Cosmic Ray Muons and the Muon Lifetime

Experiment CRM

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PHY4803L — Advanced Physics Laboratory

Objective

Four scintillation detectors and coincidence techniques are used to determine the flux and angular distribution of muons created in collisions of cosmic rays with atoms in the upper atmosphere. The muon lifetime is measured using rare events where, after passage of a muon into the scintillators is detected, its decay is also detected. Statistical techniques for low counting rate experiments are employed.

Introduction

Cosmic rays are produced in the collision of extra-terrestrial particles with nuclei in the Earth's upper atmosphere. Hydrogen nuclei (protons) make up most of the incident flux, but helium nuclei (alpha particles) and other light nuclei also are present, as are high-energy gamma rays. Nuclei that enter the atmosphere will eventually collide with an air molecule and initiate a hadronic shower—a cascade of particles (mostly pions) that may undergo further nuclear reactions. Neutral pions (π^0) immediately decay into two gamma rays, which in turn generate electromagnetic showers (e^+, e^-, γ) that are not very penetrating. Charged pions ($\pi^\pm$) that do not undergo further nuclear reactions will decay in-flight into muons and neutrinos: $\pi^+ \rightarrow \mu^+ + \nu_\mu$, $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$. Both the muon and its corresponding neutrino are classified as leptons, particles that do not participate in nuclear reactions. The neutrinos have an extremely tiny capture cross-section, and thus typically pass through the Earth without any further interactions.

Muons were discovered in cosmic rays by C. Anderson and S.H. Neddermeyer in 1937. There are two kinds of muon, the negative μ^- and its antimatter partner, the positive μ^+. They are essentially heavy versions of the electron (and its antimatter partner, the positron) with the same spin and charge, but with a mass $m_\mu = 105.6$ MeV/c^2 approximately 200 times larger. Muons are unstable—decaying into an electron (or positron) and two neutrinos: $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$, $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$ with an average lifetime $\tau_e = 2.197\,\mu$s—about 100 times longer than that of the charged pion. Because the muon undergoes a 3-body decay, the kinetic energy of the emitted electron is not fixed but has a broad distribution of values with a maximum (endpoint energy) of 53 MeV in the rest frame of the muon.[1]

The muon decay is a completely random event that does not depend on the past history

[1] This is similar to nuclear beta-decay (another 3-body decay) where a neutron inside a nucleus decays into a proton, an electron, and an anti-neutrino. In fact, the neutrino's existence was first postulated to explain why electrons from the beta-decay of a given isotope are not emitted with a fixed energy as would be predicted if the neutron decayed into only a proton and electron.

of the particle.[2] That is, the probability dP of decay in the *next* infinitesimal time interval dt is independent of how long it has lived since creation and is given by:

$$dP = \Gamma dt \tag{1}$$

where the decay rate Γ is the inverse of the lifetime: $\Gamma = 1/\tau_e$.

This decay process implies that the probability of a muon decay in the interval from t to $t+dt$ (given that the muon exists at $t = 0$) follows the exponential probability density function:

$$dP_e(t) = \Gamma e^{-\Gamma t} dt \tag{2}$$

Here, the time t represents the time for a particular decay to occur and will be called a *decay time*. In one part of this experiment, you will measure a large sample of decay times and compare with this exponential distribution.

Exercise 1 *(a) Explain the difference between dP in Eq. 1 and $dP_e(t)$ in Eq. 2. (b) Show that the expectation value for the decay time is the lifetime: $\langle t\rangle = \tau_e$. (c) Show that the muon "half-life" (the time at which half of a large sample of muons will have decayed) is given by $t_{1/2} = \tau_e \ln 2$. (d) Part (b) implies the average of a large sample of decay times will be a reliable estimate of the lifetime τ_e. However, suppose that experimental problems make short decay times unreliable—let's say all decay times shorter than 0.5 μs can't be trusted. Show that if one averages a large number of decay times ignoring any below $0.5\,\mu$s, the result will be $\tau_e + 0.5\,\mu$s.*

The differential flux of cosmic ray muons (per unit time, per unit area, per unit solid angle) at the surface of the Earth is approximately described by:

$$\frac{dN}{dA\, d\Omega\, dt} \approx I_0 \cos^k \theta \tag{3}$$

[2] See the addendum to the *Statistical Analysis of Data: Exponential Decay and Poisson Processes.*

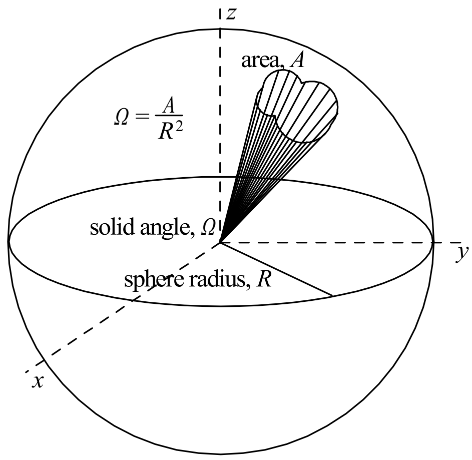

Figure 1: The solid angle Ω subtended from the origin of a sphere of radius R by an arbitrary area A on the sphere is $\Omega = A/R^2$.

where θ is the polar angle with respect to vertical, $k \approx 2$, and $I_0 \approx 100$ m^{-2}sr^{-1}s^{-1} at sea level. There is no expected dependence on the azimuthal angle ϕ. Eq. 3 is not expected to be valid for $\theta > 80°$ where the Earth's curvature becomes an important consideration.

Solid angle is a three-dimensional analog of an included angle in a two-dimensional plane. Shown in Fig. 1, an arbitrary solid angle Ω can be defined by the area A it would cover on a sphere of radius R centered at the apex of the solid angle.

$$\Omega = \frac{A}{R^2} \tag{4}$$

Solid angles are expressed in the dimensionless units of steradian, abbreviated sr.[3] One steradian is the solid angle covered by an area of 1 m^2 on a sphere with a 1 m radius. Notice that the solid angle for covering the entire

[3] The units of steradian should be dropped where inappropriate. For example, in $A = \Omega R^2$ (from Eq. 4), the units on the left are those of area (m^2) and on the right they are solid angle times length squared (sr m^2=m^2).

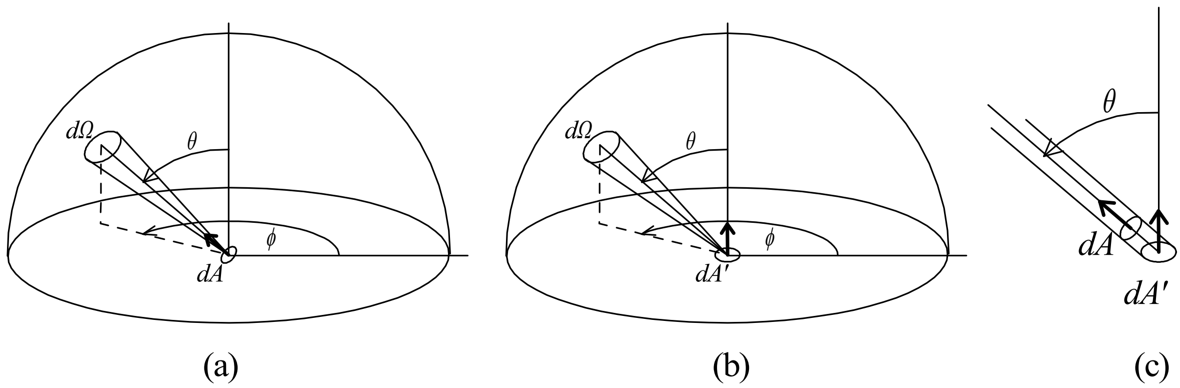

Figure 2: Muons arrive from all overhead directions (solid angles) and their flux is described as a number per unit time per unit area per unit solid angle. In (a) the area element is oriented in the direction of the incoming muons. In (b) the area element is oriented vertically. (c) shows equivalent areas for the two cases: $dA = \cos\theta dA'$

sphere (area $4\pi R^2$) is 4π sr.

Figure 2 shows the geometry for Eq. 3. $dN/dt \approx I_0 \cos^k\theta\, dA\, d\Omega$ would be the rate at which muons pass through an area dA coming from a polar angle θ within the solid angle $d\Omega$. The area dA should be considered to have its normal along the incoming direction as shown in Fig. 2a and thus the area orientation would vary as θ or ϕ varies. Alternatively, the orientation of the area element could be taken in a fixed, vertical orientation as in Fig. 2b. A comparison between equal effective areas in the two cases is demonstrated in Fig. 2c with

$$dA = dA' \cos\theta \tag{5}$$

Thus, for an area element oriented vertically, Eq. 3 would be

$$\frac{dN}{dA' d\Omega\, dt} \approx I_0 \cos^{k+1}\theta \tag{6}$$

A muon loses energy as it travels through the atmosphere and other materials. At the surface of the Earth, the typical muon energy is about 4 GeV, but with significant tails higher and lower. The mean energy loss per unit length for any charged particle traversing a block of matter is governed by the Bethe-Bloch equation:

$$\frac{dE}{dx} = -Kz^2 \frac{Z}{A}\frac{1}{\beta^2} \cdot \left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{\max}^2}{I^2} - \beta^2 - \frac{\delta}{2}\right] \tag{7}$$

Here β and γ are the usual relativistic factors, Z and A are the atomic number and mass of the medium, z is the charge of the incident particle, $T_{\max}$ is the maximum kinetic energy that may be transferred to an electron in a collision, and K, I, and δ are atomic factors. A plot of this energy loss as a function of momentum for muons incident on copper is given in Fig. 3. The energy loss is reported in units of MeV $\mathrm{cm}^2\,\mathrm{g}^{-1}$, so you must multiply by the density of the medium to get the energy loss per unit length. At low momenta, charged particles rapidly lose energy through ionization. However, once the particle becomes relativistic, the energy loss approaches a minimum

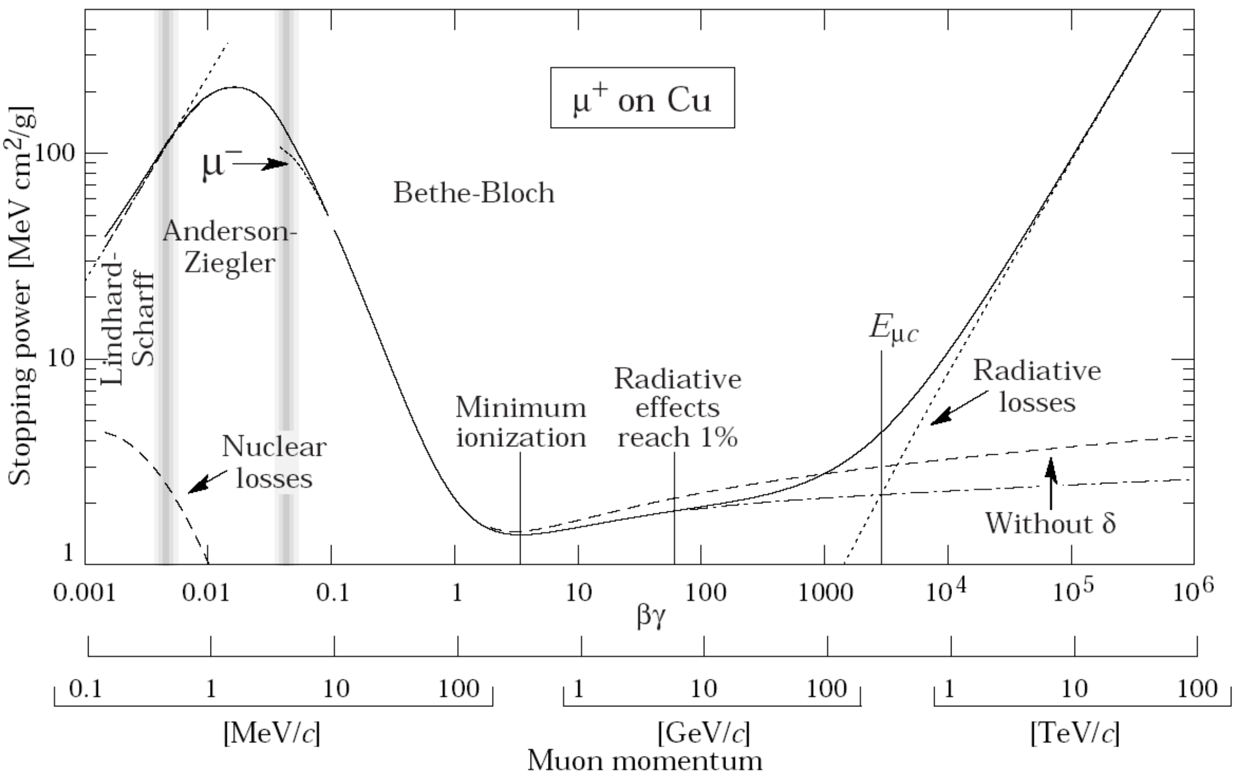

Figure 3: The mean energy loss for muons incident on copper as a function of momentum. Taken from *The Passage of Particles Through Matter* by the Particle Data Group.

with only a gradual increase toward very high energies. For most light materials, the energy loss for muons at this minimum is approximately 1.8 MeV cm^2 g^{-1}. Thus, it is clear that only muons at the very low end of the momentum spectrum will stop in a thin piece of plastic scintillator.

Exercise 2 *If a muon has a momentum such that it is at the minimum of the ionization loss curve, estimate the average energy loss when it crosses 1 cm of polystyrene ($\rho = 1.0$ g/cm^3)? What is the loss in 1 m of air ($\rho = 1.3$ kg/m^3)? 15 km of air?*

Without the effects of Einstein's Special Theory of Relativity, a muon—even if it is moving at the speed of light—would travel only 660 m before decaying in 2.2 μs. Very few would survive long enough as they travel tens of kilometers to get to the surface of the Earth. However, because of the time dilation effect of relativity, high-energy muons are able to travel much farther before decaying and many reach our detector where we can measure their flux and angular distribution.

Exercise 3 *For this exercise, assume that the muons are created in a shell 15 km above the surface of the Earth and that the Earth is approximately flat for such a shallow height. Assume the muons start off with a uniform angular distribution and that a polar angle dependence at sea level develops from muon decay and the longer time of travel for muons coming from larger polar angles. (Assume all muons have speeds near the speed of light.) (a) If time dilation did not occur, what fraction of the* *muons coming straight down would reach the ground without decaying? Despite the small size of this fraction, the observed rate at sea level might still be possible if the creation rate in the upper atmosphere were high enough. (b) Still assuming time dilation did not occur, how would this fraction depend on θ? For example, determine the ratio of the cosmic ray flux at $\theta = 30°$ to that at $0°$. How does this θ-dependence differ from that in Eq. 3?*

Once a muon comes to rest in a material, it can decay into an electron and two neutrinos with an average lifetime of 2.2 μs as already mentioned. However, for negatively charged muons (μ^-), a second decay process is possible. Negative muons, once stopped, can displace an atomic electron in the material and be bound with an atomic radius 200 times smaller than that for the displaced electron. This leads to a significant overlap of the muon wavefunction with the atomic nucleus, and muon capture by a proton is possible:

$$\mu^- + {}^A_Z X \rightarrow {}_{Z-1}^{\;\;A} X + e^- + \nu_\mu \qquad (8)$$

This converts the proton into a neutron, which transmutes the nucleus and ultimately liberates an atomic electron. The final-state isotope also may be unstable and will decay as well. The net effect is that this nuclear reaction rate adds to the free decay rate of the muon, leading to a shorter observed lifetime for negative muons. For the muon lifetime experiment performed here with plastic scintillators, which does not discriminate between μ^+ and μ^-, the observed lifetime will be shortened by approximately 0.1 μs (5%).

Procedure

Cosmic Ray Flux

This first phase of the experiment is concerned with determining the cosmic ray flux

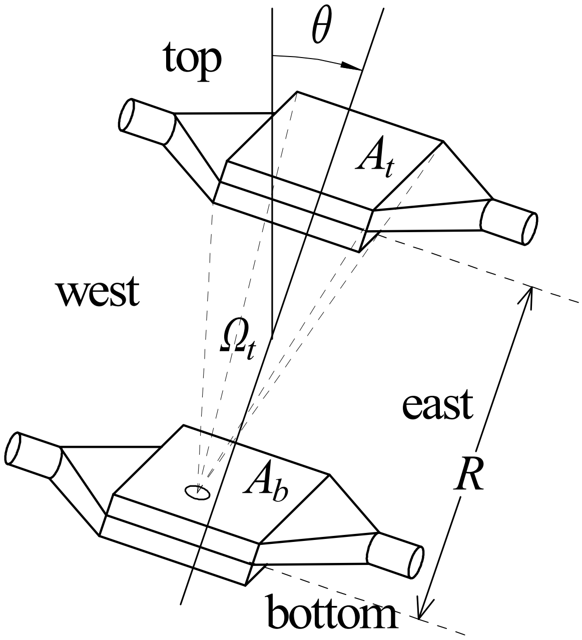

Figure 4: Geometric configuration of the four scintillation detectors. The dotted lines show the solid angle Ω_t subtended by the top detector from a point on the bottom detector. This defines the solid angle of acceptance when the apparatus is used in the telescope mode where muon passage through both the top and bottom detectors is measured. Not shown, the mount for the detectors allows the polar and azimuthal angles to be varied.

and angular dependence using four plastic scintillation detectors and discrete coincidence logic modules. The scintillation material is polystyrene doped with a fluorescent dye that emits light when a charged particle excites the medium.

As shown in Fig. 4, the four detectors are arranged as two pairs (top and bottom), with each pair having an east and west detector. The polar angle θ (from vertical) is illustrated in the figure and is adjusted by rotating the detectors about a horizontal axis. The azimuthal angle (say, measured from due south) is adjusted by rotating the apparatus on its casters about a vertical axis.

The paddle shaped scintillators within each pair are touching face to face making it likely that a muon passing through one will also pass through the other. A muon detected in both scintillators of a single pair can be assumed to have passed through the common (overlap) area having come from any polar angle.

In the "telescope mode" only those muons passing through both a top and a bottom scintillator are counted. These counts are rarer because the muon would have to come from a small range of polar angles. As shown in Fig. 4, the top detector's area defines a solid angle of acceptance Ω_t for each area element on the bottom detector.[4]

Exercise 4 *Based on an integration of Eq. 3, explain why the rate of muons passing through both detector pairs would be predicted to be:*

$$\frac{dN}{dt} = I_0 \cos^k \theta \, \frac{A_t A_b}{R^2} \tag{9}$$

where the A's are the overlap areas of the top and bottom detectors and R is the separation between them. [Hints: Assume all θ's can be taken as approximately the value for the centerline between the detectors as shown in Fig. 4. Show how the factor $A_t A_b / R^2$ arises from the integration over area and solid angle. The factor can be obtained choosing to integrate over the area of either the top or bottom detector and using the other to define the integration over solid angle.]

You will start your investigations by setting up the four detectors, determining their efficiencies, and measuring the distribution of muons as a function of the polar angle. The final investigation is to determine the muon lifetime by measuring the distribution of times between "double pulses" arising from muons which stop and then decay in the detectors. These double pulses are rare events occurring at a rate of about one per minute and consequently it will take overnight or longer runs to get suitable data. Be sure to read ahead about these measurements and get your first overnight run started on the 2nd Tuesday of the rotation. This should give you enough data by the Thursday session to check if there are any mistakes in the setup and to do a rough determination of the lifetime. You can then start a longer run from that Thursday to the third Tuesday which should produce enough data for a fairly precise lifetime determination.

1. Measure the overlap area for each pair of scintillation detectors (top and bottom). This defines the active area when the two detectors are put into coincidence. Measure the thickness of each scintillator. Measure the separation distance R between the two pairs of detectors. Calculate the approximate solid angle of acceptance. Calculate a rough range of θ values for a particular orientation of the telescope.

2. Orient the detectors vertically ($\theta = 0$) so that their faces are horizontal.

3. Have the instructor check the cable connections to the photomultipliers attached to the four scintillators before turning on the high voltage. Make sure that the high voltage is set to NEGATIVE. The operating voltage is 2000 V for the photomultiplier tubes.

4. Examine the output of each of the four detectors using the oscilloscope. Record

[4] Or, the bottom detector's area can be considered as defining the solid angle of acceptance for each area element on the top detector.

the typical pulse height and pulse duration (FWHM). The pulses should be at least 50 mV in amplitude. Make sure you terminate the signals at 50 Ω. Are there any glitches in the signal shape after the primary pulse?

5. Connect the four photomultiplier outputs to the Phillips 5-channel discriminator module. Make sure the inputs are in numerical order from the top down (the cables are labeled 1-4) and that the switch at the bottom is set for LED, which stands for leading edge discrimination. In this mode, the module puts out a short logic pulse whenever the amplitude of the input pulse from the PM tube is larger than the LLT (lower level threshold). This discrimination step prevents the processing of small noise pulses, which occur in large numbers and are often not associated with a muon event. The LLT for each of the four channels is set around 0.3 V but varies slightly because each detector has a somewhat different response and noise level. Check with the instructor if you believe an LLT needs to be adjusted. Examine the output of the discriminators using the oscilloscope. Make sure you see logic pulses with an amplitude of −1 V and a width of at least 75 ns. Try connecting both the east and west outputs of a pair of detectors to the two inputs of the scope. You should see a coincidence as a muon travels through both detectors. Look for this coincidence in both the top pair and the bottom pair. Check to see that coincidences between a top and a bottom detector are rarer.

6. Connect the output of each discriminator to the 3-channel scalar module. Measure the count rate from each scintillation detector.

7. Now connect the discriminator outputs to the LeCroy coincidence unit. You should require a coincidence between each pair of counters (east and west of the top pair, and east and west of the bottom pair). Connect the output of each of these two coincidences to the scalar module. Only use the OUT terminals. Do not use the LIN OUT terminals. Also connect the two coincidence outputs (one from each pair) back to the input of a third coincidence logic section, and send the output of this 4-fold photomultiplier coincidence to the third input of the scalar counter. Check to see that the outputs of the coincidence module also have a width of at least 75 ns using the oscilloscope.

8. With the counters still oriented vertically ($\theta = 0$), measure the coincidence count rate from each pair of counters. You should collect data for about 10 minutes. How do these rates of "coincidences" compare to the rates of "singles" from Step 6?

It is very important to appreciate why the coincidence count rate is so much lower than the singles count rate. There is a lot of background radiation that can set off a count in a single scintillator that will not penetrate and set off simultaneous counts in two scintillators. There are also "dark counts"—random pulses from the photomultipliers without any excitation of the scintillators. If two singles—one single in one detector and an unrelated single in the other detector—occur close enough in time, they will be indistinguishable from a real coincidence. However, these "random coincidences" occur at a very low rate. If the singles rates are R_1 and R_2, the random coincidence rate is predicted to be $R_1 R_2 \Delta t$ where Δt is the coincidence time window. By counting only if there are simultaneous pulses from both scintillators of a pair, it is nearly certain that the count represents the passage of a particle through both.

Detector Efficiencies

Measured count rates will need to be corrected to take into account the detector efficiencies—that they sometimes miss a muon. In the next step, you will measure the efficiency of each detector to fire its discriminator output given that a muon passed through that detector.

The best way to measure efficiencies is to measure coincidences on 3 of the 4 counters, so that it is just about certain that a muon traversed through the entire telescope and must have passed through the fourth counter. Then, the fraction of the time the fourth counter fires gives the efficiency for that counter. For example, to determine the efficiency of counter 1, you should acquire over some reasonable time interval the count N_{2+3+4} (the coincidence counts on counters 2, 3, and 4) while simultaneously acquiring $N_{1+2+3+4}$ (the coincidence counts on all four detectors). Then the efficiency of counter 1 will be

$$\epsilon_1 = \frac{N_{1+2+3+4}}{N_{2+3+4}} \tag{10}$$

C.Q. 1 *(a) Show that the statistical uncertainty in* ϵ_1 *would then be given by*

$$\sigma_{\epsilon_1} = \sqrt{\frac{\epsilon_1(1-\epsilon_1)}{N_{2+3+4}}} \tag{11}$$

Hint: Note that all counts in $N_{1+2+3+4}$ *will also be in* N_{2+3+4}*. Thus, one can consider the counts* $N_{+1} = N_{1+2+3+4}$ *as those counts in* N_{2+3+4} *that were also detected in counter 1, and* $N_{-1} = N_{2+3+4} - N_{1+2+3+4}$ *as those counts in* N_{2+3+4} *that were not also detected in counter 1. Thus,* $N_{2+3+4} = N_{+1} + N_{-1}$*. Partitioning the counts—into* N_{+1} *and* N_{-1}*—is useful because then each of them would then be statistically independent and would have a zero covariance.* N_{+1} *and* N_{-1} *may be assumed to have uncertainties (standard deviations) given by their square root.*

(b) The efficiency could also be determined by collecting the coincidence counts in the numerator and denominator of Eq. 10 over separate but equal time intervals. In this case, N_{2+3+4} *and* $N_{1+2+3+4}$ *would be statistically independent. Show that in this case the uncertainty in* ϵ_1 *would be given by*

$$\sigma_{\epsilon_1} = \epsilon_1\sqrt{\frac{1}{N_{2+3+4}} + \frac{1}{N_{1+2+3+4}}} \tag{12}$$

(c) Using the techniques in (a) and (b) above, suppose both gave 100 counts for the coincidences in the three counters and 90 counts for coincidences in all four. Thus, the fourth counter has a calculated efficiency $\epsilon = 0.90$ *in both cases. Determine the uncertainty in* ϵ *for both cases.*

(d) Explain why the partitioning in part (a) makes N_{+1} *and* N_{-1} *statistically independent whereas in part (b)* N_{2+3+4} *and* $N_{1+2+3+4}$ *would be statistically independent.*

9. Determine the efficiencies of all four detectors. Accumulate the counts for both the numerator and the denominator for calculating the efficiency ratio at the same time.[5] Because all of the events in the numerator are also in the denominator, the statistical uncertainty on the efficiency is given by Eq. 11.

10. Deduce the true integrated rate dN/dt of cosmic ray muons crossing the detector pairs using the coincidence count rates found in Step 8 and correcting for the efficiencies of the counters as determined

[5] Actually, you can measure the efficiencies of two counters at the same time using all three inputs to the scalar.

in the previous step. That is, divide each uncorrected dN/dt by the product of the two detector efficiencies. Compare the corrected dN/dt obtained from the top and bottom pair of detectors. Your comparisons should include a calculation of the propagated uncertainty for each. Average the two corrected dN/dt values which will be used later in a calculation of the muon flux constant I_0 after you have measured the angular distribution and determined the cosine exponent k.

Next, you will examine the count rate from the coincidence of all four discriminated outputs of the photomultipliers, i.e., with the apparatus in the "telescope" mode.

11. Measure the four-fold coincidence rate as a function of θ. To change θ, pull out the locking pin, **slowly** rotate the telescope to the desired setting, and then release the pin making sure it registers back into the hole. **Watch the HV and signal cables** where they connect to the base of the photomultiplier tubes. As you rotate the telescope, one set can come close to the top of mounting rack. Make sure the cables pass by cleanly without getting snagged. Adjust the mounting rack so the telescope points out the window (approximately east) when you are taking data for nonzero θ-values. Take measurements at consecutive holes of the telescope. There is a small offset in the holes so none of them line up perfectly vertical or horizontal. Take measurements over 90° from near vertical (approximately straight up through the upper floors of the building) to near horizontal (approximately straight out the window). Set the running time at each angle to get uncertainties smaller than about 10%. This may mean running for a longer time at larger angles.

For a non-zero polar angle (e.g., $\theta = 30°$), the telescope can be pointed out the window or through the building components above the lab by rotating it to points east, north, or west. The theory suggests that the muon flux is independent of the azimuthal angle (ϕ in Fig. 2), but is this what you would observe?

C.Q. 2 *(a) Why might your measurements show a dependence of the muon flux on the azimuthal angle? (b) What is the rough areal density of the atmosphere for a polar angle of 0° (straight up). What would it be at 45°? (The atmospheric areal density is the mass per unit area for a column all the way to the top of the atmosphere and should have units kg/m^2.) It can be obtained from the atmospheric pressure, ($\approx 10^5$ N/m^2), the acceleration of gravity (≈ 10 m/s^2) and the polar angle. (c) Concrete has a mass density of about 2400 kg/m^3. How thick a slab of concrete would have the same areal density as the atmosphere? (d) Assuming the physics building's floors and roof are about 40 cm thick, is the building's areal density above the lab a significant fraction of the atmosphere's?*

12. Set the telescope polar angle to 30°. In the previous step, you collected four-fold coincidence data at this angle with with the telescope pointed out the window. Now, collect data at three other azimuthal angles (every 90°, approximately east, north, and west) and try to determine whether or not there are any significant differences in the coincidence rates. Discuss your results.

CHECKPOINT: Procedure should be completed through Step 10, including the determination of the detector

efficiencies in Step 8. C.Q. 1 should be answered. An overnight run to determine the muon lifetime should be started. (Read ahead for instructions on this investigation.)

Analysis of angular distribution

Make a graph of the four-fold coincidence rate dN/dt as a function of the angle of the telescope. From what direction do the cosmic ray muons primarily come from? Fit your results to Eq. 9 (modified to include an offset I_B to account for random coincidences which should be independent of θ). That is, correct your measure coincidence rates for the four detector efficiencies to get the corrected dN/dt and then fit to $R_B + R_0 \cos^k \theta$, with R_B, R_0 (predicted to be $I_0 A_b A_t / R^2$) and k as parameters.

Perform weighted fits, taking into account the experimental uncertainty on each of your measured points. Use your value of R_0 (and detector geometry) to get an estimate of I_0.

Use the average of the corrected coincidence rate for the top and bottom detectors determined in Step 10 to get another estimate of I_0. You will need to show that the count rate would be predicted to be

$$\frac{dN}{dt} = I_0 \frac{2\pi A}{k+2} \tag{13}$$

[Hint: Does Eq. 3 or Eq. 6 apply? Why? The integration over area gives the factor A. The differential solid angle $d\Omega$ corresponding to differential variations in the polar and azimuthal angles is given by $d\Omega = \sin\theta \, d\theta d\phi$. The integration over all solid angle in the upper half plane gives the other factors.]

Compare this I_0 with that obtained from the fit to the angular distribution.

C.Q. 3 *How do your data support time dilation in Special Relativity? [Hint: compare the fitted results with the predictions of Exercise 3.]*

Muon Lifetime

In this second phase of the experiment you will measure the rate at which cosmic ray muons stop in the thin plastic scintillation counters. You will measure the time difference between an initial signal as a low-energy muon enters one of the paddles and a later signal arising from the decay of that muon into an electron. The *QuarkNet* board has multiple inputs and measure the time difference from an initiating trigger signal on any of its inputs to any later signal on any of its inputs. It provides this time difference in cycles of a 50 MHz clock (20 ns clock ticks) up to a maximum of about 1000 ticks (20 μs). All numerical inputs and outputs to the board are represented in hexadecimal.

13. Return the telescope to its vertical orientation ($\theta = 0$).

14. Plug in the 5 V wall supply to the *QuarkNet* board. Connect the (COM2) serial port of the computer to *QuarkNet* board using the ribbon cable. Do not yet plug in any photomultiplier signals. Be careful to avoid having any metal objects short out the circuitry on the board.

15. Communicate with the *QuarkNet* board using the terminal emulator program (located in the Muon folder on the desktop) connected to the COM2 port of the computer. The terminal emulator should echo your input to the monitor. If not, check that Local Echo is turned on from the setup menu. You should be able to see a prompt from the board when you hit <Enter>. Type "help" and you will see a list of all available commands.

16. The board can be made to start the clock (trigger) based on particular coincidences among its four inputs. Set the width of the coincidence time window to just one clock cycle (1 to 6 are valid) by entering the following command to the *QuarkNet* board:
`ww 1`
Only the first two inputs will be used and we want a trigger any time the board gets a pulse on either. So we enable just the first two inputs to the board and set the multiplicity level (number of coincident input pulses needed for a trigger) to just one counter by entering:
`wc 03`
The first four bits of this 8-bit control register word are represented by a "3" in hexadecimal (0011 in binary), indicating that only the first and second labeled inputs are active (a zero in a bit position indicates that an input is not used). The "0" for the fifth and sixth bits indicates that the multiplicity level is one, meaning that just one of the two inputs need to fire to initiate the board's trigger and start the time measurement. A "1" for these two bits would indicate a coincidence level of two (and so on).

17. Now connect the output from the coincidence of the top pair of scintillation counters to input 1 of the *QuarkNet* board. Connect the output from the coincidence of the bottom pair of counters to input 2 of the board. (We do not send the photomultiplier signals directly because they seem to interact adversely with the board.) You should now see a row of numbers register on the terminal screen. Each line represents a trigger (a coincidence with a multiplicity of just one as set earlier) that is satisfied. The LED display on the board is a counter of the number of triggers (in hexadecimal). Some of the lines displayed on the screen are two columns, and some are four columns. The four column lines are triggers that also had a second pulse within 20 μs of the trigger (so-called double pulses), and are thus the signature for candidate muon decays. The fourth number in these lines is the number of processor clocks (in hexadecimal) since the trigger condition occurred. Each clock cycle corresponds to 20 ns. There will be an intrinsic uncertainty of 1 cycle in the time measurement. To display only these double pulse lines on the screen, you should suppress "singles," which are triggers without the second pulse within 20 μs (the two column entries). Do this by entering the following "suppress singles" command:
`ss`
Now all entries reported should be four columns wide.

18. Allow the board to run for several minutes. Already, you may see evidence of an excess of time coincidences at small times (less than several microseconds). However, a spike at about zero should be excluded from later analysis, and is related to the board trying to measure the time-of-flight as a muon traverses from the top counters to the bottom counters. Remember that the results are displayed in hexadecimal. You can enable logging of the results to a file (File|Log on the terminal window), but this should not be necessary unless you plan on performing some special analysis of the data.

19. Exit the terminal program and start the "Histogram" LabVIEW program in the Muon folder on the computer desktop.

Start data collection from Operate|Run. For this high statistics run, you will want to collect data for at least 24 hours, and ideally for several days. Make sure to use only the "Stop" button next to the histogram to stop the data acquisition. If you don't do this, you will lose all your data! It is recommended that runs lasting more than one day are partitioned into several single day runs, where you save the output from each day to a separate file. If you don't do this, you might lose several days worth of data!

Analysis of Lifetime Data

After hitting the Stop button next to histogram, the program will save the histogram to a file which you can import into Excel or an equivalent program for analysis. Fit the time spectrum to a combination of an exponential and a flat background. You should exclude any spurious instrumental spikes or holes in the distribution, which typically occur at low channels.

You will likely encounter low statistics in your fit, where many bins have 0, 1, or 2 entries. This is problematic when using "square root statistics" and standard least-squares fits and will bias your measurement of the muon lifetime. A better approach is to use an effective chi-square function derived for data with a Poisson distribution, as described in the *Addendum: Exponential Decay and Poisson Processes* found in the Statistics area on the laboratory home page. Remember to convert your data into time units (either before or after the fit). Consecutive histogram bins are separated in time by the *QuarkNet* board's clock period of $\tau = 20$ ns, which you can assume is accurate to better than 0.1%. However, you can't trust that the first bin will correspond to $t = 0$ and without some calibrated pulser, you can't find the offset. Assume the offset t_0 may be ± 300 ns (15 bins). Thus the conversion from bin number n to the true midtime t_n for events histogrammed into that bin should be assumed to be

$$t_n = t_0 + n\tau \tag{14}$$

where $\tau = 20$ ns $(\pm 0.1)\%$ and $t_0 = 0 \pm 300$ ns. Also determine the uncertainty on the fitted lifetime.

To cross check your results, you may want to take data with only one pair of counters connected to the *QuarkNet* board (rather than both pairs). This may remove the instrumental spike at 0 time difference, but at the cost of only half the capture rate.

C.Q. 4 *(a) How does your measurement of the muon lifetime compare to the nominal value? (b) From the fit or the data, determine the total number of muons that stop in either detector pair. [Hint: this total is the sum of all counts under the exponential decay but above the background level. And they extend down to $t = 0$. Because the measurements fail in this region, you will have to figure out a way to estimate the counts you would have measured if the apparatus worked down to $t = 0$. (Assume $t = 0$ corresponds to channel 0.) Use this to calculate the rate of muons stopping in either detector pair and compare it to the total flux of muons passing through a pair of detectors, i.e., determine what fraction of muons passing into a detector decay in the detector. (c) Convert the fitted background level to a rate and compare it with a prediction for the random coincidence rate. For the prediction, treat the top and bottom pairs of paddles each as a single detector. Assume each produces pulses at a rate derived from the data taken in Step 7.*