Muon Addendum

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In the cosmic ray muon experiment, four scintillation "paddles" are arranged as two pairs. Each paddle is a separate detector having a 30 cm square by 1 cm thick scintillator. Two paddles are strapped face-to-face to make a pair and the two pairs face each on a common axis separated by about 150 cm in an alt-azimuth "telescope" mount. They and are used together to "see" muons passing through at different angles and determine their angular distribution.

The apparatus can also monitor the rare events in which a low-energy muon stops and decays inside the scintillator. The muon is an unstable particle and spontaneously decays into an electron and two neutrinos with a lifetime of 2.197 μs in vacuum. The electron's kinetic energy after the decay ranges up to 53 MeV—half the rest mass energy of the muon and both the stoppage and subsequent decay cause scintillations. The experiment measures the time intervals between consecutive scintillations and, as predicted, an overabundance of relatively short intervals is observed.

The following is a model predicting the interval distribution that can be expected based on the muon model while also taking into account experimental conditions and limitations. The data is used with the model to get an estimate of the muon lifetime. The model is for a single detector. The two-pair arrangement in the actual experiment suggests modifications that provide for a significant improvement compared to treating the paddles as four separate detectors. The modifications and their effect will be addressed after the single-detector model.

A photomultiplier tube (PMT) is optically coupled along one edge of the scintillator slab and converts the scintillation light to an electric pulse. The PMT also produces random dark pulses unrelated to any scintillations. Other non-ideal PMT behaviors include "dead time" and "after-pulsing" rendering the scintillator unreliable for the first 50 ns or so after a previous pulse.

Muon passage, stoppage and decay events can deposit different amounts of energy in the scintillator resulting in different amounts of scintillation light and is one of the main causes for the varying amplitudes for the electrical pulses from the PMT. The pulse heights are used in conjunction with a discriminator, which produces a short, uniformly-shaped logic pulse whenever the photomultiplier pulse is above some threshold size. Other pulses such as dark pulses and pulses from background radiation also come in a variety of sizes, but are often smaller than muon pulses. The discriminator level is set to reject as many background pulses as possible without losing too many muon pulses. Logic pulses from the discriminator are passed to a data acquisition system for timing their arrivals with a precision of 10 ns.

Another experimental limitation that will be addressed shortly arises from weak reflections of the PMT pulses at the ends of the coaxial cable between the PMT and the discriminator. They are due to impedance mismatch and can trigger the discriminator to produce one or more "echo" pulses at specific reflection times after an initial pulse.

Due to discriminator settings and other experimental issues, not all events lead to a logic pulse. The probability of producing a logic pulse given the physical event occurred is called the detector efficiency ϵ for that class of events. For example, our detector efficiencies are typically above 90 percent for a muon passages and may be even higher for stoppages and decays because these events typically deposit more energy in the scintillator than other events.

Background logic pulses include those arising from natural, non-muon radiation and dark counts and occur at a rate around 10-40/s. Muon passages are where a muon passes completely through the detector without stopping. Their pulses occur at rate around 10-20/s, and together with the background pulses, account for more than 99.9% of all pulses. Muon stoppages are very rare—accounting for less than 0.01% of all pulses. We will distinguish the rate at which muons actually stop R_s and their rate of logic pulses $\epsilon_s R_s$. Muon decays occur at the same rate as muon stoppage events, but will be given their own efficiency factor ϵ_d.

Muon stoppages, passages, and background events are all independent Poisson processes. The logic pulses generated by these processes occur randomly at some constant average rate. Grouped together they are again a uniform Poisson process with a rate equal to the sum of the rates in each group. One could even throw into this group ultra-rare muon decay pulses where the prior muon stoppage went undetected. The average rate of logic pulses from all these events will be taken as R_Σ.

Our apparatus uses a computerized data acquisition system equipped with a 100 MHz clock. The clock is started on one detector pulse and is stopped on the next detector pulse occurring within a $t_{\rm max} = 20\ \mu$s timeout period. If the second pulse does not occur within the timeout, the clock is rearmed and ready for another start. If a second pulse occurs within the timeout, the measured time interval is saved. The intervals will vary and are acquired over a period of several days to acquire a histogram of the frequency (or count) C_i of intervals over that period versus the interval length t_i. The complete histogram consists of the counts C_i in two thousand t_i bins separated by the bin spacing $\Delta t = 10$ ns—one clock cycle.

After-pulses and echos are relatively rare but they look just like a muon capture and decay and occur often enough to make interval data at early times unreliable. As data at these early times would have to be eliminated from analysis anyway, and there is some chance the muon decay is yet to come, it is better to simply ignore stop pulses at early times and continue seeking a later stop pulse. Thus the software ignores pulses that occur before some minimum time $t_{\rm min}$ from the last one. A setting of $t_{\rm min} = 300$ ns is about right for our set-up because, without a lower limit, the last artifact in the histogram is a small but observable excess of intervals at the fourth echo time around 280 ns. The effects of this lower limit will be included in the analysis.

Every start pulse initiates a wait for a stop pulse that can last up to the full wait time $t_{\rm max}$. In fact, almost all starts never get a stop pulse and the software almost always waits the full $t_{\rm max}$ interval. This wait time leads to a software dead-time. While the sources for all start pulses occurs at the true rate R_Σ, their rate as start pulses is slightly reduced from this value due to the dead time. We will put prime notation on start rates to indicate this reduced start rate. Thus, the rate of all start pulses is denoted R'_Σ and in some long acquisition time T, there will be $N = R'_\Sigma T$ start pulses leading to a total dead time of $Nt_{\rm max}$. Consequently, the actual time during which start pulses were sought is $T' = T - R'_\Sigma T t_{\rm max}$. We should recover the true rate of their occurrence as the measured number of counts over this dead-time corrected interval. $R_\Sigma = N/T' = R'_\Sigma T/(T - R'_\Sigma T t_{\rm max})$. Solving for R'_Σ gives the reduced start rate

$$R'_\Sigma = \frac{R_\Sigma}{1 + R_\Sigma t_{\rm max}} \tag{1}$$

In fact, the rate of pulses from any group will be reduced by the factor $1/(1 + R_\Sigma t_{\rm max})$ when those pulses are being used to start a timing interval. For R_Σ less than 100/s and $t_{\rm max} = 20\ \mu$s, the correction factor is $1/(1 + 0.002) \approx 0.998$. Thus the rate of pulses as start pulses is never more than 0.2% lower than the true rate of those pulses and thus really not worth worrying about. Nonetheless, the prime symbol will be used to indicate these dead time corrected start rates.

Start pulses from a muon stoppage occur at the rate $\epsilon_s R'_s$ and all other start pulses (from non-stoppage events) occur at the rate $R'_\Sigma - \epsilon_s R'_s$. Note that, as required, their sum is the rate of all starts. We will treat these two independent contributions to the histogram separately and sum them for the final result.

For the non-stoppage starts there is no muon in the scintillator and the interval can only be stopped by an unrelated event which are occurring with a probability per unit time R_Σ. For the next (stop) pulse to occur in the bin at t_i, there must be no pulses from $t_{\rm min}$ to t_i (with a probability $e^{-R_\Sigma(t_i - t_{\rm min})}$) followed by a pulse in the bin interval at t_i (with a probability $R_\Sigma \Delta t$).

$$e^{-R_\Sigma(t_i - t_{\rm min})} R_\Sigma \Delta t \tag{2}$$

is thus the probability of a stop pulse at t_i given the start pulse was not from a muon stopping.

The rate at which the bin at t_i fills from these events is then the start rate for non-stoppage start pulses times this probability.

$$R_i = (R'_\Sigma - \epsilon_s R'_s)\, e^{-R_\Sigma(t_i - t_{\rm min})} R_\Sigma \Delta t \tag{3}$$

This is a major component of the interval distribution and contains no information about the muon decay. It is the rate of random intervals and is effectively constant at all t_i. For R_Σ under 100/s and $t_{\rm max} = 20\ \mu$s, the exponent $R_\Sigma(t_i - t_{\rm min})$ never gets above 0.002. For exponents this low the exponential is well-approximated by the first two terms of its Taylor expansion about zero: $e^x \approx 1 + x$

$$R_i = (R'_\Sigma - \epsilon_s R'_s)\,(1 - R_\Sigma(t_i - t_{\rm min}))\, R_\Sigma \Delta t \tag{4}$$

This shows that to a good approximation R_i is nearly constant—decreasing linearly by less than 0.2% over the whole measurement range. Ignoring the tiny slope, the small dead time effect and the tiny contribution from muons stopping, R_i is very nearly $R_\Sigma^2 \Delta t$ and on the order of 10^{-5}/s.

The rest of the contributions to the histogram will then arise from start pulses due to muon stoppage events (occurring at the rate $\epsilon_s R'_s$). The muon is then inside the scintillator and has a high decay rate equal to the inverse of its lifetime and denoted Γ ($\approx 450,000$/s). The probability it survives to t_i without decaying is $e^{-\Gamma t_i}$. The probability the muon has not decayed by $t_{\rm min}$ is thus $e^{-\Gamma t_{\rm min}}$ and the probability it has decayed by $t_{\rm min}$ is $1 - e^{-\Gamma t_{\rm min}}$. The contribution from these two possibilities will be considered separately and summed for the final result.

Given the muon decayed before $t_{\rm min}$ (with a probability $1-e^{-\Gamma t_{\rm min}}$), there is again no muon in the scintillator when stops are sought beginning at $t_{\rm min}$. The next random pulse after $t_{\rm min}$ will stop the interval and these are occurring at the rate R_Σ. To get a stop at t_i there must be no pulses from $t_{\rm min}$ to t_i (with a probability $e^{-R_\Sigma(t_i-t_{\rm min})}$) and there must be a pulse in the next bin interval (with a probability $R_\Sigma \Delta t$). Consequently

$$\left(1 - e^{-\Gamma t_{\rm min}}\right) e^{-R_\Sigma(t_i-t_{\rm min})} R_\Sigma \Delta t \tag{5}$$

gives the probability that the muon decayed before $t_{\rm min}$ and the stop pulse occurred at t_i. Multiplying this probability by the start rate for muon stoppages $\epsilon_s R'_s$ gives the rate at which the bin at t_i fills for these events.

$$R_i = \epsilon_s R'_s \left(1 - e^{-\Gamma t_{\rm min}}\right) e^{-R_\Sigma(t_i-t_{\rm min})} R_\Sigma \Delta t \tag{6}$$

This contribution has the same time dependence as the random contribution from non-stoppage start pulses, Eq. 3.

The remaining contribution occurs when the muon does not decay by $t_{\rm min}$ (with a probability $e^{-\Gamma t_{\rm min}}$). There is then a muon in the scintillator at $t_{\rm min}$ and two ways to get a stop at t_i depending on whether or not the impending decay leads to a logic pulse.

If the decay does not produce a logic pulse (with a probability $1 - \epsilon_d$), there must still be no random pulse from $t_{\rm min}$ to t_i (with a probability $e^{-R_\Sigma(t_i-t_{\rm min})}$), followed by a random pulse in the bin at t_i (with a probability $R_\Sigma \Delta t$). Multiplying by the start rate $\epsilon_s R'_s$ then gives the third random contribution to the rate at which the bin at t_i fills

$$R_i = \epsilon_s R'_s e^{-\Gamma t_{\rm min}} \left(1 - \epsilon_d\right) \left(e^{-R_\Sigma(t_i-t_{\rm min})}\right) R_\Sigma \Delta t \tag{7}$$

If the decay does produce a logic pulse (with a probability ϵ_d), then, in order to get a stop at t_i there must be no decay and no random pulse during the interval from $t_{\rm min}$ to t_i (with a probability $e^{-(\Gamma+R_\Sigma)(t_i-t_{\rm min})}$) and there must be a detected stop event in the bin interval at t_i—either the decay pulse or a random non-decay pulse (with a probability $(\Gamma + R_\Sigma - \Gamma R_\Sigma \Delta t)\,\Delta t \approx (\Gamma + R_\Sigma)\,\Delta t$). Multiplying by the start rate $\epsilon_s R'_s$ then gives the rate of sought after stops in the bin at t_i that, because $\Gamma \gg R_\Sigma$, arise almost entirely from decay events.

$$\begin{aligned} R_i^{(\rm decay)} &= \epsilon_s R'_s e^{-\Gamma t_{\rm min}} \epsilon_d e^{-(\Gamma+R_\Sigma)(t_i-t_{\rm min})} \left(\Gamma + R_\Sigma\right) \Delta t \\ &= \epsilon_s \epsilon_d R'_s e^{R_\Sigma t_{\rm min}} e^{-(\Gamma+R_\Sigma)t_i} \left(\Gamma + R_\Sigma\right) \Delta t \end{aligned} \tag{8}$$

Summing the three contributions (Eqs. 3, 6 and 7) gives a total random histogram rate of

$$\begin{aligned} R_i^{(\mathrm{rnd})} &= \left(R'_\Sigma - \epsilon_s\epsilon_d R'_s e^{-\Gamma t_{\min}}\right) e^{-R_\Sigma(t_i - t_{\min})} R_\Sigma \Delta t \\ &= \left(R'_\Sigma e^{R_\Sigma t_{\min}} - \epsilon_s\epsilon_d R'_s e^{-(\Gamma - R_\Sigma)t_{\min}}\right) e^{-R_\Sigma t_i} R_\Sigma \Delta t \end{aligned} \tag{9}$$

Equations 8 and 9 should be added to get the total rate for filling each bin at t_i and multiplied by the acquisition time to get the counts C_i in each histogram bin at t_i. The bin filling is a Poisson process and each bin should fill at the random but constant average rate given by these two equations.

The fitting equation one should use to model the histogram data is actually quite simple. To the extent $\Gamma \gg R_\Sigma \gg R_s$, over the range from $t_{\min} \le t_i \le t_{\max}$

$$C_i = B + Ae^{-\Gamma t_i} \tag{10}$$

is a very good approximation with $B = TR_\Sigma^2\Delta t$ and $A = T\epsilon_s\epsilon_d R_s\Gamma\Delta t$

For the two-pair arrangement, modifications to the start and stop pulses and corrections to the predictions are discussed next. Each paddle pair is treated separately and in the exact same way. Here we describe how to handle one pair.

The upper and lower paddles are treated slightly differently. Since the efficiency for a muon stoppage is high, a stoppage in the lower paddle of a pair should have crossed the upper paddle and produced nearly simultaneous logic pulse from both paddles. Thus, a start pulse is taken when virtually simultaneous logic pulses occur from both paddles. R'_Σ in the formulas should then be interpreted as the corrected rate of such near simultaneous pulses which is about 1/2 to 1/4 of all pulses in this paddle. The decay should then be in the lower paddle and thus the stop pulse is only sought there without regard to any other pulses. Thus R_Σ should be taken as the rate of all pulses in the lower paddle.

A muon stopping in the upper paddle of a pair should not have a simultaneous pulse in the lower paddle as this is an indication the muon continued past and did not stop. R'_Σ in the formulas should then be taken as the rate of pulses from the upper paddle with no simultaneous pulse in the lower paddle, which is about 1/2 to 3/4 of all pulses in this paddle. The decay should then be in the upper paddle and thus the stop pulse is only sought there and R_Σ should be the taken as the rate of all pulses in the upper paddle.

Each of the four histograms—two from each pair—can be examined and checked separately, or they can be added together and analyzed as a group. The summation is recommended when fitting to determine the best estimate of the muon lifetime in the scintillator.