

Quanta 3

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Photoelectric Effect

Planck proposed that the energy of the atomic oscillators in a black body is **quantized**, such that $E_n = nhf$, where h is a constant of nature. The emitted radiation is the difference in energy between two such energy levels: $\Delta E = hf$. We will now see how Einstein made an even bolder proposal to explain the photoelectric effect.

The photoelectric effect is the ejection of electrons from a clean metal by electromagnetic radiation (light), first observed by Heinrich Hertz in 1887. The energy transferred from the field to the metal “kicks” out an electron. The *minimum* binding energy of an electron to a metal (it could be more for any particular electron) is called **work function** of the metal, and is denoted by the Greek letter: ϕ . It is the energy required to remove an electron from the metal, and it varies from one type of metal to another. For example, magnesium has a work function of 3.7 eV. Electrons must be bound to a metal; otherwise, they would all leak out!

A typical demonstration of the photoelectric effect requires a **photocathode** (the metal which emits electrons under a light source) and an anode (a metal which can receive the electrons). Even in the absence of a potential difference between the plates, a current can be measured between the two plates if electrons are ejected. We are free to apply a voltage between the two plates to either accelerate or stop the emitted electrons.

Experimental Facts:

1. A reverse-bias voltage ($-V_0$) can stop the emitted electrons. This is referred to as the **stopping potential**. This voltage is sufficient to stop *all* photoelectrons *regardless* of the light intensity. In classical electromagnetism, you might expect that higher intensities transfer a greater amount of energy to the electrons, thereby requiring a larger voltage to stop them.
2. The maximum kinetic energy of the emitted photoelectrons at the photocathode depends only on the frequency f , not the intensity, of the light. This maximum kinetic energy (E_K) can be measured by measuring the stopping voltage; because by energy conservation, $E_K = W = qV = eV_0$. Again, in classical electromagnetism, you might expect that higher intensities transfer a greater amount of energy to the electrons.

3. There is a **threshold frequency** (f_0), below which no electrons are emitted by the light. This threshold frequency is smaller for smaller work functions.
4. When produced, the number of ejected photoelectrons is proportional to the intensity of light.
5. The photoelectrons are ejected almost *instantly* after illumination (nano-seconds), independent of intensity. You might expect that in classical electromagnetism, that it would take a very long time (seconds) to transfer enough energy to a single electron if the intensity is low.

▷ **Classical Electromagnetism cannot explain these facts!**

Kinetic energy should depend on the intensity of light, there should be no stopping potential, and for low light intensities it should take a long time for photoelectrons to be ejected.

Einstein's Theory of the photoelectric Effect

Einstein proposed in 1905 (the same year he published Special Relativity) that the **electromagnetic radiation field is quantized**. Energy comes in discrete packets called photons, each with a definite energy. In other words, the electromagnetic field is not continuous. The energy of each photon depends only on the frequency:

$$E_g = hf$$

The constant h is the same constant Planck found to explain the black-body radiation spectrum!

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$

The photon is thus a **quantum** of energy. The radiation field consists of an integral number of photons. Since the speed of light in vacuum is c , and since $\lambda f = c$,

$$E_g = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}$$

The essence of Einstein's postulate is this:

Light exhibits both particle-like and wave-like properties

For this, and the explanation of the photoelectric effect to come, he received the Nobel prize (not for his theories of Relativity, which were viewed to radical at the time).

Now in Einstein's theory, the photoelectric effect can be explained by conservation of energy between a quantum of light (the photon) and a quantum of electricity (the electron):

Initially, the photon has energy $E_g = hf$, and the electron has energy $E_e = mc^2 - \mathbf{f}$ (in other words, it takes energy to overcome the work function and liberate the electron). The photon is completely absorbed in the process, and since energy is conserved, the electron must gain this energy: $E'_e = E_K + mc^2 = mc^2 - \mathbf{f} + hf$. Since the rest mass cancels out when we equate the initial and final energies, we have:

$$hf - \mathbf{f} = E_K$$

where E_K is the kinetic energy of the ejected electron. Note that for the kinetic energy to be positive (and thus liberate the electron), the frequency must be larger than the **threshold frequency**:

$$f_0 = \frac{\mathbf{f}}{h}$$

Now, if we apply an electric potential in reverse bias, we can overcome the kinetic energy of the electron with the electric potential energy and stop it:

$$E_K - eV_0 = 0 \quad \Rightarrow \quad E_K = eV_0$$

This gives us an equation for the **photoelectric effect** without having to measure the electron velocity, provided we determine the stopping potential:

$$eV_0 = hf - \mathbf{f} = hf - hf_0$$

So we see that the stopping potential only depends on frequency as observed, and there is a threshold frequency below which the photoelectric effect does not occur.

The work function varies from material to material. For magnesium it is 3.7 eV, so the threshold wavelength is $\mathbf{f} = hf_0 = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{1240 \text{ eV} \cdot \text{nm}}{\mathbf{f}} = 335 \text{ nm}$, which is in the ultraviolet region (thus visible light will not eject electrons from magnesium). However, **for all materials**, the stopping potential times electron charge (eV_0) always has the same slope h versus frequency. This is the same universal constant Planck proposed to explain the black-body spectrum.

Einstein's photoelectric effect is harnessed everyday in **photomultiplier tubes**, which convert and amplify light into electrical signals. These tubes can be sensitive to single photons, and are used extensively in particle experiments and hospitals—anywhere small amounts of light must be detected.