Cosmic Ray Muons and the Muon Lifetime

Experiment CRM

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PHY4803L — Advanced Physics Laboratory

Objective

Four scintillation paddles and coincidence techniques are used to determine the overall flux and angular distribution of cosmic ray muons. The muon lifetime is measured using rare events where, after passage of a muon into a scintillator is detected, its decay is also detected a short time later. The distribution of the decay times provides information about the average muon lifetime. Statistical uncertainties appropriate for Poisson variables are employed throughout the experiment.

References

1. CMS Collaboration, *Measurement of the charge ratio of atmospheric muons with the CMS detector*, Physics. Letters B Volume 692, August 2010, pp. 83-104.

Introduction

Cosmic rays are high-energy particles—mostly protons and alpha particles with a small fraction of heavier nuclei and other subatomic particles such as electrons, positrons and antiprotons. Their origins in supernovae, quasars, and other exotic astronomical events and how they acquire their sometimes colossal energy (over 10^{20} eV) is a topic of current research.

Cosmic ray muons are created when cosmic rays enter earth's atmosphere where they eventually collide with an air molecule and initiate a hadronic shower—a cascade of particles (mostly pions) that may undergo further nuclear reactions. Neutral pions (π^0) decay in into two gamma rays with a very short lifetime less than 10^{-17} s, which in turn generate electromagnetic showers (e^+, e^-, γ) that are not very penetrating. Charged pions (π^+, π^-) that do not undergo further nuclear reactions will decay in-flight into muons (μ^+, μ^-) and neutrinos (ν_μ, $\overline{\nu}_\mu$): $\pi^+ \to \mu^+ + \nu_\mu$, $\pi^- \to \mu^- + \overline{\nu}_\mu$ with a lifetime of 26 nanoseconds. Both the muon and its corresponding neutrino are classified as leptons — particles that do not participate in nuclear reactions. The neutrinos have an extremely tiny capture cross-section, and typically pass through the Earth without any further interactions.

Since most cosmic rays and the nuclei they interact with are positive, positive cosmic muons are more abundant than negative muons. Studies of cosmic ray muons below a momentum of 100 GeV/c using the Compact Muon Solenoid at CERN (see reference 1) found the ratio of positive to negative muons to be 1.28.

Muons were discovered in cosmic rays by C. Anderson and S.H. Neddermeyer in 1937. There are two kinds of muon, the negative μ^- and its antimatter partner, the positive μ^+. They are essentially heavy versions of the electron and its antimatter partner, the positron, having the same spin and charge, but with a mass $m_\mu = 105.66$ MeV/c^2 approximately 207 times larger than the electron. Muons are unstable — decaying into an electron or positron and two neutrinos: $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$, $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$ with an average lifetime $\tau_\mu = 2.197\,\mu$s — about 100 times longer than that of the charged pion.

Because the muon undergoes a 3-body decay, the kinetic energy of the emitted electron or positron is not fixed but has a broad distribution of values with a maximum (endpoint energy) of 53 MeV in the rest frame of the muon. This kind of energy spectrum is similar to nuclear beta-decay (another 3-body decay) where a neutron inside a nucleus decays into a proton, an electron, and an anti-neutrino. In fact, the neutrino's existence was first postulated to explain why electrons from beta-decay are not emitted with a fixed energy as would be predicted if the neutron decayed into only a proton and electron.

Once created, the muon decay is a completely random event that does not depend on its past history. The probability dP of decay in the *next* infinitesimal time interval dt is independent of how long it has lived since creation and is given by:

$$dP = \Gamma dt \tag{1}$$

where the decay rate Γ is the inverse of the lifetime: $\Gamma = 1/\tau_\mu$.

This decay process implies that the probability of a muon decay in the interval from t to $t+dt$ (given that the muon exists at $t = 0$) follows the exponential probability density function:

$$dP_e(t) = \Gamma e^{-\Gamma t} dt \tag{2}$$

Here, the time t represents the time for a particular decay to occur and will be called a *decay time*. In one part of this experiment, you will measure a large sample of decay times and compare with this exponential distribution.

Exercise 1 *(a) Explain the difference between dP in Eq. 1 and $dP_e(t)$ in Eq. 2. (b) Show that the expectation value for the decay time $\langle t \rangle$ — defined as the muon lifetime τ_μ — is equal to $1/\Gamma$. (c) Show that the muon "half-life" (the time at which half of a large sample of muons will have decayed) is given by $t_{1/2} = \tau_\mu \ln 2$.*

The differential flux of cosmic ray muons (per unit time, per unit area, per unit solid angle) at the surface of the Earth is approximately described by:

$$\frac{dN}{dA\,d\Omega\,dt} \approx I_0 \cos^k \theta \tag{3}$$

where θ is the polar angle with respect to vertical, $k \approx 2$, and $I_0 \approx 100$ m^{-2}sr^{-1}s^{-1} at sea level. I_0 can vary by a few percent with latitude and altitude as well as with atmospheric temperature and pressure. There is no expected dependence on the azimuthal angle ϕ. Eq. 3 is not expected to be valid for $\theta > 80^\circ$ where the Earth's curvature becomes an important consideration.

Solid angle is a three-dimensional analog of an included angle in a two-dimensional plane. Shown in Fig. 1, an arbitrary solid angle Ω can be defined by the area A it would cover on a sphere of radius R centered at the apex of the solid angle.

$$\Omega = \frac{A}{R^2} \tag{4}$$

Solid angles are expressed in the dimensionless units of steradian, abbreviated sr.[1] One steradian is the solid angle covered by an area of 1 m^2 on a sphere with a 1 m radius. Notice that the solid angle for covering the entire sphere (area $4\pi R^2$) is 4π sr.

[1]The units of steradian should be dropped where inappropriate; for example, in $A = \Omega R^2$ (from Eq. 4), the units on the left are those of area (m^2) and on the right they are solid angle times length squared (sr m^2=m^2).

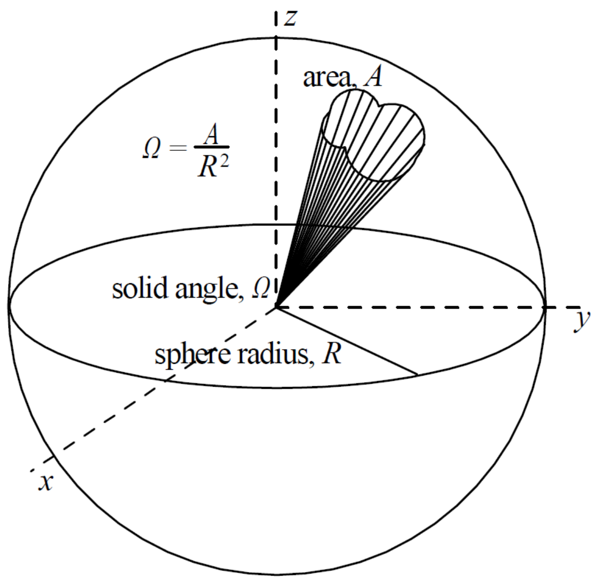

Figure 1: The solid angle Ω subtended from the origin of a sphere of radius R by an arbitrary area A on the sphere is $\Omega = A/R^2$.

Figure 2 shows the geometry for Eq. 3. $dN/dt \approx I_0 \cos^k\theta\, dA\, d\Omega$ would be the rate at which muons pass through an area dA coming from a polar angle θ within the solid angle $d\Omega$. The area dA should be considered to have its normal along the incoming direction as shown in Fig. 2a and thus the area orientation would vary as θ or ϕ varies. Experimentally, the area element is sometimes fixed in the horizontal plane with the area normal oriented vertically as in Fig. 2b. A comparison between equal effective areas in the two cases is demonstrated in Fig. 2c with

$$dA = dA' \cos\theta \tag{5}$$

Thus, for an area element in a horizontal plane, Eq. 3 would be

$$\frac{dN}{dA' d\Omega\, dt} \approx I_0 \cos^{k+1}\theta \tag{6}$$

where $0 \le \theta \le \pi/2$, i.e., the muons only come from the upper half plane.

Muons lose energy as they travel through the atmosphere and other materials. The mean energy loss per unit length (called the *stopping power*) for any charged particle traversing a block of matter is governed by the Bethe-Bloch equation:

$$\frac{dE}{dx} = -Kz^2\frac{Z}{A}\frac{1}{\beta^2}\cdot \left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{\max}^2}{I^2} - \beta^2 - \frac{\delta}{2}\right] \tag{7}$$

Here $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$ are the usual relativistic factors, Z and A are the atomic number and mass of the medium, z is the charge of the incident particle, $T_{\max}$ is the maximum kinetic energy that may be transferred to an electron in a collision, and K, I, and δ are atomic factors.

A scaled version of the stopping power is given as a function of momentum for muons incident on copper in Fig. 3. For reasons to be discussed shortly, the values are in units of MeV cm^2/g and must be multiplied by the density of copper (8.94 g/cm^3) to get the stopping power in MeV/cm.

The general shape of this graph is common to charged particles other than muons. At low momentum, charged particles rapidly lose energy as they ionize atoms in the medium and the stopping power is high. The stopping power decreases with increasing momentum and approaches a minimum as the particle momentum gets into the relativistic regime. It then increases only gradually from the minimum as the particle momentum continues to increase.

Figure 3 can also be used for materials other than copper. The scaling principle is that the actual energy lost (not the stopping power) should be roughly the same for passage through different materials as long as the product of the travel length and the density of the material is the same — passage through

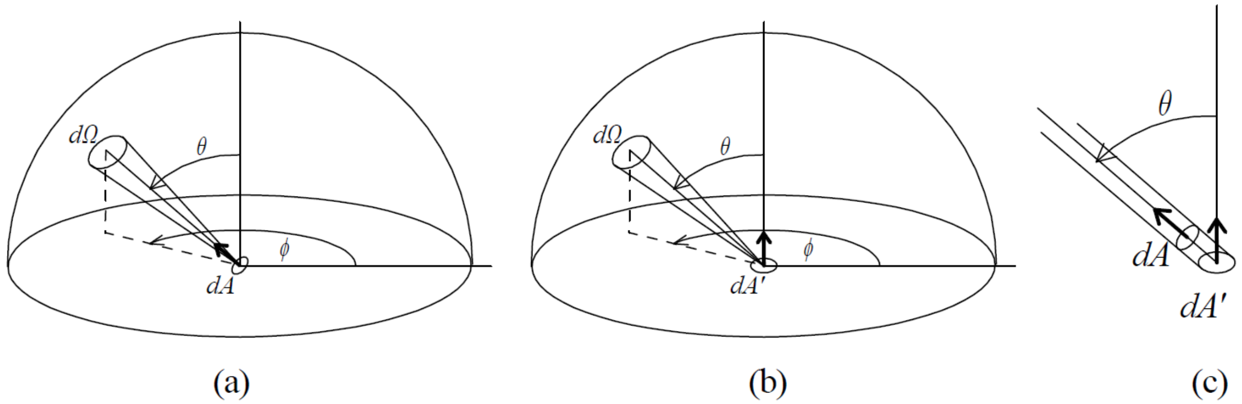

Figure 2: Muons arrive from all overhead directions (solid angles) and their flux is described as a number per unit time per unit area per unit solid angle. In (a) the area element is oriented in the direction of the incoming muons. In (b) the area element is oriented vertically. (c) shows equivalent areas for the two cases: $dA = \cos\theta dA'$

one meter of a copper (with a density around 9 g/cm^3) would lead to roughly the same average energy loss as passage through 9 meters of water (which has a density of 1 g/cm^3). Consequently, to use Fig. 3 for another material simply multiply by that material's density rather than copper's.

As long as it remains small compared to the muon kinetic energy, the actual energy loss is then calculated as the product of the stopping power, the material density, and the distance traveled in the material. The calculation becomes more complicated if the energy loss calculated this way leads to a final muon energy where the stopping power has changed significantly. In this case, one would have to take into account the energy loss in smaller slices of the material and integrate.

Due to the randomness of individual scattering events, as the muon energy decreases, angular scattering and variations in energy loss increase. And, of course, at the lowest energies, the muon will ultimately stop inside the material.

Exercise 2 *Muons reaching the earth's surface have an average energy around 4 GeV with a significant but reduced flux at both higher and lower energies. (a) What is the average energy loss for a 4 GeV muon passing through 1 m of air (ρ = 1.3 kg/m^3)? 15 km of air? 1 cm of plastic scintillator (ρ = 1.0 g/cm^3)? (b) Roughly, what is the largest muon momentum such that the muon has a reasonable chance of stopping in 1 cm of scintillator? (Hint: Where would the energy loss in one centimeter of scintillator be of the same order of magnitude as the muon's initial kinetic energy?)*

Without the effects of Einstein's Special Theory of Relativity, a muon — even if it is moving at the speed of light — would travel only 660 m before decaying in 2.2 μs. Very few would survive long enough as they travel tens of kilometers to get to the surface of the Earth. However, because of the time dilation effect of relativity, high-energy muons are able to travel much farther before decaying and many reach our detector where we can measure their flux and angular distribution.

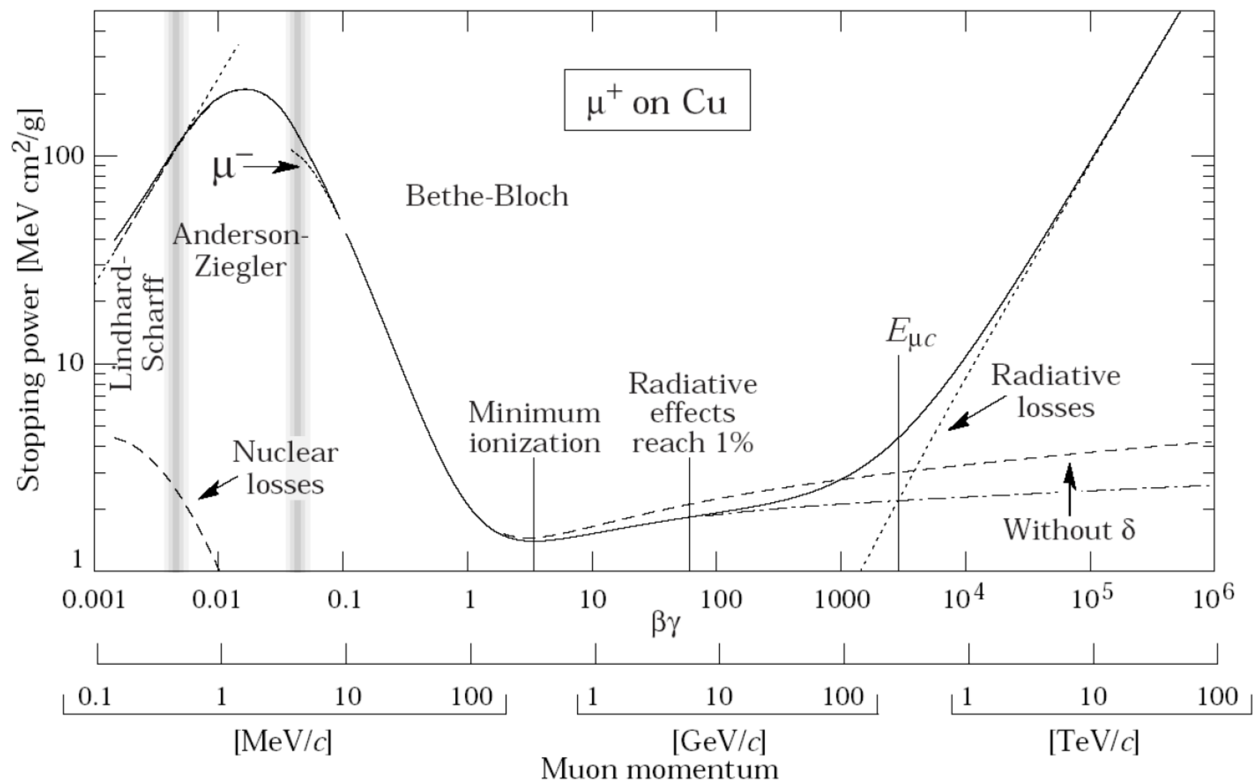

Figure 3: The "scaled" stopping power for muons incident on copper as a function of momentum. The value on the vertical axis must be multiplied by the density of copper in g/cm^3 to get the stopping power in MeV/cm of travel. The vertical axis can be multiplied by the density of other materials to get the approximate stopping power for that material. Taken from *The Passage of Particles Through Matter* by the Particle Data Group.

Exercise 3 *For this exercise, assume that the muons are created in a shell 15 km above the surface of the Earth and that the Earth is approximately flat for such a shallow height. Assume the muons start off with a uniform angular distribution and that a polar angle dependence at sea level develops due to muon decay and due to the longer time of travel for muons coming from larger polar angles. (Assume all muons have speeds near the speed of light.) (a) If time dilation did not occur, what fraction of the muons coming straight down would reach the ground without decaying? Despite the small size of this fraction, the observed rate at sea level might still be possible if the creation rate in the upper atmosphere were high enough. (b) Still assuming time dilation did not occur, how would this fraction depend on θ? For example, determine the ratio of the cosmic ray flux at $\theta = 30°$ to that at $0°$. How does this θ-dependence differ from that in Eq. 3?*

Occasionally, a low energy muon will come to rest in one of the scintillators where it can then decay into an electron or a positron and two neutrinos. Negatively charged muons can also decay inside a nucleus of one of the scintillator atoms. The μ^- first displaces an atomic electron in the atom and because it is

207 times more massive, its orbit is 207 times smaller than that of the displaced electron. The muon wave function has significant probability inside the nucleus where capture by a proton is possible:

$$\mu^- + {}^A_Z X \rightarrow {}_{Z-1}^{\ \ A} X + \nu_\mu \tag{8}$$

This muon reaction converts a proton into a neutron, transmuting the nucleus and releasing roughly the muon rest mass energy to the neutrino, nucleus and atomic electrons. The final nuclear state may also be unstable and decay. Muon capture inside nuclei is a topic rich in experimental and theoretical physics which you are encouraged to explore. The net effect is that this muon reaction rate adds to the vacuum decay rate for the μ^- and leads to a shorter lifetime for negative muons. In high-Z nuclei, this additional decay mode can significantly shorten the average muon lifetime. However, the muon capture rate scales as Z^4 and in our apparatus, where the plastic scintillator is largely made of carbon and hydrogen atoms, the effect is fairly small.

Particularly when the time constants are of similar size, fitting multi-exponential functions presents difficulties that are discussed in the literature. It turns out that the data from our muon lifetime experiment will be well modeled as a single exponential with an average lifetime that will be shorter than the vacuum value by approximately 5%.

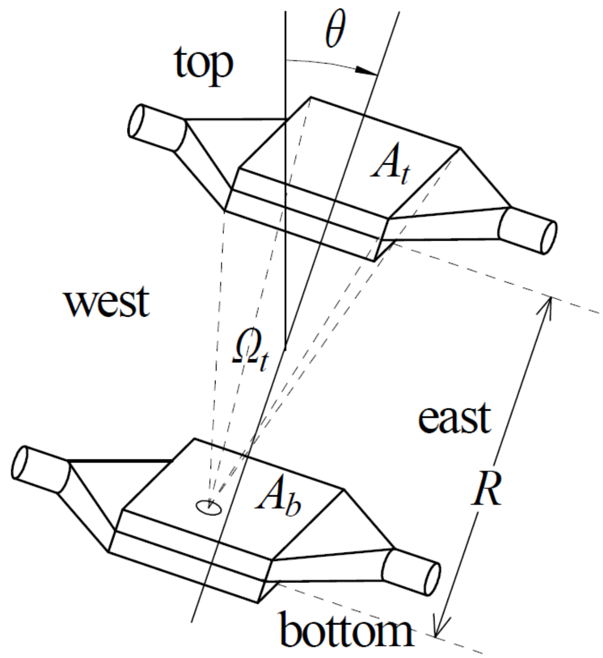

Figure 4: Geometric configuration of the four scintillation detectors. The dotted lines show the solid angle Ω_t subtended by the top scintillator from a point on the bottom scintillator. This defines the solid angle of acceptance when the apparatus is used in the telescope mode where muon passage through both the top and bottom pair is measured. Not shown, the mount for the detectors allows the polar and azimuthal angles to be varied.

Measurements

The first phase of the experiment is concerned with determining the angular distribution and overall flux of muons using four plastic scintillation detectors. As shown in Fig. 4, the detectors are arranged as two pairs — a top pair and a bottom pair. The polar angle θ (from vertical) is illustrated in the figure and is adjusted by rotating the detectors about a horizontal axis. The azimuthal angle is adjusted by rotating the apparatus on its casters about a vertical axis.

The active volume of each paddle-shaped detector is the rectangular slab called the scintillator. A specially shaped optical coupler transports light from the edge of a scintillator at one end to the face of a photomultiplier tube (PMT) at the other end (the cylinders in Fig. 4). Each paddle is wrapped in a light-tight material with a highly reflective film on the inside surface.

The scintillation material is a transparent plastic doped with a fluorescent dye. When a charged particle, such as a muon, passes through the material, it excites and ionizes atoms in the scintillator medium with the fluorescent molecules there to enhance the production of photons. The decay of the excited states via spontaneous emission takes only about ten nanoseconds and the pulse width from the PMT is likewise of this order. Roughly one photon is created for each 100 eV of energy loss in the scintillator.

The two scintillators in a pair are mounted face to face making it highly likely that a muon passing through one will also pass through the other. An event in which a muon passage is detected simultaneously (within a few tens of nanoseconds) in two scintillators is called a "double." For a paddle pair oriented horizontally, the muon responsible for a double can pass into any area element in the upper scintillator and can be moving in almost any direction from straight overhead to nearly horizontally, i.e., from within the 2π steradians of the upper hemisphere. For muons hitting near the edges of a scintillator, the possible muon directions leading to a double become limited as some passage directions would not cross into the other scintillator. If the volume near an edge — say within the thickness of the scintillator — is a small fraction of the total, this effect should be small. Our paddles have a edge fraction over 10%. Nonetheless, as a first approximation, we will assume, in effect, that the paddles are infinitesimally thin (have an edge fraction of zero) when modeling certain aspects of the apparatus. Take note when this assumption is being used and how it may affect any conclusions. It surely would have small systematic effects when determining the overall muon flux.

A four-fold coincidence or "quad" event is one in which all four scintillators detect a muon passage simultaneously. Quads are rarer than doubles because they occur only if the muon comes from a small range of solid angles passing through both the top and bottom pair. As shown in Fig. 4, the top detector's area defines a solid angle of acceptance Ω_t for each area element on the bottom detector. Determining the rate of quads as you vary the polar angle θ provides information about the angular distribution that can be compared to predictions based on Eq. 3.

Exercise 4 *Based on an approximate integration of Eq. 3, explain why the rate of muons passing through both detector pairs would be predicted to be:*

$$\frac{dN}{dt} = I_0 \cos^k \theta \, \frac{A_t A_b}{R^2} \tag{9}$$

where the A's are the areas of the top and bottom detectors and R is the separation between them. Hints: Assume all θ's can be taken as approximately the value for the center line between the detectors as shown in Fig. 4. Show how the factor $A_t A_b / R^2$ arises from the integration over area and solid angle. The factor can be obtained choosing to integrate over the area of either the top or bottom detector and using the other to define the integration over solid angle.

You will start your investigations by setting up the four detectors and determining their efficiencies for detecting the passage of a muon. Photons from a muon passage are channeled through the optical coupler onto the cathode of the photomultiplier tube for that paddle. Via the photoelectric effect, these photons liberate electrons, which are then accelerated to an energy around 100 eV onto the first PMT "dynode." Each incident electron loses its energy near the surface of the dynode and in the process ejects around 10 electrons. Each of these electrons is then accelerated to the next dynode where the multiplication repeats.

There are around 10 dynodes in the PMT — the last of which is called the anode. The chain of accelerations and electron ejections — called a cascade — leads to a large pulse of charge on the anode that raises its voltage for a few nanoseconds before decaying away.

The pulse amplitude depends on many factors. It is typically large from a muon passage which can induce many scintillation photons. A thermionic electron is one that randomly jumps out of a metal overcoming the potential barrier associated with metal's work function via its thermal energy. Such electrons emitted from the cathode or a dynode can also initiate a cascade, but the pulse is typically smaller. A room light photon leaking into the detector can also initiate a cascade leading to a detectable pulse. All pulses not arising from a muon will be called background pulses.

Because the pulses from the PMT vary in size, they are called analog pulses. They are transformed into uniformly shaped digital pulses used for computer processing by a *discriminator* module. The Phillips model 730 five-channel discriminator we use has five independent discriminators. Four are used — one for each detector. A digital output pulse is created only if the analog input pulse height exceeds some user-adjustable minimum, called the lower level threshold or LLT. The LLT is adjusted separately for each detector to eliminate the small background pulses which occur in large numbers.

You will determine the overall muon flux at the earth's surface and its angular distribution. You will also determine the muon lifetime by measuring the distribution of time intervals between double pulses in the same detector. These are not the coincident doubles arising from a single muon passing through two scintillators. They are time-separated pulses — the first occurring as a muon enters and stops in a scintillator and the second when the muon later decays in that same scintillator. The decay time is the interval between these pulses and is predicted to vary randomly according to the exponential distribution discussed in the introduction. Muon decays are rare events occurring at a rate of about two per minute and consequently it will take overnight or longer runs to get suitable data. Be sure to get one of these long runs started as soon as possible.

Data acquisition

Four counters in a National Instruments USB-6341 multifunction data acquisition module — together with a LabVIEW *Muon* program — process and display data about the pulses from the four detectors.

All four counters are started simultaneously at the beginning of a run and continually increment on each pulse from a 100 MHz clock. Thus the count in each counter at any point in time is the same for all four counters — the time since starting in units of 10^{-8} seconds (10 ns).

The logic pulses directly from the discriminator do not have the correct voltage levels to drive the counters and so they are passed through a home-made, four-channel "level adapter" before they are connected to the corresponding counter's *gate* input. As each pulse arrives at the gate, the clock count is latched and saved to a buffer.

The *Muon* program reads and saves these clock counts or "timestamps" to a set of four arrays — one for each detector — containing continually increasing timestamps giving the arrival time of each photon detected in that channel. Thus, if a muon passes through and "lights up" scintillators 1 and 2 simultaneously, then the timestamp when that happened would show up in the two corresponding arrays.

The timestamps do not have to be exactly the same for the program to tag them as coincident. The user can adjust the allowed timestamp separation for which a coincidence will be recorded. Setting this "coincidence time" to 0 would mean the timestamps must be exactly the same. Setting it to 1 would mean they can differ by up to 1 clock pulse (10 ns), 2 would mean 20 ns, etc. You should check how the coincidence rate depends on the coincidence time but the default value of 2 should work well.

The output signal from the level adapter has a sharp leading edge that reliably triggers a timestamp reading with very little jitter (shot to shot time differences) relative to the true time the muon lights up the detector. However, the output signal from the level adapter shows some significant oscillations for about 80 ns and often triggers a timestamp twice. The software ignores these second timestamps if they are closer than the *debounce count* — a user-adjustable value in clock cycles that has a default value of 8 producing an 80 ns "dead time" after each pulse during which a real second pulse would go undetected.

The software continually scans the four timestamp arrays as they fill and finds the earliest in each array. The earliest of these four is then compared with the other three and any within the coincidence time are noted by the software as detectors that fired (or lit) simultaneously. This information is used to update various counters as described next. The timestamps of the lit detectors are then deleted from the arrays and the process repeats.

Based on the lit detectors, the software constructs a four bit tag with each bit, 0-3, taking on the value of 1 or 0 depending on whether detector, 0-3, fired or not. There are 16 values for a 4-bit datum (called a "nibble" or hexadecimal digit). Here, the zero value (no detectors fired) is not used. The muon program increments **exactly one** of the 15 counters associated with that 4-bit tag, which can be described follows:

Independent singles: when exactly one detector fires — an array of four integers for the four detectors in the order 0, 1, 2 and 3.

Independent doubles: when exactly two detectors fire simultaneously — an array of six integers for the six pairs in the order 01, 02, 03, 12, 13, 23.

Independent triples: when exactly three detectors fire simultaneously — an array of four integers for the four triples in the order 123, 023, 013, 012.

Quads: when all four detectors fire simultaneously — a single integer.

The counts above would be statistically independent Poisson random variables. They are Poisson variables because they occur randomly with a fixed probability per unit time and they are statistically independent because they have no counts in common.

There is a second group of counts, called "full" counts, that can be derived from and has a one-to-one correspondence with the "independent" group. For example, the full singles count for detector 0 is the total number of times detector 0 fired, regardless of whether any others fired in coincidence. The full triples count for detectors 0, 1, and 2 would by the number of times those three fired simultaneously whether or not detector 3 also fired. Counts in the independent group will always be labeled with the independent qualifier. Counts in the full group will not be statistically independent and will normally be referred to without a qualifier. In terms of the counters in the independent group, they are:

Quads: The number of times all four detectors fired simultaneously — a single integer. (This is the same counter as in the independent group.)

Triples: The number of times any three detectors fired simultaneously — an array of four integers for the four triples in the order 123, 023, 013, 012. Each triples count is the sum of the corresponding independent triples plus the quads (because every quad is also a triple for any combination of detectors).

Doubles: The number of times each pair of detectors fired simultaneously — an array of six integers in the order 01, 02, 03, 12, 13, 23. Each doubles count is the sum of the corresponding independent doubles plus the appropriate two of the four independent triples (in which the double is also included) plus the quads (because every quad is also a double for any two detectors).

Singles: The number of times each detector fired — an array of four integers for the four detectors in the order 0, 1, 2 and 3. Each singles count is the sum of the corresponding independent singles plus three of the six independent doubles (that include the single) plus three of the four independent triples (that include the single) plus the quads (which always include any single).

The full counts are not statistically independent because they have common counts. For example, any two triples counts have the quad counts in common; any two singles counts have their corresponding doubles count in common. The covariance between any two counters is the variance of the common counts.

Exercise 5 *Consider three independent Poisson random variables: n'_1, n'_2, and n_c having means μ'_1, μ'_2, and μ_c, respectively. Each sample of these three variables is used to construct two new variables $n_1 = n'_1 + n_c$ and $n_2 = n'_2 + n_c$ so that n_1 and n_2 have n_c counts in common. Show that n_1 and n_2 will have means $\langle n_i \rangle = \mu_i = \mu'_i + \mu_c$ and variances $\langle (n_i - \mu_i)^2 \rangle = \mu_i$, for $i = 1, 2$ and a covariance $\langle (n_1 - \mu_1)(n_2 - \mu_2) \rangle = \mu_c$, i.e., equal to the common counts.*

Recall that the uncertainty of any calculated quantity derived from correlated random variables must take into account their covariances in addition to their variances. Exercise 5 shows that the variance of n_1 and n_2 is their distribution's mean and the covariance is the mean of the distribution for the common counts. Also recall that "square root statistics" are appropriate when a sample from a Poisson distribution is greater than 30 or so. The sample value will then be close enough to its mean to justify using that sample value as an estimate of the variance, i.e., the uncertainty is the square root of the count. The same will be true of the covariance. If the common counts are greater than 30 or so, their sample value will also be a good approximation to the covariance.

Procedure

Set-up and initial measurements

1. Measure the dimensions of the scintillators and the separation distance R between the two pairs of detectors. Calculate the approximate solid angle of acceptance. Calculate a rough range of θ values for a particular orientation of the telescope.

2. Orient the detectors vertically ($\theta = 0$) so that the scintillators are horizontal.

3. Have the instructor check the cable connections to the photomultipliers before turning on the high voltage. Make sure that the high voltage is set to NEGATIVE. The operating voltage is 2000 V for the photomultiplier tubes.

4. Examine the pulses from one of the PMTs using the oscilloscope. Make sure to put a 50 Ω terminator at the oscilloscope input using a "tee." Record the typical range of pulse heights and pulse duration (FWHM). Can you see any cable reflections in the signal after the pulse? How does it change when the 50 Ω terminator is removed?

5. Connect the four photomultiplier outputs to the bottom four channels of the Phillips five-channel discriminator module. (The input LEMO connector on the top channel is a bit flaky.) Make sure the inputs are in numerical order from the top down (the cables are labeled 0-3) and that the switch at the bottom is set for LED, which stands for leading edge discrimination. In this mode, the module puts out a short logic pulse whenever the amplitude of the input pulse from the PMT is larger than the LLT (lower level threshold). This discrimination step prevents the processing of smaller pulses, which occur in large numbers from the background with only a small number due to a muon event. The LLT for each of the four channels is set around 0.1 V but varies somewhat because each detector has a somewhat different response and noise level.

6. Examine the output of a discriminator using the oscilloscope. You should see logic pulses with an amplitude of -1 V and a width around 80 ns. Connect the discriminator outputs from the top or bottom pair of detectors to the two inputs of the scope. Triggering on one, you should see an occasional coincidence whenever a muon travels through both detectors.

7. Connect the output of each discriminator in numerical order to the input (LEMO connector) of the 4-channel level adapter. Connect the outputs (BNC connectors) of the level adapter to the gates of the four counters on the data acquisition module (labeled 0-3) in numerical order.

8. Load the *Muon* program and hit the LabVIEW run button to start it. Select a file name where the spreadsheet-compatible data will be saved as a text file. With the detectors still oriented vertically ($\theta = 0$), collect data for a few minutes.

Detector efficiencies

Detector efficiencies are needed in order to determine the true rates at which muons pass through the scintillators from the measured rates. A muon passage through the scintillator sometimes does not result in a logic pulse. In the next step, you will determine the efficiency or probability for a detector to fire given that a muon passed through that detector.

The best way to determine efficiencies is to measure triples and quads over the same time interval. Due to the detector arrangement, any triple is almost certain evidence that a muon traversed through the entire telescope and must have passed through all four scintillators. Thus, the fraction of the time the fourth counter fires given the other three did is the efficiency for that counter. For example, to determine the efficiency of counter 0, you should record (over some reasonable time interval) the full number of triples — simultaneous counts in counters 1, 2, and 3 (N_{123}) and the number of quads (N_{0123}). Our hypothesis is that N_{123} is the total number of muons that passed through scintillator 0, whether it fired or not. Out of that many chances, N_{0123} is the number of times detector 0 fired. The efficiency of the detector is thus

$$\epsilon_0 = \frac{N_{0123}}{N_{123}} \tag{10}$$

C.Q. 1 *Use propagation of errors to show that the statistical uncertainty in ϵ_0 would then be given by*

$$\sigma_{\epsilon_0} = \sqrt{\frac{\epsilon_0(1-\epsilon_0)}{N_{123}}} \tag{11}$$

To apply POE directly to Eq. 10, you will need the variances for N_{0123} and N_{123} and their covariance since it is nonzero. Assume the counts are large enough to use square-root statistics. These two variables have the quads count in common, so that square root statistics imply the covariance between N_{0123} and N_{123} can be taken as N_{0123}. (a) Determine the uncertainty in ϵ_0, using propagation of error on Eq. 10 including these variances and covariance.

Alternatively, one can rewrite the formula for ϵ_0 in terms of the two independent counts: the quads count N_{0123} and the independent triples $N'_{123} = N_{123} - N_{0123}$ — triples that were not in the quads count. The efficiency formula is then the same as Eq. 10,

$$\epsilon_0 = \frac{N_{0123}}{N'_{123} + N_{0123}} \tag{12}$$

but now written in terms of two independent variables with no covariance between them. (b) Use propagation of error for independent variables on this equation — again using square root statistics — to show that this method gives the same result.

(c) The efficiency could also be determined by collecting the coincidence counts in the numerator and denominator of Eq. 10 over separate but equal time intervals. In this case, N_{123} and N_{0123} would be statistically independent. Show that in this case the uncertainty in ϵ_0 would be given by

$$\sigma_{\epsilon_0} = \epsilon_0\sqrt{\frac{1}{N_{123}} + \frac{1}{N_{0123}}} \tag{13}$$

(d) Using simultaneously collected counts for which Eq. 11 applies or independently collected counts for which Eq. 13 applies, suppose both gave 100 triples and 90 quads. Thus, counter 0 has a calculated efficiency $\epsilon_0 = 0.90$ in both cases. Determine the uncertainty for both cases.

(e) The efficiencies are needed to translate theoretically expected rates assuming perfect efficiencies to actual rates. For example, the 4-fold coincidence rate for muon passage for $\theta = 0$ is predicted by Eq. 9 to be $R = I_0 A_t A_b / R^2$. The measured rate would then be predicted to be smaller than this by the factor of $E = \epsilon_0\epsilon_1\epsilon_2\epsilon_3$. To find the uncertainty in I_0 from the uncertainty in the measured rate then requires finding the uncertainty σ_E in this product efficiency. If done directly from the definition of E as a product of efficiencies, one would have to take into account the covariances between between each pair of efficiencies. Alternatively, the product efficiencies could be written in terms of independent counts and the propagation of error without covariance terms could be used. For example, the 2-fold efficiency $E = \epsilon_0\epsilon_1$ (for paddles 0 and 1) is needed to determine I_0 from measured 2-fold coincidences. To use propagation of error without covariance terms, the product efficiency, say for paddles 0 and 1, would be written in terms of independent counts as:

$$E = N_{0123}^2 \left(N'_{123} + N_{0123}\right)^{-1} \left(N'_{023} + N_{0123}\right)^{-1} \tag{14}$$

For the 4-fold efficiency one would use

$$\begin{aligned} E \;=\; & N_{0123}^4 \left(N'_{123} + N_{0123}\right)^{-1} \left(N'_{023} + N_{0123}\right)^{-1} \\ & \left(N'_{013} + N_{0123}\right)^{-1} \left(N'_{012} + N_{0123}\right)^{-1} \end{aligned} \tag{15}$$

Show that the fractional variance in the 2-fold efficiency product $E = \epsilon_0\epsilon_1$ is given by

$$\begin{aligned}\frac{\sigma_E^2}{E^2} &= \Big(\frac{2}{N_{0123}} - \left(N_{0123}+N'_{123}\right)^{-1} \\ &\qquad - \left(N_{0123}+N'_{023}\right)^{-1}\Big)^2 N_{0123} \\ &\qquad + \left(N_{0123}+N'_{123}\right)^{-2} N'_{123} \\ &\qquad + \left(N_{0123}+N'_{023}\right)^{-2} N'_{023} \\ &= \frac{1}{N_{0123}^2}\Big((2-\epsilon_0-\epsilon_1)^2 N_{0123} \\ &\qquad + \epsilon_0^2 N'_{123} + \epsilon_1^2 N'_{023}\Big) \qquad (16)\end{aligned}$$

Similarly, show that the fractional variance of the 4-fold efficiency product $E = \epsilon_0\epsilon_1\epsilon_2\epsilon_3$ is given by

$$\begin{aligned}\frac{\sigma_E^2}{E^2} &= \frac{1}{N_{0123}^2}\Big((4-\epsilon_0-\epsilon_1-\epsilon_2-\epsilon_3)^2 N_{0123} \\ &\quad + \epsilon_0^2 N'_{123} + \epsilon_1^2 N'_{023} + \epsilon_2^2 N'_{013} + \epsilon_3^3 N'_{012}\Big) \\ &\qquad (17)\end{aligned}$$

9. Calculate the efficiencies of all four detectors and check for agreement with the values on the LabVIEW program tab page labeled Rates/Efficiencies. Once verified, feel free to use this page.

10. Deduce the true integrated rate dN/dt of cosmic ray muons crossing the top and bottom pairs — the measured full doubles rates divided by the product of the two detector efficiencies. Compare the corrected dN/dt obtained from the top and bottom pair of detectors. Your comparisons should include a calculation of the propagated uncertainty for each.

C.Q. 2 *Use the data from a long run (with horizontal detectors) to answer such questions as:*

What is the approximate rate at which muons pass through a top or bottom scintillator? What is the approximate rate at which muons pass thorough both the top and bottom scintillators? Why is one so much smaller than the other?

What is the rate of background pulses for each detector? You should obviously start from the independent singles to eliminate counts from doubles, triples or quads, but you should also consider the possibility that independent singles arise not only from background pulses but also from muons passing through the top or bottom pair of scintillators and then one of the detectors fires but the other does not. How big a contribution does this kind of event make to the singles rates? What fraction of the full singles from a detector are due to muons?

How do you account for the number of doubles in non-adjacent detectors, i.e., with one detector in the top pair and one in the bottom pair? They are far rarer than doubles for the two adjacent pairs (top or bottom). Most are from triple or quad events. If these are subtracted out, however, the remaining independent doubles are even rarer. As discussed next, some of these independent doubles are due to random coincidences while others arise from a muon passage.

A random double would be created, for example, when two background pulses — one in one detector and one in the other detector — occur by chance within the coincidence time Δt. As long as Δt is short enough, background rates of R_1 and R_2 lead to a random doubles rate of $R_1R_2\Delta t$.[2] *Explain why Δt for a coincidence count of two clock cycles is 50 ns. These random coincidences would occur for both adjacent and non-adjacent detector pairs. Provide an estimate for these rates. Is it a significant contribution to even the non-adjacent,*

[2]The logic behind this formula is that it is the rate in one detector, say R_1, multiplied by the probability $R_2\Delta t$ (which must be small compared to 1) that the other detector pulse will occur in a time window Δt such as to produce a coincidence.

independent doubles rates?

A double in non-adjacent detectors can also arise from a muon passing through all four scintillators. But rather than producing a quad (if all four detectors fire) or even a triple (if one detector does not fire), an independent double will be created if exactly two detectors do not fire. Estimated this rate from the data. Is it a significant contribution to the non-adjacent independent doubles rates? Are there other contributions to this rate?

Angular distribution

Next, you will examine the quad rate for coincidences in all four detectors, i.e., with the apparatus in the "telescope" mode.

11. Measure the four-fold coincidence rate as a function of θ. Be sure to record total counts and the acquisition time so you will also be able to determine the uncertainty in any rates.

    To change θ, pull out the locking pin, **slowly** rotate the telescope to the desired setting, and then release the pin making sure it registers back into the hole. Keep an eye out for the HV and signal cables to the photomultiplier tubes. Rotating the telescope brings the cables close to the top and bottom of the mounting rack. Make sure they pass by cleanly. As you rotate or move the telescope, they can get snagged on something and damage the PMT or its connectors.

    Adjust the mounting rack so the telescope points out the window (approximately east) when you are taking data for nonzero θ-values. Take measurements at consecutive holes of the telescope. There is a small offset in the holes so none of them line up perfectly vertical or horizontal. Take measurements over 90° from near vertical (approximately straight up through the upper floors of the building) to near horizontal (approximately straight out the window). **For $\boldsymbol{\theta = 90°}$, be sure to also record the doubles count for each paddle pair for use in C.Q. 7.** Use an acquisition time at each angle to get uncertainties smaller than about 10% in the quads rate. This will require running for a longer time at larger angles.

For a non-zero polar angle (e.g., $\theta = 30°$), the telescope can be pointed out the window or through the building components above the lab by rotating it to points east, north, west, or south or angles in between. While the theory, as presented, does not include a dependence on the azimuthal angle — ϕ in Fig. 2 — a weak ϕ-dependence is actually expected.

C.Q. 3 *(a) Why might your measurements show a dependence of the muon flux on the azimuthal angle? For example, could the earth's magnetic field have a steering effect? Would there need to be a difference in positive and negative muon production to see such an effect? Should there be a measurable attenuation of the muons that must go through the upper floors of the physics building when compared with those that must only go through the window?*[3] *(b) What is the rough areal density of the atmosphere for a polar angle of* $0°$ *(straight up). What would it be at* $45°$*? (The atmospheric areal density is the mass per unit area for a column all the way to the top of the atmosphere and should have units of kg/m*^{2}*.) It can be obtained from the atmospheric pressure, (*$\approx 10^5$ *N/m*^{2}*), the acceleration of gravity (*≈ 10 *m/s*^{2}*) and the polar angle.*

[3] Luis W. Alvarez used cosmic ray muon detection as a probe to locate the inner chambers of Egyptian pyramids. See http://www.lns.mit.edu/fisherp/AlvarezPyramids.pdf for details.

(c) Concrete has a mass density of about 2400 kg/m^3. How thick a slab of concrete would have the same areal density as the atmosphere?
(d) The two concrete slabs above the lab have a total thickness around 25 cm. Is the building's areal density above the lab a significant fraction of the atmosphere's?

12. Set the telescope polar angle to 30°. In the previous step, you collected quads at this angle with the telescope pointed east out the window. Now, collect data at other azimuthal angles, perhaps every 90° — approximately north, west, and south. Discuss your results. For example, determine whether or not there are any significant differences in the coincidence rates at different azimuthal angles. Make additional measurements with better statistics or more angles, if needed. Considering the possible effects of concrete in the path of the muons and muon interactions with the Earth's magnetic field, does the data support/refute the contention that there is no dependence of the flux on the azimuthal angle. Based on the χ^2, can you conclude if a fit to a constant flux is reasonable or not?

CHECKPOINT: Procedure should be completed through Step 10, including the determination of the detector efficiencies. C.Q. 3 should be answered. An overnight run to determine the muon lifetime should be started. (Read ahead for instructions on this investigation.)

Analysis of angular distribution

Make a graph of the four-fold coincidence rate dN/dt as a function of the polar angle. Equation 9 predicts the true rate at which muons pass through both detector pairs and must be corrected for experimental issues. A logical fitting function becomes:

$$\frac{dN}{dt} = R_B + R_0 \cos^k \theta \tag{18}$$

where dN/dt values are the measured four-fold coincident rates at each angle — uncorrected for detector efficiencies. Their standard deviations thus depend only on the statistical uncertainty in the counts acquired to determine them. (The uncertainty in the acquisition time is very small and should not contribute significantly to any rate uncertainties.)

Compared with Eq. 9, experimental issues require adding the R_B term to take into account background counts. Perform a properly weighted fit of the raw dN/dt taking into account statistical uncertainties only and determine the best-fit values and uncertainties for R_B, R_0, and k. Use the fitted R_0, and its relationship to I_0, detector geometry parameters, and detector efficiencies to get an estimate of I_0 and its uncertainty. Is k or its uncertainty affected by detector efficiencies or their uncertainties?

C.Q. 4 *Use the doubles rates from a long run at $\theta = 0$ to get another estimate of I_0. You will need to show that the true rate of muons passing through each pair would be predicted to be*

$$\frac{dN}{dt} = I_0 \frac{2\pi A}{k+2} \tag{19}$$

[Hint: Does Eq. 3 or Eq. 6 apply? Why? The integration over area gives the factor A. The differential solid angle $d\Omega$ corresponding to differential variations in the polar and azimuthal angles is given by $d\Omega = \sin\theta\, d\theta\, d\phi$. The integration over all solid angles in the upper hemisphere gives the other factors.] Correct for detector efficiencies to determine I_0 from each pair's measured coincidence rate

and compare with the I_0 obtained from the fit to the angular distribution.

C.Q. 5 *How do your data support time dilation in Special Relativity? [Hint: compare the fitted results with the predictions of Exercise 3.]*

Muon Lifetime

In this part of the experiment you will measure the distribution of decay times for muons that stop inside one of the scintillators. You will measure the time difference between an initial PMT pulse created as a low-energy muon enters and stops inside one of the scintillators and a later pulse arising from the decay of that muon into an electron and two neutrinos.

Energy must be conserved as the muon's 106 MeV rest mass energy is transformed in the decay. A small amount (0.511 MeV) goes into the rest mass of the electron or positron created. Much of the energy goes undetected to the neutrino's kinetic energy. The energy relied on for detection is the kinetic energy of the electron or positron created in the decay. This quantity ranges up to a maximum around 53 MeV but has a broad range of values below. Like a muon, the electron or positron ionizes atoms inside the scintillator material and many scintillation photons are created. Because of its relatively low energy, the stopping power in the scintillator is relatively high and the detection probability should be as good or better than it is for a muon passage.

The algorithm to find muons decaying inside a scintillator treats the top and bottom paddle pairs independently. For each pair, the program looks for a "start" event. A good choice would be a double in that pair. Most of the time, this represents a muon passing completely through both scintillators and no decay event will be found. Rarely but measurably, however, this start event will be a low energy muon traversing the upper scintillator of a pair and stopping in the lower. Based on this assumption for the start event, we should then expect the lower scintillator to occasionally light up within a few microseconds when the stopped muon decays.

It is just about as likely that a low energy muon will stop in the upper scintillator — never making it to the lower scintillator. Thus, we also take a start event as an independent single in the upper scintillator (one not in coincidence with a pulse in the lower scintillator). Most of the time this start event will be a background pulse, but in rare instances it will be a muon stopping in the upper scintillator. In this case, we should then expect the decay scintillation to occur in the upper scintillator.

Thus, after saving the timestamp of the start event — either a double or an independent single in the upper scintillator, the program switches to looking for a stop event — a single in the lower or upper scintillator, respectively, depending on the start event. If the program finds a stop event, it saves the timestamp for this event and starts looking for a new start event.

The timestamp difference between the start and stop event then increments that channel in a 2000-bin frequency histogram. The spacing between bins is 1 clock cycle (10 ns). That is, if the timestamp difference is 100 clock cycles (1 μs), channel 100 in the frequency histogram is incremented.

If a stop event is not detected after 2000 clock pulses (20 μs), the start pulse is ignored (no bin is incremented) and a new start pulse is sought.

13. Set the telescope to its vertical orientation ($\theta = 0$). Real muon decays and random coincidences occur at around one or two per minute. Much lower or much

higher rates probably indicate a problem. To get enough events for an analysis, collect data for at least 24 hours, and ideally for several days. Processed data, **not the four timestamp arrays**, are saved every 43 seconds and when you hit the stop button. Look at this file to see the obvious pattern starting with the total time (in seconds) followed by the 15 independent counter values. Following the counters values there will be four side-by-side columns giving the histogram frequencies for time intervals associated with possible muon decays (i.e. stop pulses) in the upper and lower paddles of the top pair followed by two more columns for stop pulses in the upper and lower paddles of the bottom pair.

The interval measuring part of the software that makes the histograms has a minimum time that defaults to 30 clock cycles (300 ns). It is user adjustable. If you set this minimum to zero and collect data for a long run, your histograms would show statistically significant and relatively narrow spikes at times that correspond to cable reflections. In fact, up to four spikes can be observed spaced every 65 ns from the previous one. (65 ns is the time it takes for a pulse to reflect from the discriminator input, reflect again at the PMT, and return to the discriminator input where it might trigger another logic pulse.) The histogram spikes can be observed out to a time of around 270 ns (four reflections) and render data in this range and below corrupted by the spikes and unusable for analysis. Hence the 300 ns (30 clock cycle) min time. Of course, it is difficult to understand how a stop pulse from a fourth reflection could be observed in the histogram. Why wasn't the interval already stopped by a logic pulse from a prior reflection? In any case, there might be a real muon decay stop pulse that will arrive after this time. Thus stop pulses before the min time are ignored and the software continues looking for a stop pulse for the current start pulse.

Analysis of Lifetime Data

Sum all four histogram columns to get a single histogram for a fit. Fit the decay time histogram to a combination of an exponential and a flat background. That is, the predicted mean for each histogram bin i is given by:

$$y_i^{\rm fit} = A + Be^{-t_i/\tau} \tag{20}$$

where A, B, and τ are the fitting parameters and t_i is the time corresponding to the bin. The justification for this fitting function can be found in the addendum: *Muon Lifetime Measurement* with a link on the course web page for this experiment.

Consecutive histogram bins are separated by the 10 ns clock period, which you can assume is highly accurate and should not be a factor in the error analysis for the muon lifetime. As discussed in the statistical analysis book, to perform the fit, you can either maximize the Poisson log-likelihood function or, equivalently, use iterated least squares. For the latter technique, the chi-square is minimized with respect to A, B and τ using σ_i^2 fixed at the value $y_i^{\rm fit}$ from the prior minimization. The σ_i^2 should be updated after the fit to the new $y_i^{\rm fit}$ and fixed there for the next chi-square minimization. Continue iterating until the $y_i^{\rm fit}$ converge. When finished, calculate the fitting parameter covariance matrix or use the $\Delta\chi^2 = 1$ rule to determine the uncertainty in the muon lifetime.

If you set the min time below the default 30 clock cycle value, you will have to make sure the chi-square or log-likelihood sum excludes the first few channels which will all be zero due to the debouncing part of the software algorithm. And you should also exclude the two or three groups of channels near the front of the histogram which will have frequencies well above the predictions and are due to one or more reflections of pulses at the ends of the cables. They can be easily eliminated from the fit by starting the sum just after the enhancement due to the last reflection. If you used the default min time, all counts below 30 will be zero, and your sum should start after that.

C.Q. 6 *(a) How does your measurement of the muon lifetime compare to expectations? (b) Convert the fitted background level (parameter A of the fit) to a rate (by dividing by the acquisition time) and compare it with a prediction that this rate arises from random events not associated with muon decay.*

For the prediction, treat each paddle pair separately and add the results. Recall that for each pair, there are two possible start/stop event pairs and each possibility adds a component to the background rate. Only a tiny fraction of start events will ever get a stop event within the 20 μs limit and so the random component rate in any bin associated with either start/stop possibility is equal to the rate of start events (either a full double or an independent single in the upper detector) times the probability for a random stop event (a full single in either the lower or upper detector, respectively) to occur in the time interval associated with that bin. The probability of a random stop event occurring in any 10 ns bin interval is the rate of that stop event times 10 ns.

(c) Use the fit parameters B and τ (for the histogram sum for both paddle pairs) to determine the number of muons that decay in the either scintillator of either pair. Hint: the fitted number of counts in the exponential component of the histogram at any t_i is $n(t_i) = A\exp(-t_i/\tau)$. Sum this quantity over all t_i by converting the sum to an integral

$$\begin{aligned}\sum n(t_i) &= \frac{1}{\Delta t}\sum n(t_i)\Delta t \\ &= \frac{1}{\Delta t}\int n(t_i)\,dt \qquad (21)\end{aligned}$$

where Δt is the bin size. Use this number and the acquisition time to calculate the rate of muons decaying in any scintillator. Express this rate as a fraction of the rate at which muons pass into either scintillator pair (the sum of the doubles rate for each pair) This fraction is then an estimate of the probability that a muon passing into a scintillator pair will stop and decay in that pair.

C.Q. 7 *Optional conversation question: "How often do muons decay in your body?" Or: "How often do positrons and electrons annihilate one another in your body?" Both answers will be a rate. Because roughly half the muons (the μ^+) decay into positrons (which then find an electron to annihilate with), the answer to the second question is half the answer to the first. Hints: Assume your body is a suitably-sized rectangular solid. Use your telescope-mode doubles measurements taken at $\theta = 0$ (for the top rectangle) and $\theta = \pi/2$ (for the side rectangles) to determine the rate at which muons enter through the top and the four sides. Why don't any enter from the bottom rectangle? Note that at $\theta = \pi/2$ the scintillator surface is vertical and muons will pass though from either side. Thus, the measured doubles rate in this orientation will be twice the rate at which muons pass in from one side only. The number which decay in your body is some fraction of the number that enter. To estimate this fraction use the results of C.Q. 6c, but because the muon paths will, on average, be longer in your body than in a paddle pair, this fraction will be a few times larger (a factor of five, say?) for your body than for a paddle.*