

Visible Spectrometer

Experiment VIS

University of Florida — Department of Physics
PHY4803L — Advanced Physics Laboratory

Objective

The Balmer spectral lines from a hydrogen discharge lamp are observed with a transmission grating spectrometer and analyzed to obtain the Rydberg constant. Wavelength calibration is achieved by measuring diffraction angles for spectral lines of “known” wavelengths from a mercury and helium discharge tube and fitting this data to a grating equation. The wavelengths of the hydrogen lines are then determined and fit to the Rydberg formula. Basic spectroscopic and statistical analysis techniques are covered.

Theory

The Hydrogen Spectrum

Both the semi-classical Bohr model (circular electron orbit with quantized angular momentum) and the solutions to the spinless Schroedinger equation for the hydrogen atom lead to discrete energy levels that can be expressed in terms of a principal quantum number n

$$E_n = -\frac{\mu e^4}{8\epsilon_0^2 h^2 n^2} \quad (1)$$

where e is the electron charge, ϵ_0 is the permittivity of free space, h is Planck's constant, and μ is the reduced mass of the electron-proton

m_e - m_p system,

$$\mu = \frac{m_e m_p}{m_e + m_p} \quad (2)$$

A photon is emitted when the electron makes a transition from a higher energy level to a lower level with the photon carrying away the excess energy $\Delta E = E_{n_i} - E_{n_f}$. Because the energy of a photon and its wavelength λ in vacuum are related by $\lambda = hc/\Delta E$, Eq. 1 predicts the relation

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (3)$$

where

$$R_H = \frac{\mu e^4}{8\epsilon_0^2 h^3 c} \quad (4)$$

is called the *reduced mass Rydberg*. If the nucleus were infinitely heavy, the reduced mass μ becomes m_e , the mass of the electron. The combination of physical constants

$$R_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} \quad (5)$$

is called the *infinite mass Rydberg* or simply the Rydberg. Although R_H and R_∞ differ by less than 0.1%, with care, the measurements you make should be just accurate enough to distinguish the difference between them.

Sets of wavelengths (series) are categorized by the quantum number n_f of the lower level of

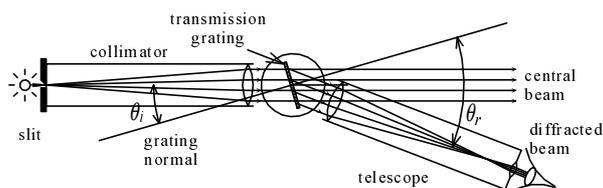


Figure 1: Top view of a transmission grating spectrometer. Note that the incident angle θ_i and the diffraction angle θ_r are relative to the grating normal.

the transition. The Lyman series is obtained for $n_f = 1$, $n_i = 2, 3, \dots$ and is in the ultraviolet part of the spectrum; the Balmer series corresponds to $n_f = 2$, $n_i = 3, 4, \dots$ and is in the visible; the Paschen series, for $n_f = 3$, is in the infrared; etc.

The Diffraction Grating

Wavelength measurements in this experiment are based on the interference of a large number of waves scattered from the grooves of a transmission grating illuminated by incident plane waves of various wavelengths. The geometry is shown in Fig. 1.

Exercise 1 Draw a figure showing adjacent grating grooves (spaced d apart) and show that constructive interference occurs for

$$m\lambda = d(\sin \theta_r - \sin \theta_i) \quad (6)$$

where the incident angle θ_i and the diffraction angle θ_r are measured relative to the grating normal and have the same sign when they lie on opposite sides of the grating normal (as in Fig. 1). The diffraction order m is a positive integer ($\theta_r > \theta_i$) negative integer ($\theta_r < \theta_i$) or zero ($\theta_r = \theta_i$).

Dispersion and Resolution

Two important spectrometer properties are dispersion and resolution. Dispersion is a mea-

sure of the rate of change of spectral line position with λ . With higher dispersion, the spectral lines are more separated from each other or more spread out. With our spectrometer, spectral line positions are measured as an angle θ_r and dispersion is best represented by the value of $d\theta_r/d\lambda$.

Exercise 2 (a) Obtain an expression for $d\theta_r/d\lambda$ from the grating equation (Eq. 6) in terms of d , m , and θ_r . Show that it increases for larger order number m , smaller d , and larger θ_r . (b) For a given λ , m , and d , will the dispersion increase, decrease, or remain the same as θ_i increases? Hint for part b: As θ_i increases, θ_r must change as well. Figure out how θ_r must change. Then from part (a) you will know how the dispersion will change. The end result is that the answer to part b will depend on the sign of m , i.e., to which side of the grating normal you are measuring. The dispersion will increase on one side and decrease on the other. You should derive and explain this dependence.

Exercise 3 If θ_i were 25° and the grating had 600 lines/mm, find all angles θ_r where a red line at 650 nm and a blue line at 450 nm should appear in first order ($m = \pm 1$) or second order ($m = \pm 2$). Hint: You should get only seven angles not eight. Which one isn't possible? Assume you made measurements at these seven angles with an uncertainty in θ_r of 4 minutes of arc ($4/60^\circ$) for all of them. Provide an analytic expression for the propagated uncertainty in λ and determine its value for each measurement.

Resolution describes the ability of the spectrometer to separate two nearby spectral lines. Imagine the wavelength separation between the two spectral lines becoming smaller. The observed lines will begin to overlap each other and at some small separation $\Delta\lambda$ the ability to

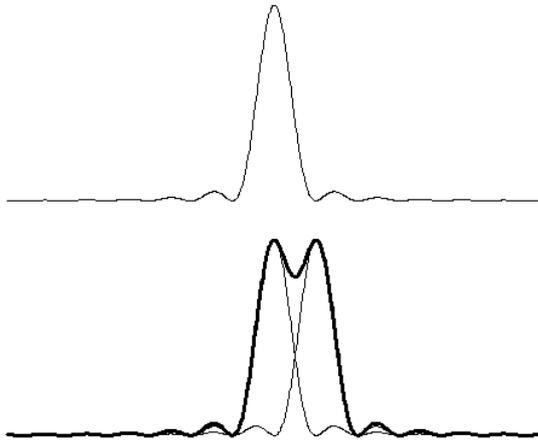


Figure 2: Top: Many-slit diffraction pattern for a single wavelength. Bottom: The thick line is the sum of two diffraction patterns for two different wavelengths separated by the Rayleigh criterion.

discern the lines as separate will be lost. Resolution is a measure of the smallest discernable $\Delta\lambda$. When the spectral linewidths are dominated by spectrometer settings, such as slit width and focusing quality, higher dispersion generally implies higher resolution. But the narrowest linewidths are ultimately limited by the number of grating grooves illuminated. As the number of grooves contributing to the diffraction increases, the angular width of the diffraction pattern (and hence the minimum linewidth) decreases. (Large gratings are used when high resolution is needed.)

The Rayleigh criterion gives a generally accepted minimum “just resolvable” $\Delta\lambda$. The criterion is that the peak of the diffraction pattern of one line is at the first zero of the diffraction pattern of the other. See Fig. 2. Diffraction theory can be used to show this condition occurs for two lines of wavelengths $\lambda \pm \Delta\lambda/2$ when

$$\frac{\lambda}{\Delta\lambda} = mN \quad (7)$$

where N is the number of grooves illuminated, and m is the order of diffraction in which they

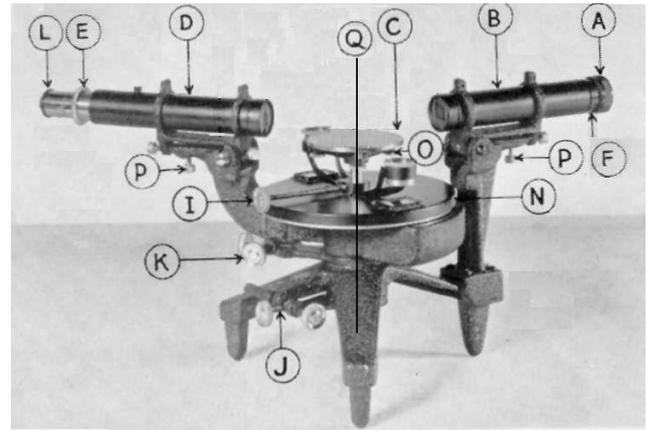


Figure 3: The spectrometer for this experiment. The various components are described in the text.

are observed. $\lambda/\Delta\lambda$ is called the resolving power.

Spectrometer

Fig. 3 shows the location of the various spectrometer adjustment mechanisms. Familiarize yourself with them. Components in *italics* below will be referred to throughout the write-up.

Note the two sets of *rotation adjustments* (J and K) near the legs of the spectrometer. The upper one affects rotations of the *telescope* (D), and the lower one affects rotations of the *table base* (N) on which the actual *table* (C) (holding a prism or grating) is inserted. Not shown is a mounting post on the table. Both rotational motions are about the *main spectrometer axis* (Q) which is vertical and through the center of the instrument. Each rotation adjustment has a locking screw and a tangent screw. When the locking screw is loose, the corresponding element (telescope or table base) can be rotated freely by hand. When tightened, the corresponding element can be rotated small amounts with fine control using the tangent screw. There are two pre-

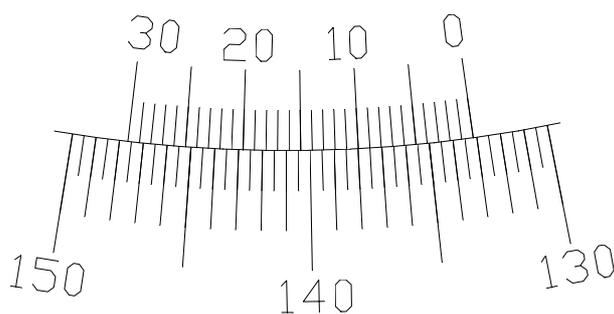


Figure 4: The angular reading is $133^{\circ} 9'$. The 0 mark is just after the 133° line on the main scale. The $9'$ mark is directly aligned with a mark on the main scale.

precision machined circles (called divided circles) associated with these elements. The outer circle is rigidly attached to the telescope and has an angular scale from 0 to 360° in 0.5° increments. The adjacent inner circle is rigidly attached to the table base and has two 0 to $30'$ (minutes of arc) vernier scales on opposite sides.

When measuring angles it is important to take readings at both vernier scales. Because of manufacturing tolerances, the two readings will not always be exactly 180° apart. By using them both, more accurate angular measurements are obtained. See Fig. 4 for an example of a reading at one vernier scale.

The table is mounted on a post that is inserted into the table base and locked into place with the *table locking screw* (I). Three *tilt adjustment screws* (O) on the table allow the pitch and yaw of the table to be varied.

A *cross hair* inside the telescope is brought into focus by sliding the *eyepiece* (L) in or out. The telescope is focused by rotating the *telescope focusing ring* (E). The *collimator* (B) (on which the *entrance slit* (A) is located) is adjusted for focus by sliding the inner collimator tube in or out. An *index ring* (F) on the collimator tube allows the focusing posi-

tion to be maintained once found. The ring has a V-shaped protuberance that fits into one of the V-shaped cutouts spaced 90° apart on the outer collimator tube. Making registration with one of the V's once the proper focusing and slit orientation is obtained, the ring is then tightened into place. This permits the slit orientation to be changed 90° without losing the focus by sliding the tube out slightly, rotating it 90° , and pushing it back into the other V-shaped cutout.

The telescope and collimator also have *leveling screws* (P) that allow their optical axes to tilt up or down. These adjustments are used in a special procedure to orient both optic axes in a common plane perpendicular to the main spectrometer axis. To use them, loosen the upper (locking) screw and rotate the lower (adjustment) screw to tilt the optic axis up or down. When properly adjusted, finger tighten both screws while maintaining the axis orientation.

A grating mounted on a glass plate with a grating spacing $d \approx 1/600$ mm will be used for the measurements. The grating should only be handled by the edges of the glass plate. Do not damage the grating by touching it.

Alignment Procedure

This spectrometer is a semi-precision instrument that can be damaged with improper use. Parts should not be removed for any reason without first checking with an instructor.

1. Carefully take the spectrometer out into the hallway and rest it on a portable table or lab stool. Look through the telescope at the bulletin board near the *Student Services* offices at the end of the corridor. This is far enough to be an effectively infinite object distance. With both eyes open and the unaided eye focused on the distant object, simultaneously focus

- on the cross hair by sliding the eyepiece in or out and on the distant object using the telescope focus. Both the telescope and cross hair images should now be focused at infinity and there should be no parallax. As you move your eye slightly from side to side, the cross hair should not move relative to the image of the bulletin board. If it does move, the image is not at the cross hair and the telescope focus still needs adjustment. It may help to defocus the telescope and then readjust the eyepiece to focus only on the cross hair before trying to refocus the telescope. Once both the cross hair and bulletin board are focussed and show no parallax, bring the spectrometer back to the lab bench.
2. Loosen the index ring, move it back against the entrance slit assembly and retighten it. You should now be able grab the index ring to move the collimator tube smoothly in and out and rotate it. Line up the telescope to look into the collimator and place a incandescent bulb behind the entrance slit. Looking through the telescope, move the collimator tube in or out until the open slit is in sharp focus. **Do not touch the telescope focus ring. It must be left where it was from the previous step.** Adjust the entrance slit for a narrow width and orient it horizontally. Check the slit orientation by rotating the telescope slightly and verifying that the cross hair moves parallel to the slit. Don't be concerned if the slit is slightly high or low relative to the cross hair. Carefully—**without moving the collimator tube**—loosen the index ring completely. Then gently move it into one of the V-grooves and retighten it—**again, without moving the collimator tube.** At the end of this step you should have
 - a horizontal and sharp image of the entrance slit.
 3. Using the telescope leveling screws, get the cross hair centered on the narrow (and still horizontal) entrance slit. Tighten the leveling screws while maintaining the vertical alignment. This ensures the telescope and collimator optical axes are aligned to one another but not necessarily perpendicular to the main rotation axis.
 4. Rotate the telescope until it makes an 80-90° angle with the collimator. Mount an optical flat (a prism face) near the center of the table and along a diameter and tighten it under the clamp. Adjust the table height and the table base rotation angle as necessary to see (through the telescope) the reflection of the entrance slit off the optical flat. Tighten the table and table base locking screws and adjust the table base rotation and the three table tilt adjustment screws to center the reflected image of the slit on the cross hairs. This step guarantees the optical flat normal is perpendicular to the main rotation axis. Try to appreciate why this works!
 5. Rotate the telescope to half-way to the collimator so that the telescope is approximately perpendicular to the optical flat. Shine a light into the side hole near the telescope eyepiece and further rotate the telescope until you can see light reflected off the optical flat. As you get close to perpendicularity, a partial circle of light will appear in the telescope that will probably not be centered vertically. Again, adjust the telescope leveling screws to center the circle of light. If the focusing is correct, you should be able to see a reflected cross hair. If not, adjust the telescope focus to get the re-

flected cross hair in focus with no parallax between the two cross hair images. Further adjust the telescope rotation angle and the leveling screws to align (overlap) the two cross hairs with no parallax. Overlapping the cross hair and its reflection is called *autocollimation*, and will be performed in other steps. In this step, only the vertical alignment is important; it ensures the telescope axis is parallel to the reflecting surface normal and thus perpendicular to the main rotation axis. Tighten the leveling screws while maintaining the vertical alignment.

6. Loosen the table locking screw and carefully remove the table and prism, setting them aside without upsetting the position of the prism relative to the table. Align the telescope and collimator to directly view the entrance slit. Adjust the telescope focus, if necessary, to remove parallax. Adjust the collimator leveling screws so that the telescope cross hair is in vertical alignment with the entrance slit, and then retighten them. This step makes the collimator axis parallel to the telescope axis and thus perpendicular to the main rotation axis.
7. Go back to Step 4 and refine your alignment.

The spectrometer should now be in proper alignment. The telescope and collimator focus should be at infinity. Both the cross hair and slit should be in sharp focus with no parallax between them and they should be vertically aligned. The telescope and collimator optical axes should be perpendicular to the spectrometer rotation axis.

8. Remove the prism. Mount the grating on the table and tighten it under the

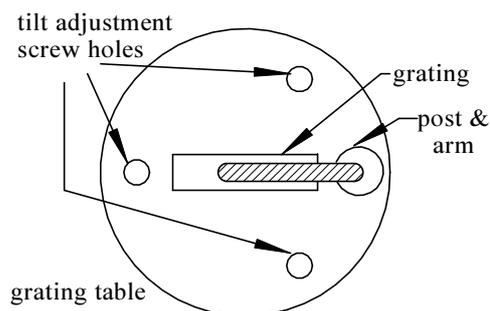


Figure 5: Schematic top-view of the grating placement relative to the three table tilt screws.

clamp. Make sure that the grating is well centered and in line with the mounting post and the opposite one tilt adjustment screw, bisecting the other two tilt adjustment screws as shown in Fig. 5. Also try to make it reasonably vertical. If it isn't nearly vertical, you will have trouble when you later attempt to find the autocollimation reflection.

9. Adjust the table height by looking through the grating at the collimator.
10. Rotate the table and the table base to get an angle of incidence θ_i around $25\text{--}30^\circ$. The open end of the grating mount should face the collimator. Otherwise, the mount will block the spectral lines at large angles on one side. Also, adjust the vernier scales so they are roughly 90° to the collimator (and thus easy to view). Tighten the table and table base locking screws.
11. Again, shine a light into the telescope side hole and adjust the telescope rotation and the table tilt screws to obtain an autocollimation reflection from the grating surface. This step ensures the grating normal is perpendicular to the main rotation axis.
12. Place an incandescent light source behind

the entrance slit. Rotate the telescope to one side and the other and find the “rainbows” on each side. Adjust the grating dispersion plane using the inline table tilt adjustment screw (opposite the post in Fig. 5) so that the rainbows on each side remain aligned with the cross hairs.

13. Repeat from Step 11 until no further adjustments are necessary.

At this point the spectrometer is ready for measurements.

Measurements

Make sure the table base locking screw is tightened and that you do not accidentally use the tangent screw. This would cause the incidence angle to change and it should remain fixed throughout the experiment.

The measurement of angular positions consists of lining up a feature with the cross hair and recording angular readings from both verniers. Do not make any subtractions or averages before recording. Label column headings A for one vernier and B for the other.

A common spectroscopic technique for measuring unknown wavelengths from one source is to first measure known wavelengths from some other sources. These initial measurements are used to calibrate the spectrometer (determine constants such as the groove spacing d and the angle of incidence θ_i) and are called calibration measurements. While it is perhaps a bit artificial, in this experiment the hydrogen wavelengths will be considered unknown and those from any other sources will be considered the known calibration wavelengths. It is recommended that both mercury and helium be used as calibration sources, but feel free to use other sources as you see fit. The following measurement and analysis steps outline how this is done.

14. Record the telescope angles A_n and B_n where the autocollimation signal from the grating reflection occurred in Step 11. This is a measure of the grating normal direction.
15. Turn the entrance slit vertical and make it reasonably narrow (a few tenths of a millimeter). You will have to make some tradeoffs between the slit width and the ability to see weak spectral features. The narrower the slit, the sharper the lines. But they also get harder to see.
16. Place a calibration source (e.g., helium or mercury discharge lamp) just behind and nearly touching the entrance slit. Adjust its position for maximum brightness while viewing a spectral line. Measure and record the angular reading A_i and B_i for the straight-through, zero order (all wavelengths) image. This reading is a measure of the incidence direction. Note the “ghost” lines from imperfections in the grating. These ghost lines may also be seen on stronger spectral lines. Ignore them.
17. Make a table with columns for the color of the line and both vernier readings A_r and B_r which would then be a measure of the diffraction angle. Record the readings for the brighter lines of the calibration sources in all orders attainable on both sides of the zero order image.
18. Place a hydrogen discharge lamp behind the entrance slit and adjust its position as for the calibration lamps. The discharge should have a bright red section in the middle of the tube. If the discharge is all or mostly pink (less than a couple of centimeters of red in the middle), it is time to change the tube. Make a similar table of color, A_r , and B_r for the

observable lines of this spectrum. You should be able to see the Balmer lines corresponding to $n_i = 3, 4, 5,$ and 6 in several orders. They appear as a violet (weak, sometimes extremely weak), blue violet, blue green, and red. Feel free to use the video camera (if available) to see the weaker lines of hydrogen. Just point it in where you put your eye and focus it. With the camera aperture fully open, the lines for $n_i = 6$ and 7 (and perhaps higher) should be measurable in first (and perhaps higher) order.

CHECKPOINT: The procedure should be complete through the prior step, and analysis should be complete through Step 2.

19. Use the sodium lamp and the narrowest possible entrance slit. Observe the sodium doublet lines at 589.0 and 589.6 nm in first order ($m = 1$). They should be easily resolved — appearing as two separate yellow lines. Now place the auxiliary slit over the collimator objective and orient it vertically. Slowly decrease its width. Since the light leaving the collimator is a parallel beam, as the auxiliary slit is narrowed, less and less grating grooves will be illuminated. Narrow the slit until the doublet is no longer resolved and appears as a single line. Measure the auxiliary slit width at the point where the sodium lines are no longer resolved. Determine how many grating grooves are illuminated. Do the lines then become resolvable when viewed in second order? Discuss the significance of this mini-experiment. Be quantitative. What is the resolving power of the spectrometer in first order assuming diffrac-

tion limited performance when the auxiliary slit is removed?

Data Analysis

Calibration

The first step is to reduce each pair of angular readings A and B to a single value H by averaging the readings for A and $B \pm 180^\circ$. Choose the sign in $B \pm 180^\circ$ such that this term is near that of the A reading. For example, with $A = 32^\circ 33'$ and $B = 212^\circ 30'$, use the $-$ sign; but for $A = 325^\circ 15'$ and $B = 145^\circ 17'$, use the $+$ sign.

1. Add a column to your data table converting the A_r and B_r for each spectral line to an H_r . Also convert the incidence angle readings A_i and B_i to an H_i and the grating normal readings A_n and B_n to an H_n .

Recall that the incidence angle θ_i and the diffraction angles θ_r are relative to the grating normal and are thus given by

$$\theta_i = H_i - H_n \quad (8)$$

and

$$\theta_r = H_r - H_n \quad (9)$$

Equation 6 in terms of H -readings becomes:

$$m\lambda = d [\sin(H_r - H_n) - \sin(H_i - H_n)] \quad (10)$$

The values d , H_i , and H_n are constants in the fit and all three can be determined from a linear regression.

C.Q. 1 (a) Use the trigonometric identity $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$, but only for the term $\sin(H - H_n)$, to show that Eq. 10 can be written:

$$m\lambda = D_c \sin H_r + D_s \cos H_r + D_0 \quad (11)$$

where

$$\begin{aligned} D_c &= d \cos H_n \\ D_s &= -d \sin H_n \\ D_0 &= -d \sin(H_i - H_n) \end{aligned} \quad (12)$$

Equation 11 is in the form of a linear regression of $m\lambda$ on the terms $\sin H_r$, $\cos H_r$, and a constant. The numerical values for the three coefficients: D_c , D_s , and D_0 obtained from the fit can then be used to determine d , H_n and H_i . (b) Show that d and H_n can be determined from

$$\begin{aligned} d &= \sqrt{D_c^2 + D_s^2} \\ H_n &= \text{ATAN2}(D_c, -D_s) \end{aligned} \quad (13)$$

and that H_i can then be found using these values and

$$H_i = -\sin^{-1}(D_0/d) + H_n \quad (14)$$

The $\text{ATAN2}(x, y)$ inverse tangent function is available on Excel and guarantees the returned angle θ is in the correct quadrant such that $x = r \cos \theta$ and $y = r \sin \theta$ (with $r^2 = x^2 + y^2$) will both be correctly signed.

2. Make side-by-side columns for $\sin H_r$ and $\cos H_r$. Keep in mind that Excel's trig functions need arguments in radians and the inverse trig functions return angles in radians. The conversion factor is $\pi/180$ and Excel has a $\text{PI}()$ function for the value of π . Perform a linear regression of $m\lambda$ on both of these columns (plus a constant). Then use the fitted coefficients to determine d , H_n and H_i . Also record the rms deviation of the fit.
3. Make a plot of $m\lambda$ vs. H_r . Also plot the residuals: $m\lambda - (D_c \sin H_r + D_s \cos H_r + D_0)$ vs. H_r . Misidentified wavelengths or bad angular measurements should be

obvious from the residuals, which should show only random deviations of less than a nanometer centered around zero. Bad points should be rechecked for possible errors in H_r , m or λ . If you cannot reconcile a bad point, remeasure it.

Hydrogen Data

Next, you will use the calibration results to determine the wavelengths of hydrogen lines. And then you will use these wavelengths to determine the hydrogen Rydberg constant.

4. Make a spreadsheet table with a row for each measured hydrogen line having raw data columns for A_r and B_r , a column for H_r and one for $m\lambda$ based on the regression fit, i.e., using Eq. 11 with the three fit parameters D_c , D_s and D_0 from the calibration data.
5. Add a column for the order m and use this with the column for $m\lambda$ to determine the wavelength associated with the line.
6. Make any necessary corrections for the index of refraction of air, (see the CRC handbook) to obtain vacuum wavelengths for the hydrogen lines.
7. Create a second set of columns for determining a fit of the hydrogen wavelengths to the Rydberg formula (Eq. 3). You will need a column for $y = 1/\lambda$ and one for $x = 1/n_f^2 - 1/n_i^2$. Perform a linear regression of y on x and plot y vs. x . Equation 3 predicts that the slope is R_H and that the intercept is zero. Thus, while it is instructive to check if the fitted constant comes out zero to within its uncertainty, the correct statistical procedure is to perform the regression fit with the **Constant is zero** box checked.

Comprehension Questions

2. Compare the fitted value of H_n and H_i with their measured values and comment on any discrepancy. How close was the fitted d to the manufacturer's specification? Why might they differ?
3. Compare the rms deviation of the fit to expectations based on estimates of the uncertainties in the angular measurements.
4. Estimate the rms deviation expected for the fit to the Rydberg formula (assuming, of course, that that formula correctly describes the data). Comment on the results of the fit to y vs. x with and without a fitted intercept. Discuss the fitted intercept and the overall agreement of the fits with the Rydberg formula.
5. Compare your value for the hydrogen Rydberg R_H with a reference value. Does your result test the difference between R_H and R_∞ ? Is the correction for the index of refraction of air significant at the level of your uncertainties? Explain.
6. Is the Bohr theory accurate enough for these measurements? What is the largest correction to the theory and how does it compare to your experimental uncertainties?