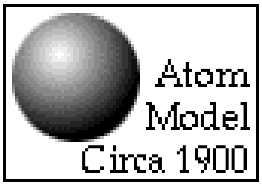
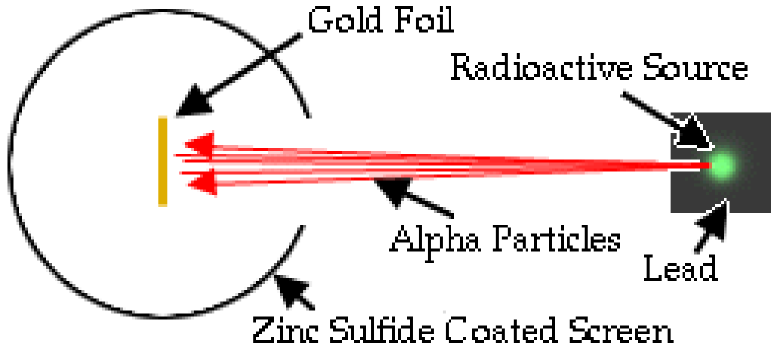
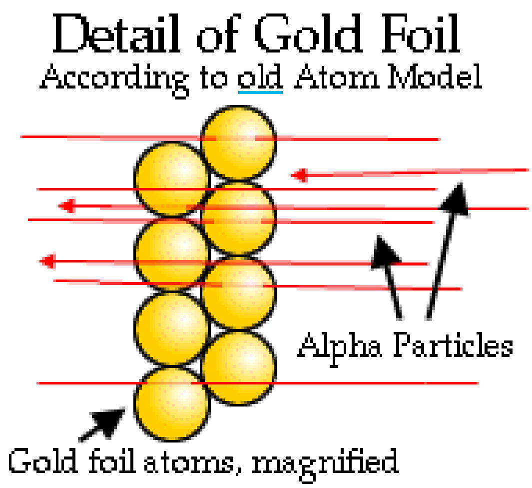
The Nucleus

PHY 3101
D. Acosta

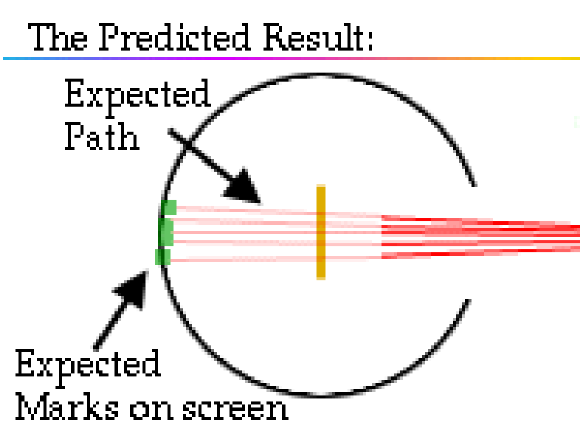
Rutherford Scattering

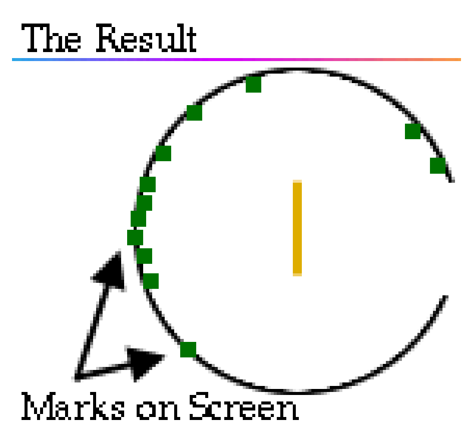




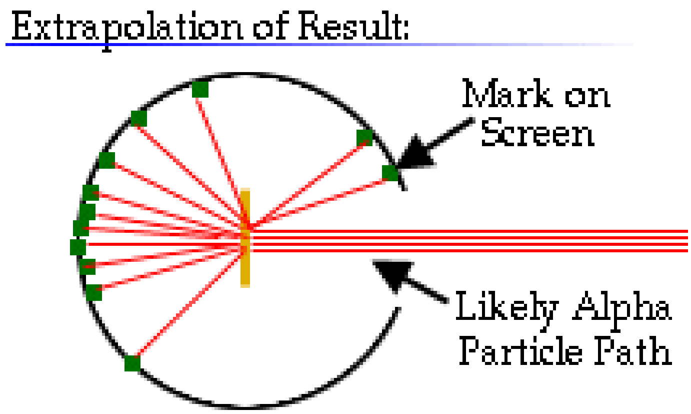


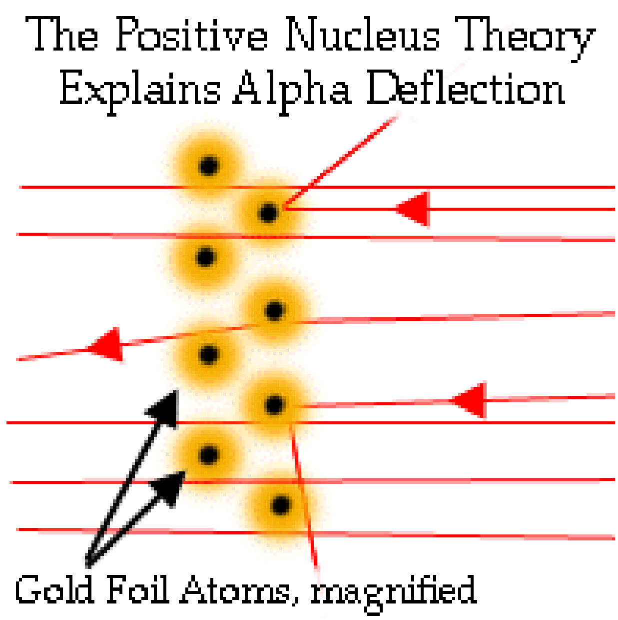
Experiments by Geiger & Marsden in 1909



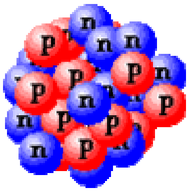


Rutherford Model of the Atom





Conclusion: the atom contains a positive nucleus < 10 fm in size (1 fm = 10^{-15} m)

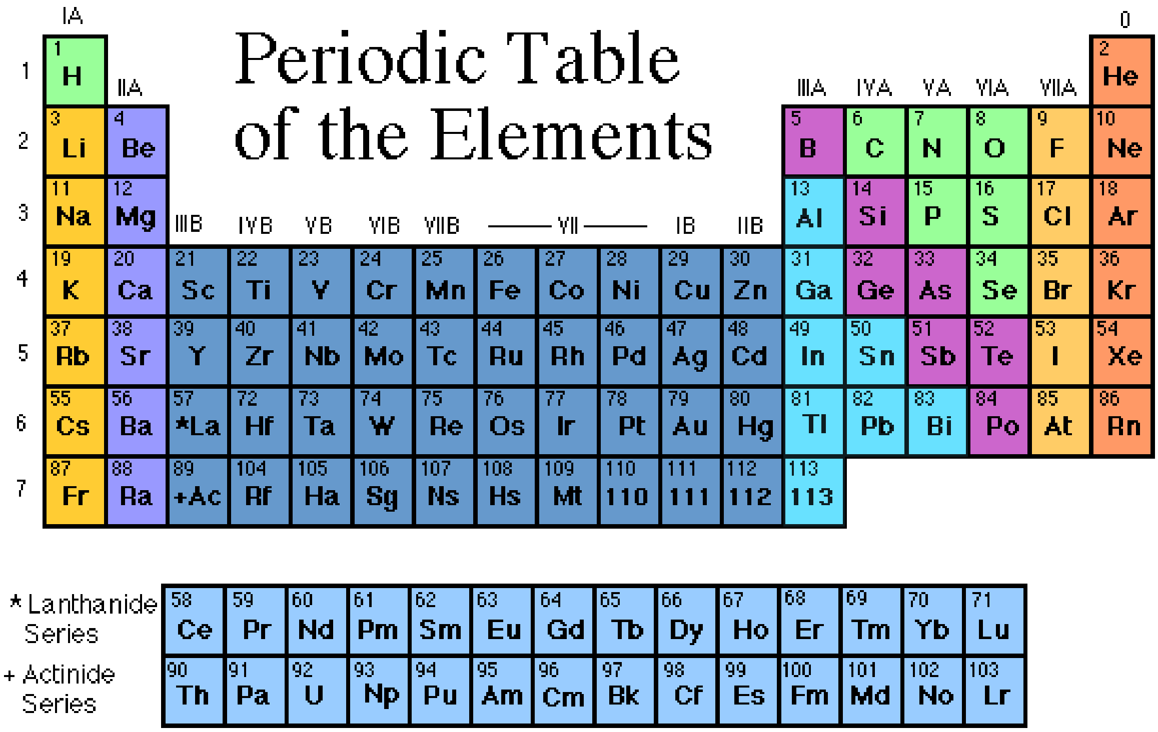


The Neutron

- The neutron was discovered in 1932 by James Chadwick
  - α-particles accelerated in a small accelerator and collided with Be nuclei
  - Neutral, very penetrating radiation
  - Found by elastic scattering off protons in paraffin wax

- By the way, the positron (anti-electron) also was discovered in 1932 by Carl Anderson in cosmic rays
  - Anti-matter predicted by P.A.M. Dirac in his relativistic version of the Schrodinger Equation

The Periodic Table



- All elements composed of just electrons, neutrons, and protons
- Elements of the same group have nearly the same chemical property
- Chemical periodicity depends on the atomic number *Z*
- Any other fundamental particles? Next chapter…

Nomenclature

$$^{A}_{Z}X$$

- X is the element
- A is the atomic mass (Z+N)
- Z is the atomic number (number of protons)
- N is the number of neutrons

- Atoms are neutral. Number of electrons equals number of protons = Z
- Chemical properties depend on Z
  - Ordering of Periodic Table given by valence configuration of electrons

- Isotopes:
  - Same Z, different A $\quad ^{4}_{2}\mathrm{He}\quad ^{3}_{2}\mathrm{He}$
- Isobars:
  - Same A, different Z $\quad ^{3}_{1}\mathrm{H}\quad ^{3}_{2}\mathrm{He}$
- Isotones:
  - Same N, different A $\quad ^{13}_{6}\mathrm{C}\quad ^{14}_{7}\mathrm{N}$

Atomic Mass Units (u)

$$\text{mass of } ^{12}C \equiv 12 \text{ u}$$

- The atomic mass is the mass of an atomic isotope, including electrons

$$1 \text{ u} = 1.66054 \times 10^{-27} \text{kg} = 931.49 \text{ MeV} / c^2$$

$$m_p = 1.00727647 \text{ u} \quad = \quad 938.27 \text{ MeV} / c^2$$

$$m_n = 1.00866490 \text{ u} \quad = \quad 939.57 \text{ MeV} / c^2$$

$$m_e = 5.4858 \times 10^{-4} \text{ u} \quad = \quad 0.511 \text{ MeV} / c^2$$

- Note that mass of ^{12}C is $6 m_p + 6 m_n + 6 m_e = 12.1 \text{ u} > 12.0 \text{ u}$

- The nucleus is bound
  - Binding energy is 0.1 u = 90 MeV
  - It takes energy to liberate all particles

- Should not think of mass as measuring the number of particles, only the rest energy of the system:
  - Mass is a measure of inertia (a = F/m) not contents

Binding Energy

$$B = \left[m(\text{separate}) - m(\text{combined}) \right] \times c^2$$

- Take the mass of all particles individually, including electrons, and subtract the mass of the combined system
- A system is bound if the binding energy is positive.
- Example: Deuterium
  - Note that e^- mass cancels

$$\begin{aligned} B &= \left[M({}^1_1\text{H}) + M({}^1_0\text{n}) - M({}^2_1\text{H}) \right] c^2 \\ &= [1.007825u + 1.008665u - 2.014102u]c^2 \\ &= 0.002388u \cdot 931.5 \text{ MeV}/u \\ &= 2.224 \text{ MeV} \end{aligned}$$

- If the binding energy is negative, the system will decay. The energy released is

$$Q = \left[m(\text{combined}) - m(\text{separate}) \right] \times c^2 = -B$$

Atomic Binding Energies

- The Coulomb potential for an electron in a hydrogen-like atom can be written in terms of the dimensionless fine structure constant

$$V(r) = \frac{-\alpha(\hbar c)Z}{r} \qquad \alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$$

- The energy levels are given by

$$E_n = -\frac{1}{2}\alpha^2\,\mu c^2\,\frac{Z^2}{n^2} \qquad \mu = \left(\frac{1}{m_e} + \frac{1}{m_N}\right)^{-1}$$

- Hydrogen:

$$\mu = m_e \quad \Rightarrow \quad E_1 = -13.6\text{ eV}$$

- Positronium (e^+e^-):

$$\mu = \frac{m_e}{2} \quad \Rightarrow \quad E_1 = -6.8\text{ eV}$$

- These are the binding energies!
  - e.g. mass of H is less than mass of e+p

- The Bohr radii are

$$r_n = \frac{\hbar}{\mu c}\frac{1}{\alpha}n^2 \quad \Rightarrow \quad r_1 = 0.53\times 10^{-10}\text{ m}$$

Nuclear Binding Energies

- Consider the binding energy of the deuteron
  - proton-neutron bound state

- The binding potential is roughly similar to that of the Coulomb potential, but with a dimensionless constant characteristic of the Strong Nuclear Force rather than EM

$$V(r) = \frac{-\alpha_s(\hbar c)}{r} \qquad \alpha_s = \frac{q_s{}^2}{4\pi\varepsilon_0\hbar c} \approx 0.1 \quad > 10\alpha$$

- The energy levels are given by

$$E_n = -\frac{1}{2}\alpha_s^2\ \mu c^2 \frac{1}{n^2}$$

$$\mu = \left(\frac{1}{m_p} + \frac{1}{m_n}\right)^{-1} \approx \frac{m_p}{2} = 470\ \text{MeV}/c^2$$

$$E_1 = -\frac{1}{2}(470\ \text{MeV})(0.1)^2 \approx 2.3\ \text{MeV}$$

- Agrees with measured value of 2.2 MeV
- 1 million times larger than atomic energies!
- Nuclear radius is 10,000 times smaller:

$$r_n = \frac{\hbar}{\mu c}\frac{1}{\alpha_s}n^2 \quad \Rightarrow \quad r_1 = 4.2\times 10^{-15}\ \text{m}$$

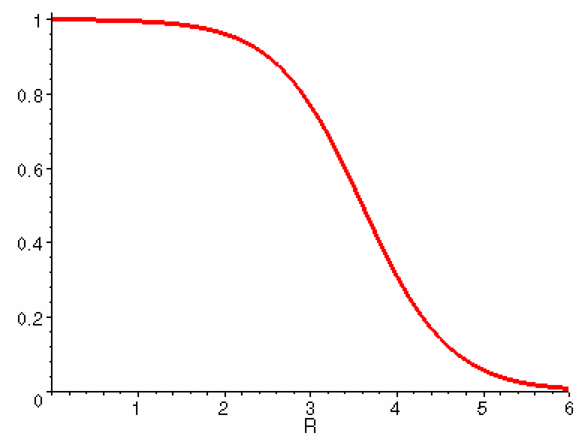
Nuclear Potential Well

- Rutherford concludes from Geiger and Marsden that the range of the Strong Nuclear Force is < 10^{-14} m
  - No deviation in the scattering rate of the highest-energy α-particles off nuclei from that predicted by electromagnetic Coulomb scattering
- Thus, the Strong Nuclear Force is short-ranged, and does not extend to infinity
- To probe the size of nuclei, need higher energies than α-particles from radioactive decay
- The nuclear potential well resembles a semi-infinite potential well
- α-particles inside the nucleus must tunnel to escape! Higher rate for higher energy α-particles

Size of Nuclei

- Robert Hofstadter performs experiment at Stanford using a new linear accelerator for electrons in 1950s
- E = 100 -- 500 MeV
- $\lambda = h / p = 2.5$ fm
- The proton is not a point! (Deviation of *elastic* scattering rate from Rutherford Scattering prediction)
- Proton and nuclei have extended charge distributions
- Nobel prize in 1961

nucleus



$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R)/a]}$$

$$R \approx r_0 A^{1/3} \qquad V = \frac{4}{3}\pi R^3 \propto A = \#\text{nucleons}$$

$$r_0 = 1.2 \times 10^{-15}\ \text{m} = 1.2\ \text{fm}$$

$$a = 0.5\ \text{fm}$$