt{N_i}$ , show that propagation of errors dictates that their sum  $N_T = \sum N_i$  will have  $\sigma_{N_T} = \sqrt{N_T}$ . (c) Show how cube root statistics would not be self consistent.
- For the mini-experiment on the attenuation of gammas by lead: (a) Use your fitted  $\mu$  values and the density  $\rho = 11.3 \text{ g/cm}^3$  of lead to determine the mass attenuation coefficients  $\mu/\rho$  at the two energies. (b) Compare your results with reference values, e.g., those from the graph in the auxiliary material for this experiment of mass attenuation coefficients for various elements in K.Z. Morgan and J.E. Turner, eds., *Principles of Radiation Protection*. (c) Determine the fraction  $\kappa$  of 0.511 MeV gammas predicted to get through the thickness for the lead aperture shielding.
- Print, compare, and explain the spectra taken in Steps 8, 12, 17, and 26-28.

4. Compare the measured random coincidence rate (coincidence rate at large angles) with the value predicted by Eq. 19. Discuss any significant discrepancy.
5. Discuss the results of the fit to the coincidence data. In particular, check if any of the parameters are unrealistic. Rework the fit until all fitting parameters are reasonable.
6. Use the results of fit to the coincidence data to determine  $C_{\max}$ . Next, use the fit results,  $C_{\max}$ , and the singles rates  $S_m$  and  $S_f$  (the net photopeak count rates as measured in Procedure Steps 10 and 26) to determine (a) the raw flux  $\Phi$  at a distance  $R$  from the detector and (b) the geometric mean of the photopeak efficiencies  $\sqrt{f_m f_f}$ . Determine the source activity  $\alpha$  from  $\Phi$  and the distance  $R$  using Eq. 2. Use your value for  $\alpha$  and the known initial activity of the source ( $10 \mu\text{C}$ ) to determine the age of the source.
7. What does a non-zero value of  $\theta_0$  indicate? How well do your results “prove” that the gammas are emitted  $180^\circ$  apart? Suppose the positronium “atoms” decay while moving in random directions with an average, non-zero lab frame kinetic energy,  $E_{\text{av}}$ . Assuming  $E_{\text{av}}$  was large enough to have an effect, how might the shape of the graph of measured counts vs. angle change? Would it become broader, covering a wider range of angles? Would its center shift to a new angle? Approximately (order of magnitude) how large would  $E_{\text{av}}$  have to be to have an observable effect on the data?