Nuclear Spin and Stability

PHY 3101
D. Acosta

Nuclear Spin

- neutrons and protons have $s = \frac{1}{2}$ ($m_s = \pm \frac{1}{2}$) so they are fermions and obey the Pauli-Exclusion Principle

- The nuclear magneton is

$$m_N = \frac{e\hbar}{2m_p} = \frac{m_e}{m_p}\frac{e\hbar}{2m_e} = \frac{1}{1840}m_B$$

- The proton magnetic moment would be

$$m_p = g_s m_N m_s = 2m_N \frac{1}{2} = m_N$$

- But actually,

$$m_p = 2.79 m_N \qquad \neq m_B \text{ like the electron}$$

$$m_n = -1.91 m_N \qquad \neq 0 \qquad \text{even with } q = 0$$

- Thus, the neutron and proton have complicated charge distributions
  - They are not fundamental particles…

Zeeman Effect

- In the atom, the orbital angular momentum of the electrons gives rise to a magnetic dipole moment which interacts with external magnetic fields

$$V = -m \cdot \mathbf{B} = -m_B B\, m_\ell$$

- In the normal Zeeman effect, atomic states with angular momentum have split energy levels in the presence of a magnetic field:

$$\Delta E = m_B B\, \Delta m_\ell$$

- Similarly, a single particle with intrinsic angular momentum (spin) can interact with an external magnetic field, with different energy configurations

- There can be transitions between these different energy levels. A photon is emitted or absorbed in the process

$$\Delta E = E_g = hf = m_B B$$

Nuclear Magnetic Resonance

- Now consider an H-atom nucleus (proton) in a 1 T magnetic field. Since the nuclear magnetic moment is smaller than the Bohr magneton, the transition energy is smaller

$$\Delta E = g_s m_p B \Delta m_s$$

$$\Delta E = 2\times 2.79 m_N B = 2\times 2.79 \frac{m_B}{1840} B = 2\times 2.79 \frac{5.79\times 10^{-5}\ \text{eV}}{1840}$$

$$\Delta E = 1.76\times 10^{-7}\ \text{eV}$$

- The frequency of emission/absorption is

$$f = \frac{\Delta E}{h} = 42\ \text{MHz} \qquad \text{(radio spectrum)}$$

- A sample of protons (like the human body) will resonate at this frequency.
  - Radio waves at this frequency induce transitions
  - Strong absorption of radio waves if there is a population difference between the two energy states (thermal distribution)

- Studied by I.I.Rabi in 1938 (Nobel in 1944)
- Now applied to medical imaging:
  - Magnetic Resonance Imaging (MRI)

Nuclear Shell Model

- Recall the electron configuration of atoms:

  Kr: $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6$

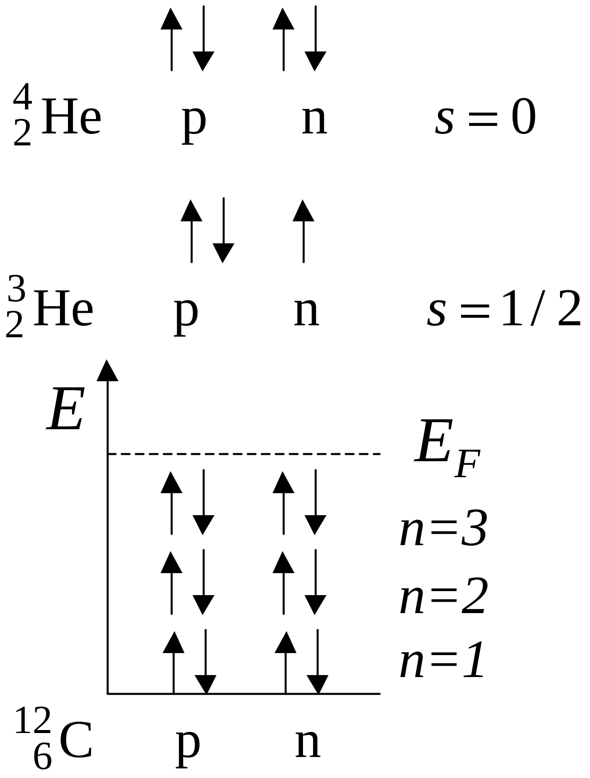
- Closed subshells occur after the following number of electrons:

  Z = 2, 10, 18, 36, 54, 80, 86

- Refer to these as atomic magic numbers
- Configuration is very stable
  - Large ionization energy ⇒ inert gases
- Similar behavior is seen in the binding energy of nuclei
  - Plot of B.E./A
- Since neutrons and protons are fermions, the Pauli-Exclusion Principle applies
- Nuclei also have shell structure

Nuclear Structure

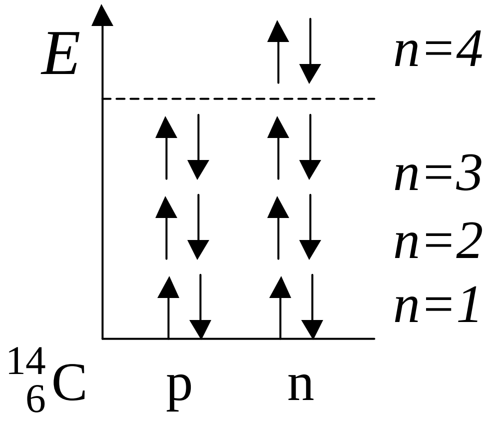
- The Pauli-Exclusion Principle applies to neutrons and protons separately
  - Not together because n, p are distinguishable from each other
- Ground state configurations



- Tend to want to have equal numbers of neutrons and protons to minimize the total energy of the nucleus
- Too many n's or p's causes binding energy to be negative -- unstable

Nuclear Magic Numbers

- Carbon-14 is unstable (radioactive)



- Some nuclei never form: $^{2}_{0}n$, $^{2}_{2}\mathrm{He}$
- Certain configurations are very stable
  - Z or N = 2, 8, 20, 28, 50, 82, 126

- These are the Nuclear Magic Numbers
  - Apply separately to neutrons and protons
  - These nuclei have large binding energies
  - Correspond to closed nuclear subshells

- "Doubly Magic" nuclei are extremely stable:

$^{4}_{2}\mathrm{He} \equiv a$ - particle has $Z = 2, N = 2$

$^{16}_{8}\mathrm{O}$ has $Z = 8, N = 8$

$^{208}_{82}\mathrm{Pb}$ (lead) has $Z = 82, N = 126$

Stability

- A nucleus is stable if the mass of the decay products is greater than the mass of the nucleus
- A nucleus is unstable if the mass of the decay products are less than the mass of the nucleus
- Consider the α-particle:

$$M\left({}^4_2\mathrm{He}\right) = 4.002602\ \mathrm{u}$$

$$M\left({}^1_0 n\right) = 1.008665\ \mathrm{u}$$

$$M\left({}^1_1\mathrm{H}\right) = 1.007825\ \mathrm{u}$$

includes electrons

$$2\times M\left({}^1_0 n + {}^1_1\mathrm{H}\right) = 4.033\ \mathrm{u} > M\left({}^4_2\mathrm{He}\right)$$

$$B = 0.0293\ \mathrm{u} \times 931.5\ \mathrm{MeV/u} = 28.3\ \mathrm{MeV}$$

$$B/A = 7.07\ \mathrm{MeV} \qquad \text{(binding energy per nucleon)}$$

- If a nucleus is unstable, it is only a matter of time before it decays into a different nucleus

Radioactivity

- Radioactive Decay Law

$$dN = -l\ N\,dt$$

$N =$ number of nuclei at time t

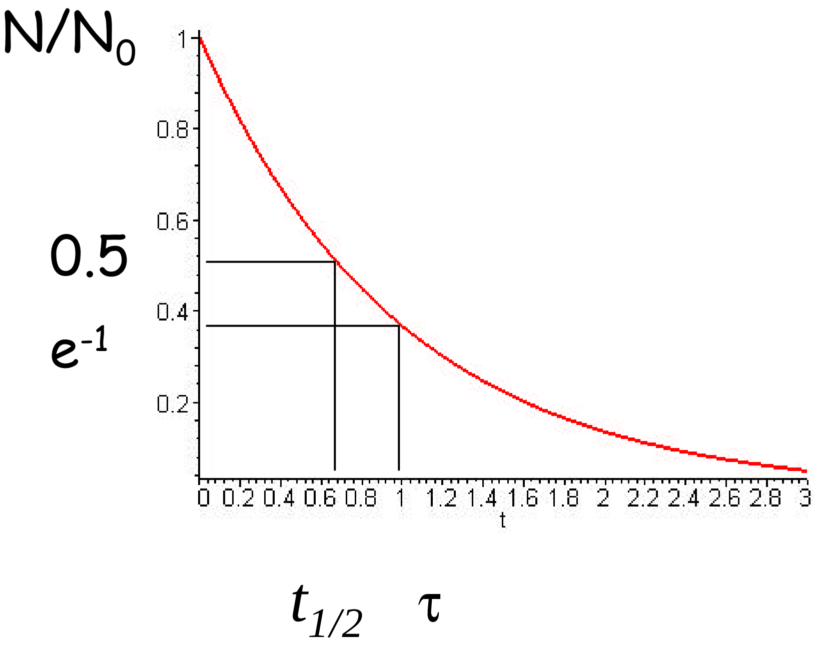
$l =$ decay constant

$$\frac{dN}{dt} = -l\,N$$

$$\boxed{\Rightarrow N(t) = N_0 e^{-lt}}$$

- Mean lifetime $\equiv \tau = 1/\lambda$

- Half-life $\equiv t_{1/2} = \ln 2 / \lambda = 0.693\ \tau$



Activity

$$\text{Activity} \equiv R = -\frac{dN}{dt} = l N(t)$$

- Decreases exponentially
- Units: disintegrations per second
  $\equiv$ Bequerel (Bq)
  - Bequerel discovered radioactivity in uranium in 1896
- Older unit: Curie (Ci) = 3.7×10^{10} Bq
  - Decay rate in 1g of radium

- Various decay modes possible:
  - α, γ, β^-, β^+, as well as electron-capture

Beta Decay

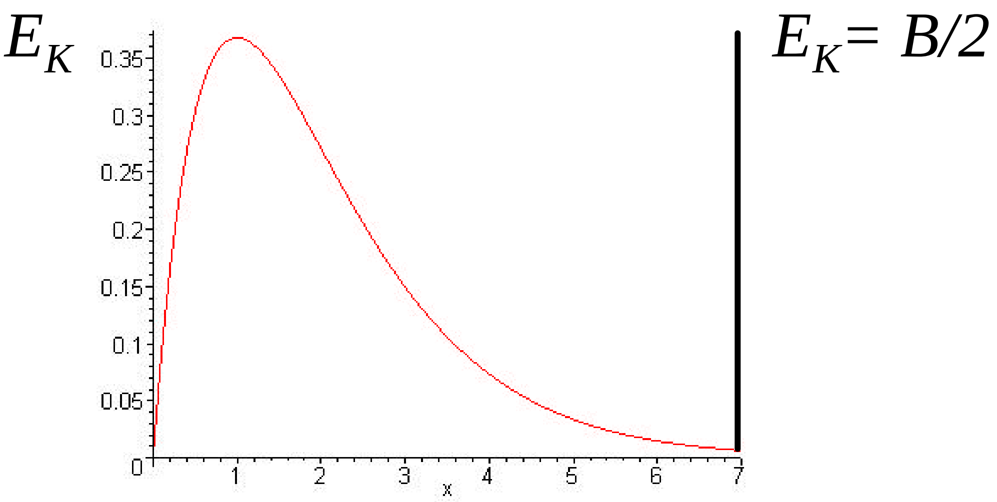
- Examples:

$$n \rightarrow p + \overset{\overset{e^-}{\swarrow}}{b^-} + \bar{n}_e$$

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + b^- + \bar{n}_e$$

$$\boxed{^{A}_{Z}X \rightarrow {}^{\;\;A}_{Z+1}X + b^- + \bar{n}_e} \quad \text{in general}$$

- Why the neutrino?
  - In the rest frame of the nucleus, the decay products would otherwise have a fixed energy of ½ × binding energy
  - This not observed:



- Either momentum is not conserved, or else there is an invisible third particle

The Neutrino

- Pauli proposes such an invisible particle in 1930 in a letter to a conference
- Enrico Fermi develops this in a theory, and calls it the neutrino – little neutral one
- The neutrino has spin ½, no charge, and nearly zero mass
- It does not interact via the electromagnetic force or the strong nuclear force, only the weak nuclear force
- The weak force is the only force which changes one particle into another
- Because of such a weak interaction, a neutrino can penetrate 10 light-years of steel !
- Must produce huge numbers of neutrinos to observe one:
  - Nuclear explosion
  - Nuclear reactor
- Cowen & Reines observe the neutrino in 1956 at a reactor (Nobel prize: 1995)