Quartz Crystal Tuning Fork in Superfluid Helium

Experiment TFH

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Objective

A quartz crystal tuning fork, designed for sharp resonant oscillations when operated in vacuum at room temperature, is immersed in liquid helium instead. The tuning fork behavior is affected by the surrounding fluid and varies as the helium temperature is lowered through the superfluid transition. A “suck stick” cryostat is used to get liquid helium to temperatures ranging from 1.6 to 4.2 K. The tuning fork’s frequency and transient responses are measured in that environment and compared with predictions based on simple models for the tuning fork and liquid helium.

References


Introduction

Mechanically, quartz crystal tuning forks are highly tuned resonators with low damping. They are shaped like the normal tuning forks used for checking pitch in musical instruments,
but are miniaturized and operate at ultrasonic frequencies. The ones used in this experiment are about 3 mm long and have a nominal frequency of 32768 Hz. See Fig. 1.

Electrically, the tuning fork is a two-terminal device, having thin film electrodes on each tine with leads for external connections. Quartz’s piezoelectric properties are exploited in construction so that tuning fork vibrations create alternating charges on the two electrodes. The same physics ensures that an applied voltage of one polarity or the other squeezes the tines together or forces them apart.

Figure 2 shows two ways to characterize tuning fork behavior. The left graph shows the steady-state oscillation amplitude of a driven tuning fork as the drive frequency is slowly scanned over the resonance. Note the extremely narrow full width at half maximum (FWHM); the amplitude rises and falls quickly near the resonant frequency $f_0$. The right graph shows ring down behavior as the oscillations in an undriven, but previously excited, tuning fork exponentially damp away.

This data set is from a tuning fork still sealed in its vacuum canister. The top of the canister is cut away in our apparatus to expose the tuning fork to the surrounding medium. When operated in a liquid or a gas, the medium’s viscosity and density strongly affect the fork’s damping and resonant frequency. You will study this dependence with the tuning fork immersed in gaseous and liquid helium.

The apparatus used to create the bath of low-temperature liquid helium is called a “suck stick” and will allow the liquid to be brought to temperatures as low as 1.6 K. Liquid helium has a temperature of 4.2 K at atmospheric pressure, but becomes colder as the pressure above it is reduced by a vacuum pump. It becomes a superfluid below the critical temperature near 2.2 K. This transition toward a state with zero viscosity causes significant changes in the tuning fork behavior.

**Phasor notation and relations**

Phasors are complex representations of sinusoidally oscillating quantities and tremendously useful for the kinds of analyses needed in this experiment.

In this write-up, sinusoidally varying quantities will be typeset using traditional math fonts, e.g., a voltage $v$ might be expressed

$$v = V \cos(\omega t + \delta)$$

where $V$ is the amplitude, $\delta$ is the phase constant, $\omega$ is the angular frequency and $t$ is the time.

The phasor associated with such a time-dependent quantity will be typeset in a bold-face math font and is the complex number having that amplitude and phase constant. For example, the phasor representing the source voltage above would be

$$\mathbf{v} = V e^{j\delta}$$

where $j = \sqrt{-1}$.

A phasor can also be represented using its real and imaginary components.

$$v = V_x + jV_y$$

where $V_x = \Re \{v\}$, and $V_y = \Im \{v\}$ are signed scalars and the functions $\Re \{z\}$ and $\Im \{z\}$ take the real and imaginary parts of a complex number $z$.

Euler’s equation

$$e^{j\theta} = \cos \theta + j \sin \theta$$

provides the key relationship between the two representations. For the voltage example, it gives

$$V_x = V \cos \delta$$
$$V_y = V \sin \delta$$
Exercise 1  (a) Show that a phasor \( v = V e^{j\delta} \) and its associated time dependent quantity \( v = V \cos(\omega t + \delta) \) satisfy
\[
v = \Re\{v e^{j\omega t}\}. \tag{6}
\]

(b) Use \( v = V_x + jV_y \) in Eq. 6 to show that
\[
v = V_x \cos \omega t - V_y \sin \omega t \tag{7}
\]

(c) Use Eqs. 5 to show that Eq. 7 is consistent with Eq. 1.

As you will see throughout this experiment, the simple idea of assigning an oscillating quantity to the real part of a complex oscillation can turn cumbersome equations into nearly trivial ones. Basically, phasors allow a general oscillation with an arbitrary phase constant \( A \cos(\omega t + \delta) = \Re\{A e^{j(\omega t + \delta)}\} = \Re\{A e^{j\delta} e^{j\omega t}\} \) to be separated into a constant part \( A e^{j\delta} \) (the phasor) and an oscillating part \( e^{j\omega t} \). Euler’s equation guarantees everything works out. To see what Euler’s equation did in Ex. 1, use it on each of the three exponentials in the equation \( e^{j(a+b)} = e^{ja} e^{jb} \) and then equate the real and imaginary parts on each side.

The Tuning Fork Model

The tuning fork is basically two parallel tines attached at their base to a bridge—all part of a single quartz crystal. There are many normal modes of oscillation for such a complex structure. The lowest modes have each tine vibrating with a node at the bridge and an antinode at the tip—the fundamental mode for a single tine. For the low-loss mode that our tuning forks operate in, the two tines move out of phase—approaching and receding from one another on alternate halves of a cycle.

For small amplitude oscillations, the motion of all its parts are proportional to one another and the description of the fork as a three dimensional solid can be reduced to a single coordinate, here taken as the position \( x \) of the tip of one tine (along a line between the tips).

With a sinusoidally forced tuning fork, the equation of motion for this coordinate takes the familiar driven harmonic oscillator form
\[
m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F \cos(\omega t + \delta) \tag{8}
\]

The effective driving force \( F \cos(\omega t + \delta) \) arises from a sinusoidal voltage across the tuning...

Figure 2: Left: Typical resonance response of a tuning fork in vacuum as the drive frequency is scanned. Right: The decaying oscillations of the tuning fork with the drive turned off. (The natural frequency is too high to see the individual oscillations.)
fork electrodes. It is specified in Eq. 8 with an arbitrary amplitude \( F \) and phase constant \( \delta \) that will later be related to that voltage.

In Eq. 8, \( m \) is the effective mass of one tine. It depends on the fork geometry and is predicted to be approximately

\[
m = 0.243 \rho_q V
\]  
(9)

where \( V = DWL \) is the leg volume and \( \rho_q \) is the density of quartz. The effective spring constant \( k \) is proportional to the Young’s modulus \( Y \), with the approximate relation:

\[
k = \frac{Y}{4 W \left( \frac{D}{L} \right)^3} \]  
(10)

The effective damping constant \( b \) arises from internal energy losses which are very low in pure quartz. In actual devices, additional energy loss mechanism arise, for example, from the attached electrodes and from tuning fork interactions with its environment. Manufacturing variations among similar forks are larger in this parameter than in the other two.

Dividing through by \( m \) gets Eq. 8 into the another common form

\[
d\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F}{m} \cos(\omega t + \delta)
\]  
(11)

where \( \omega_0 \) is the natural frequency

\[
\omega_0 = \sqrt{\frac{k}{m}}
\]  
(12)

and \( \gamma \) is the damping coefficient

\[
\gamma = \frac{b}{m}
\]  
(13)

After a time, the coordinate \( x \) satisfying Eq. 11 settles into oscillations at the drive frequency that have a fixed amplitude and a fixed phase offset from the driving force. This long-time, settled motion is called the steady state motion. If the oscillator is momentarily perturbed from the steady state motion, it returns to it after some time interval during which it executes non-steady state or transient motion.

The difference between the transient motion and the steady state motion gradually decays to zero and is referred to as the transient response.

With all else fixed, as the drive frequency varies, the steady state oscillation amplitude and phase offset vary. This dependence is called the frequency response.

Equations for both the transient and frequency response arise when finding general solutions to Eq. 11—a non-homogeneous differential equation. The general solution is the sum of any particular solution \( x_p \) satisfying that equation plus the general solution \( x_h \) satisfying the corresponding homogeneous equation:

\[
\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0
\]  
(14)

This is the differential equation for an un-driven tuning fork and has solutions \( x_h \) that take on the familiar form of exponentially damped oscillations. These decaying oscillations are the transient response. Steady state motion will satisfy Eq. 11 and will be the particular solution \( x_p \).

**The homogeneous solution**

Solutions to Eq. 14 are readily derived assuming \( x_h \) is the real part of a complex solution

\[
x_h = \Re \{ a e^{i\omega t} \}
\]  
(15)

where \( a \) and \( \omega \) are complex constants to be determined by substituting Eq. 15 as a trial solution into Eq. 14. The resulting equation is just the real part of the complex equation

\[
\left\{ \frac{d}{dt^2} + \gamma \frac{d}{dt} + \omega_0^2 \right\} a e^{i\omega t} = 0
\]  
(16)
Taking the derivatives and then dividing through by \(-a e^{j\omega t}\) leaves the characteristic equation
\[
\omega^2 - j\omega\gamma - \omega_0^2 = 0
\]
(17)

Considering only the underdamped case appropriate for our tuning forks (for which \(\omega_0 > \gamma/2\)), the characteristic equation is satisfied by two values for \(\omega\)
\[
\omega = \frac{j\gamma}{2} \pm \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}
\]
(18)

The general transient solution is then a linear superposition of the two trial solutions—one for each value of \(\omega\)—with arbitrary coefficients. Defining the free oscillation frequency
\[
\omega'_0 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}
\]
(19)

this superposition gives the general solution for \(x_h\) as
\[
x_h = \Re \left\{ a_1 e^{\frac{j\gamma}{2}t} + a_2 e^{-\frac{j\gamma}{2}t} \right\}
\]
(20)

Equation 19 shows that damping pulls the free oscillation frequency somewhat below the natural resonance frequency \(\omega_0\). However, for the small damping \(\gamma \ll \omega_0\) associated with our tuning fork, \(\omega'_0\) is indistinguishable from \(\omega_0\).

Using Euler’s equation and trigonometric identities, it is not hard to show that this solution can also be expressed in the equivalent form
\[
x_h = A e^{-\gamma t/2} \cos(\omega'_0 t + \delta_h)
\]
(21)

This form is more readily recognizable as exponentially damped oscillations.

Transient motion in undriven systems occurs only after an external excitation provides a non-zero initial displacement and/or velocity; the values for \(A\) and \(\delta_h\) would be chosen to meet those initial conditions.

**The steady state solution**

The steady state solution is constant amplitude oscillations at the drive frequency. These solutions can be expressed
\[
x_p = A \cos(\omega t + \delta_p)
\]
(22)
or
\[
x_p = \Re \left\{ x_pe^{j\omega t} \right\}
\]
(23)

where \(x_p = Ae^{j\delta_p}\) is the phasor associated with those oscillations.

With the driving force related to its phasor \(f = Fe^{j\delta}\) by
\[
F \cos(\omega t + \delta) = \Re \left\{ fe^{j\omega t} \right\}
\]
(24)

Eq. 11 with \(x_p\) as a trial solution is just the real part of the equation
\[
\left\{ \frac{d^2}{dt^2} + \gamma \frac{d}{dt} + \omega_0^2 \right\} x_pe^{j\omega t} = \frac{f}{m}e^{j\omega t}
\]
(25)

The derivatives now act only on the oscillatory factor \(e^{j\omega t}\) giving
\[
\left\{ -\omega^2 + j\omega\gamma + \omega_0^2 \right\} x_pe^{j\omega t} = \frac{f}{m}e^{j\omega t}
\]
(26)

and after canceling \(e^{j\omega t}\) shows that the solution for the position phasor \(x_p\) is simply
\[
x_p = \frac{f/m}{-\omega^2 + j\omega\gamma + \omega_0^2}
\]
(27)

A lot of physics is tied up in Eq. 27. For example, from Eq. 23, the steady state solution is
\[
x_p = \Re \left\{ \frac{f/m}{-\omega^2 + j\omega\gamma + \omega_0^2} \cdot e^{j\omega t} \right\}
\]
(28)

**Exercise 2** (a) Show that the oscillation amplitude is given by
\[
A = \frac{F/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}}
\]
(29)
Hint: Use $A^2 = x_p x_p^*$ where $x_p^*$ is the complex conjugate of Eq. 27. (b) Show that the phase difference between the displacement and the driving force is given by

$$\delta_p - \delta = \tan^{-1} \left( \frac{-\gamma \omega}{\omega_0^2 - \omega^2} \right)$$  \hspace{1cm} (30)

and is in quadrant 3 or 4., i.e., $-\pi < \delta_p - \delta < 0$. Hint: Multiply both sides of Eq. 27 by $e^{-j\delta}$ and then multiply the right side numerator and denominator by the complex conjugate of the denominator. For the quadrant, keep track of the signs of the resultant’s real and imaginary parts. (c) Show that the instantaneous power $p = f \, dx/dt$ supplied by the force oscillates at a frequency $2\omega$ with an amplitude $\omega AF/2$ and has a positive long-term average $\langle p \rangle = -(\omega AF/2) \sin(\delta_p - \delta)$. Hint: Use $f = \Re\{fe^{j\omega t}\}$ and $x = \Re\{x_p e^{j\omega t}\}$ and then note that for any complex number $z$, $\Re\{z\} = (z + z^*)/2$. (d) Show that the velocity on resonance is in phase with the force and that the power is given by $\omega AF \cos^2 \phi$ (where $\phi = \omega t + \delta$ is the drive phase) and thus has an time-averaged value $\omega AF/2$.

The transient solution $x_h$ dies away, but starts back up after any changes in the conditions, for example, after turning on the drive or changing its frequency or amplitude. After the change, a new steady state solution will be applicable. The position and velocity just before the change become the initial conditions for the new solution after the change. A transient solution of the form of Eq. 21 is then again required with non-zero amplitude $A$ and a phase $\delta_h$ chosen to make the general solution meet those initial conditions.

**Exercise 3** (a) Look up the density of quartz and its Young’s modulus and use those values with the data in the caption to Fig. 1 to predict the effective mass $m$, the spring constant $k$ and the resonant frequency $f_0$. Tuning forks are made from crystal quartz, not fused quartz. Also quartz has two different Young’s moduli, depending on whether the strain/strain is along the z-axis of the crystal or perpendicular to it. It turns out our fork’s stress/strain will be perpendicular. (b) Show that for $\gamma << \omega_0$ the amplitude of $x$ vs. frequency $f = \omega/2\pi$ should have a FWHM $= \gamma \sqrt{3}/2\pi$ (c) Use the FWHM of 0.25 Hz given in Fig. 2 to determine the damping constant $b$.

**Electrical-mechanical relations**

The electrodes have one polarity—say $(q,-q)$—when the tines are nearest one another and reverses—becomes $(-q,q)$—when they are farthest apart. The instantaneous $q$ is proportional to the tip displacement $x$.

$$q = \kappa x$$  \hspace{1cm} (31)

The behavior arises from the piezoelectric properties of quartz. The tuning fork constant $\kappa$ and the electrode polarity depend on the tuning fork geometry, the cut of the tuning fork relative to the crystal axes of the quartz, and the electrode shape and placement.

As with a capacitor, variations in the electrode charge $q$ lead to a current “through” the device given by $i = dq/dt$, or from Eq. 31

$$i = \kappa \frac{dx}{dt}$$  \hspace{1cm} (32)

Mechanically, the instantaneous power dissipated in each tine is the product of the effective drive force $f$ and the velocity $dx/dt$. The forces and velocities are opposite in the two tines and the total power dissipated is

$$p = 2f \frac{dx}{dt}$$  \hspace{1cm} (33)

Electrically, the power dissipated in a device is the product of the current through it and
voltage $v$ across it.

$$p = iv = \frac{\kappa}{\kappa} \frac{dx}{dt} v$$  \hspace{1cm} (34)$$

Equations 33 and 34 are only consistent if the effective driving force is associated with the source voltage according to

$$f = \frac{\kappa}{2} v$$  \hspace{1cm} (35)$$

Substituting Eqs. 31 (and its first and second derivatives) and Eq. 35 into Eq. 11 gives

$$\frac{m}{\kappa} \frac{d^2 q}{dt^2} + \frac{b}{\kappa} \frac{dq}{dt} + \frac{k}{\kappa} q = \frac{\kappa}{2} V \cos(\omega t + \delta)$$  \hspace{1cm} (36)$$

where $V \cos(\omega t + \delta)$ is now the voltage across the tuning fork. With the following associations:

$$L = \frac{2m}{\kappa^2}$$  \hspace{1cm} (37)$$
$$R = \frac{2b}{\kappa^2}$$  \hspace{1cm} (38)$$
$$\frac{1}{C} = \frac{2k}{\kappa^2}$$  \hspace{1cm} (39)$$

Eq. 36 becomes

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = V \cos(\omega t + \delta)$$  \hspace{1cm} (40)$$

This should be recognizable as the equation for a harmonically driven series RLC circuit with $L$, $R$, and $C$ thus construed as mechanically-associated inductance, resistance, and capacitance.

Modeling the tuning fork mechanical properties as a series RLC circuit leads to the electrical model of a quartz tuning fork as two arms in parallel as shown in Fig. 3. The equivalent circuit has (1) a series RLC arm (called the mechanical arm) arising from the piezoelectric/mechanical properties of the fork and described by Eq. 40 and (2) a parallel capacitance $C_p$ from the electrodes and connections (called the electrical arm).

Recall that steady state behavior in a.c. circuits can be determined from extensions of Kirchhoff’s rules for d.c. circuits. The d.c. voltages and currents are replaced with their phasor counterparts and resistance is replaced with impedance: $j\omega L$ for an inductor, $1/j\omega C$ for a capacitor, and $R$ for a resistor.

For example, the impedance of the mechanical arm, with the resistor, capacitor and inductor in series is just the sum of each elements’ impedance: $R + 1/j\omega C + j\omega L$. Additionally, the admittance (inverse of impedance) of two parallel branches add—giving an overall admittance for the tuning fork

$$\frac{1}{Z_f} = j\omega C_p + \frac{1}{R + 1/j\omega C + j\omega L}$$  \hspace{1cm} (41)$$

The current phasor in a circuit branch is the voltage phasor across that branch divided by the branch’s impedance. Consequently, the electrical arm carries a current $i_p = v_s \cdot j\omega C_p$ with a relatively weak $\omega$ dependence over the small frequency range explored in a resonance scan. The mechanical arm carries a current $i_m = v_s / (R + 1/j\omega C + j\omega L)$ and has a sharp resonant behavior. It peaks at a value
Exercise 4  The resonant behavior is more readily obvious if the resistance is scaled from the mechanical arm admittance. Show that this leads to the equivalent expression

\[
\frac{1}{Z_f} = j\omega C_p + \frac{1}{R} \cdot \frac{1}{1 + j(\omega^2 - \omega_0^2)/\omega \gamma} \tag{42}
\]

This equivalent parameterization, in terms of \( R, \omega_0, \gamma, \) and \( C_p \) is better suited for comparison with experimental data.

Exercise 5  A typical tuning fork operating in vacuum, might be determined to have a mechanical resistance \( R \approx 18 \, k\Omega \), a damping constant \( \gamma \approx 1.6 \, /s \), and a natural frequency \( f_0 = \omega_0/2\pi \approx 33 \, kHz \). (a) Find the mechanical inductance \( L \) and capacitance \( C \). (b) A determination of the tuning fork constant \( \kappa \) would require some measure of the tip displacement—something we cannot get with our apparatus. However, \( \kappa \) can be estimated based on the given values and an estimate of the effective mass. Estimate \( \kappa \) and then determine, for a typical current amplitude of \( 1 \mu A \) what the corresponding maximum tip displacement, velocity, and acceleration would be.

Effects of a viscous medium

Equation 11 is the equation of motion for a tuning fork in vacuum and must be modified if the tuning fork is operated in a gas or liquid. The vibrating tines cause motion in the medium which in turn creates additional forces on the tuning fork. Where the medium is in direct contact the fork, its motion is entrained with that of the fork. Far from the fork and other bounding surfaces, the fluid velocity field can be expressed as the gradient of a potential. The two behaviors merge over a penetration layer of thickness

\[
\lambda = \sqrt{\frac{2\eta}{\rho \omega}} \tag{43}
\]

where \( \eta \) is the medium’s viscosity and \( \rho \) is its density.

As long as the penetration depth and the overall vibration amplitude are small compared to the fork dimensions, the surrounding medium can be treated as producing an additional force with a “drag” component proportional to and directed opposite the velocity and a “mass enhancement” component proportional to and directed opposite the acceleration. The additional force \( F_m \) due to the medium can be expressed

\[
F_m = -b^* \frac{dx}{dt} - m^* \frac{d^2x}{dt^2} \tag{44}
\]

Both of these forces are calculable from hydrodynamic equations for the flow field around the oscillating tines, which predict that the drag constant is given by

\[
b^* = \sqrt{\frac{\rho \eta \omega^2}{2}} CS \tag{45}
\]

and the mass enhancement is given by

\[
m^* = \beta \rho V + B \rho S \lambda \tag{46}
\]

where \( S = 2(D + W)L \) is the surface area of a tine and \( C, \beta \) and \( B \) are all geometry-dependent factors of order unity. The first term in the mass enhancement arises from the potential flow and does not depend on the medium’s viscosity, i.e., it must be included even for a superfluid. The second term arises from the boundary layer entrained with the fork motion and, via the penetration depth, depends on the viscosity and thus goes to zero for a superfluid.

Adding the additional force (Eq. 44) to the right side of Eq. 11 and then bringing it over
to the left side gives

\[ m' \frac{d^2 x}{dt^2} + b' \frac{dx}{dt} + kx = F \cos(\omega t + \delta_f) \]  

(47)

where

\[ m' = m + m^* \]  

(48)

and

\[ b' = b + b^* \]  

(49)

Thus, the form of the solution remains largely the same, but the resonance frequency and width change. The increased mass decreases the resonance frequency and the increased damping broadens the resonance. Variations of \( b^* \), \( m^* \) and \( \lambda \) with \( \omega \) can be ignored as there would be only very small variations over the narrow range of a resonance. Thus \( \omega \) will be replaced with \( \omega_0 \) in those equations and \( b^* \) and \( m^* \) will be treated as constants over the range of a resonance scan.

\textbf{Liquid helium model}

Helium makes a transition to a superfluid state at \( T_\lambda = 2.1768 \text{ K} \) where the specific heat capacity has a discontinuity in the shape of the Greek letter \( \lambda \). Above this temperature, liquid helium is a normal fluid with a density around 0.14 g/cm\(^3\) (about 1/7th that of water) and a viscosity around \( 3.3 \times 10^{-6} \text{ Pa}\cdot\text{s} \) (about 1/300th that of water).

The two-fluid model is used for temperatures below \( T_\lambda \) where the liquid helium behaves as if it were a mixture of a normal fluid and a superfluid with the proportion of each a function of temperature. The size of the two fractions is illustrated in Fig. 4 where the solid line gives the superfluid density \( \rho_s \) and the dashed line gives the density \( \rho_n \) of the normal component. The total density \( \rho \) is the sum of the two

\[ \rho = \rho_n + \rho_s \]  

(50)

Figure 4: The two-fluid model of liquid helium. The graph shows the density of the normal fluid, the superfluid, and their sum. Below 1 K, helium is virtually all superfluid. Above \( T_\lambda \) it is all normal fluid. From reference 2.

Figure 5 shows the viscosity of liquid helium as a function of temperature. The normal fluid has viscosity while the superfluid does not. Consequently, the superfluid component contributes only via the \( \beta \rho V \) mass enhancement term. The normal fluid contributes to both terms of the mass enhancement and to the additional damping. Thus, the two-fluid

\textbf{Figure 5:} The viscosity of liquid helium. The two-fluid model attributes the viscosity to the normal component only. From reference 2.
model gives
\[ b^* = \sqrt{\frac{p_n \rho_n^2}{2}} CS \]  
(51)
and
\[ m^* = \beta \rho V + BS \sqrt{\frac{2 \eta \rho_n}{\omega}} \]  
(52)

**Temperature dependence**

Measurements and fits of the transient response and of the frequency response will be described shortly that will provide values for \( R, \omega_0, \gamma \) and other parameters. These parameters will be obtained as the temperature of the liquid helium is varied.

Determining the temperature dependence of \( \omega_0 \) and \( \gamma \) are the basic goals of the experiment. It turns out useful to compare these two parameters against their vacuum values, which values will now be given an additional 0 subscript
\[ \omega_{00}^2 = \frac{k}{m} \]  
(53)
and
\[ \gamma_0 = \frac{b}{m} \]  
(54)

The square of the ratio of the resonance frequency in vacuum to that in the media then gives:
\[ \left( \frac{\omega_{00}}{\omega_0} \right)^2 = \frac{m'}{m} \]
\[ = 1 + \frac{m^*}{m} \]  
(55)

It is recommended that the experimental results for the resonance frequencies be plotted as the function
\[ F = (\omega_{00}/\omega_0)^2 - 1 \]  
(56)
which is then \( m^*/m \) and thus predicted from Eq. 52
\[ F = \frac{\beta \rho V}{m} + \frac{BS}{m} \sqrt{\frac{2 \eta \rho_n}{\omega_0}} \]  
(57)

To see how the damping parameter \( \gamma = b'/m' \) should vary, first note that \( m'/m = (\omega_{00}/\omega_0)^2 \) giving
\[ \gamma(\omega_{00}/\omega_0)^2 = \frac{b'}{m} = (b + b^*)/m = \gamma_0 + b^*/m. \] Thus, if we define
\[ G = \gamma \left( \frac{\omega_{00}}{\omega_0} \right)^2 - \gamma_0 \]  
(58)

it is then \( b^*/m \) and thus predicted from Eq. 51
\[ G = \sqrt{\frac{p_n \rho_0^2 CS}{2}} \]  
(59)

**The Data Acquisition System**

The data acquisition computer for this experiment is equipped with a National Instruments PCI-GPIB+ IEEE-488 interface card used to communicate with a function generator and a dual phase lock-in amplifier. These two instruments are used to determine the tuning fork’s impedance as the drive frequency is varied through the resonance. The computer also has a National Instruments PCI-MIO-16E-4 multifunction data acquisition (DAQ) card for transient response measurements and for temperature measurements using a low-temperature thermometer installed in the cryostat. The important features of these components and their use in the tuning fork measurement circuit are presented in this section.

**Function generator and circuit**

Figure 6 is a schematic of the circuit for measuring the tuning fork behavior.

The function generator is the Stanford Research Systems DS340 with features similar to others, e.g., an adjustable frequency and amplitude and a circuit model consisting of an ideal voltage source and a 50 Ω series resistance. The output is labeled **FUNC OUT** over the bnc connector on the front panel.
The function generator output is only available after its 50 Ω output impedance. At this point in the circuit, the voltage would depend on the current, which in turn depends on the load impedance and thus cannot be specified ahead of time. The function generator output is specified irrespective of the load at the (inaccessible) source point labeled $v_0$ in Fig. 6. This voltage can be expressed

$$v_0 = V_0 \cos(\omega t + \delta_0)$$

or equivalently as the phasor

$$v_0 = V_0 e^{j\delta_0}$$

where $\delta_0$ is relative to the sync signal (described next).

The voltage waveform before the 50 Ω resistor, while inaccessible, could be measured by using a high impedance probe with no other load attached. Be sure the DS340 is set in the High-Z mode so that $V_0$ is shown on the function generator’s display. The display will be low by a factor of two if the DS340 is set to 50Ω mode. Look for the High-Z/50 Ω indicator under the output BNC connector and look for the units indicator on the right side of the front panel. The amplitude can be set or read as peak-to-peak values ($2V_0$) or as rms values $V_0/\sqrt{2}$

A second common function generator output is the sync signal. In the DS340 it is a square wave synchronized with the voltage source described above. It is labeled SYNC OUT over its bnc connector. The sync signal’s rising edges have a fixed phase difference with the positive-going zero-crossings of $v_0$ and will be used as a reference for determining the phase of any voltage measured by the lock-in. The ability to measure the phase of the current in the tuning fork relative to the source voltage is required to determine the tuning fork impedance.

The minimum amplitude from the function generator is generally too big for directly driving the tuning fork. To get the smaller excitation voltages required, a shunt resistor $R_s$ of either 0.5 Ω or 5.6 Ω is placed across its output as shown in the figure. According to Thévinin’s theorem, the shunt resistor reduces the output impedance to the parallel combination

$$R_s' = \frac{R_s \cdot 50 \Omega}{R_s + 50 \Omega}$$

which is just a bit below $R_s$. The shunt resistor also reduces the output voltage to

$$v_s' = \epsilon_s v_0$$
where the reduction factor \( \epsilon_s \) is given by
\[
\epsilon_s = \frac{R_s}{R_s + 50 \, \Omega}
\]  
(64)

and is around 0.1 for \( R_s = 5.6 \, \Omega \) and around 0.01 for \( R_s = 0.5 \, \Omega \).

Coaxial cables connect to and from the tuning fork. Coax is normally modeled as a transmission line, but for the relatively low frequencies involved in this experiment, the simpler model of the coax as a lumped capacitance to ground is appropriate. Approximately 2 m of LakeShore type SS cryogenic coax cable (capacitance about 174 pf/m) connect each tine of the tuning fork at the bottom of the cryostat to the two feedthroughs at the top. About 2 m of RG58 coax cable (capacitance about 80 pf/m) connect the function generator to one feedthrough and a similar cable connects the other feedthrough to the transimpedance amplifier. Thus, \( C_s \approx 500 \, \text{pf} \) on the source side of the circuit and \( C_d \approx 500 \, \text{pf} \) on the detector side.

**Exercise 6** According to Thévinin’s theorem, \( C_s \) can also be modeled as part of the source. Show that adding a parallel capacitance to ground (a) changes the Thévinin source impedance from \( R_s' \) to
\[
Z_s = \frac{R_s'}{1 + j\omega\tau_s}
\]  
(65)

and (b) changes the Thévinin source voltage to
\[
v_s = v_s' \cdot \frac{1}{1 + j\omega\tau_s}
\]  
(66)

where \( \tau_s = R_s'C_s \) is an effective source time constant. (c) Evaluate the time constant for the \( R_s = 5.6 \, \Omega \) shunt. (d) Noting that \( 1 + x \approx e^x \) for \( x << 1 \), show that for \( f \) around 33 kHz, \( v_s \) is effectively phase shifted from \( v_s' \) and find the size of the shift in degrees.

The previous exercise should have demonstrated that because of the low output impedance of the source, the coax capacitance \( C_s \) should have no bearing on the measurements.

The coax from the other side of the tuning fork connects to the virtual ground input of the transimpedance amplifier (current-to-voltage converter). Because of the near-zero input impedance of this amplifier, the coax capacitance \( C_d \) on this side can also be neglected. The 10 kΩ transimpedance then gives the amplifier’s output as
\[
v_d = -i \cdot 10 \, \text{kΩ}
\]  
(67)

where \( i \) is the current in the circuit and is given by
\[
i = \frac{v_s}{Z_s + Z_f}
\]  
(68)

Because it is negligible compared to \( Z_f \), the source impedance \( Z_s \) can be dropped from the analysis. The factor of -1 in Eq. 67 arises from the inverting behavior of the op amp in the transimpedance amplifier.

When the low source and detector impedances are neglected, the final result for the phasor at the amplifier output is
\[
v_d = -\epsilon_s v_0 \frac{10^4 \, \Omega}{Z_f}
\]  
(69)

**The Lock-in Amplifier**

The transimpedance amplifier output is connected to the lock-in input for measurement of \( v_d \). Consult the user manual for detailed information on our Stanford Research Systems SR830 lock-in amplifier. Here we only need to appreciate that it analyzes an oscillating voltage across its input
\[
v = V \cos(\omega t + \phi)
\]  
(70)

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and returns two signed quantities $V_x$ and $V_y$ given by

$$
V_x = V \cos \phi \\
V_y = V \sin \phi
$$

(71)

$V_x$ is called the in-phase component and $V_y$ is called the quadrature component. They are given by the lock-in as rms values. The lock-in can also provide the amplitude $V$ (again, an rms value) and the phase constant $\phi$. Note that $V_x$ and $V_y$ are just the real and imaginary parts of the phasor $\mathbf{v} = V e^{j\phi}$. In effect, the lock-in can be considered to provide the phasor associated with its input.

The lock-in determines the phase constant $\phi$ relative to the primary reference signal connected to its reference input—in our case, the sync signal. The lock-in adds a user-adjustable offset to the phase of the primary reference and uses that phase to create two secondary sinusoidal reference signals at the primary frequency—one for each of the $V_x$ and $V_y$ output circuits—that are 90° out of phase with one another. Each secondary reference is multiplied with the input signal, scaled, and time averaged to generate the $V_x$ and $V_y$ outputs. Unwanted noise in the signal that is not at the reference frequency is largely filtered out while the signal at the reference frequency remains.

The phase offset adjustment will be used to take into account the phase offset between the sync signal and the source waveform $v_0$. This phase offset will be measured and 180° will be added to it before it is applied via the lock-in phase offset adjustment. This makes the phase constant $\delta_0 = \pi$ in the source voltage (Eq. 60) so it now becomes

$$
v_0 = V_0 \cos(\omega t + \pi) = -V_0 \cos \omega t
$$

(72)

i.e., its phasor becomes

$$
\mathbf{v}_0 = -V_0
$$

(73)

Figure 7: The manufacturer calibration for our Cernox solid state thermometer for temperatures from 1.4 to 100 K and crudely extended above this range as shown.

and Eq. 69 for the measured lock-in phasor becomes

$$
\mathbf{v}_d = \epsilon_s V_0 \frac{10^4 \Omega}{Z_f}
$$

(74)

By making $\delta_0 = 180^\circ$, only the amplitude $V_0$ of the function generator source voltage now appears and the overall negative sign from the transimpedance amplifier inversion is gone. A measured lock-in phase of $\phi = 0$ for $\mathbf{v}_d$ ($V_x > 0$, $V_y = 0$) would then imply that $\mathbf{v}_d$ is a positive real quantity, that the circuit current is in-phase with $v_0$, and that $Z_f$ is a positive real quantity (resistive) with no imaginary (capacitive or inductive) component. A measured lock-in phase of 90° for $\mathbf{v}_d$ ($V_x = 0$, $V_y > 0$) would imply that $\mathbf{v}_d$ is a positive imaginary quantity, that the current leads the source voltage by 90°, and that the impedance $Z_f$ has a negative imaginary part and no real part, i.e., it is capacitive.

**Thermometry**

A Cernox solid state thermometer is positioned next to the tuning fork. Its resistance $R_{th}$ near room temperature is about 60 Ω and

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The Cernox thermometer is a four-wire resistor with two leads for supplying an excitation current and two leads for measuring the voltage generated by the current. It is placed in a voltage divider circuit as shown in Fig. 8 with an $R_{\text{cal}} = 1.00 \ \Omega$ 1% series resistor. The current through the series resistors is driven by a sinusoidal voltage generated by a 12-bit digital-to-analog converter (DAC) on the DAQ card installed in the computer. The sinusoidal voltage waveform across the thermometer is measured by a 12-bit analog-to-digital converter (ADC) also on the DAQ card.

The Cernox thermometer is a delicate sensor that must never be driven by voltages large enough to cause power dissipation above 2 mW. The 1.00 MΩ series resistor should prevent any possibility of an overdrive situation.

The a.c. drive waveform $\nu_{\text{dac}}$ is at 19 Hz and its $V_{\text{dac}} = 10$ V amplitude is set from a second DAC available on the DAQ card. The resulting waveform across the thermometer $\nu_{\text{adc}}$ is measured with the ADC on the DAQ card in synchrony with the output waveform $\nu_{\text{dac}}$.

The lock-in technique used to determine the amplitude of the resistor voltage $V_{\text{adc}}$ is similar to that of the SR830. The computer multiplies the measured $\nu_{\text{adc}}$ waveform by a sine and cosine waveform of unit amplitude and at the exact frequency of the $\nu_{\text{dac}}$ waveform and then the computer averages the result for each product over many periods as specified by a user selected time interval. The square root of the sum of the squares of the sine and cosine components (properly normalized) gives the amplitude $V_{\text{adc}}$ of the ADC waveform at the frequency of the DAC waveform while averaging away most of the noise.

Because the 19 Hz frequency is so low, cable and other capacitance have virtually no effect and d.c. equations can be used to relate the voltage amplitudes and resistances involved. The measured thermometer resistance $R_{\text{th}}$ is then given by

$$R_{\text{th}} = R_{\text{cal}} \frac{V_{\text{dac}}}{V_{\text{dac}} - V_{\text{adc}}}$$

The data acquisition programs that report temperature use this formula along with the thermometer manufacturer’s calibration formula (see the auxiliary material for the details) to convert resistance to temperature.

Because the amplitudes of both the drive waveform and the signal waveform are determined relative to the same internal reference voltage on the DAQ card, inaccuracy in this reference value plays no role on the ratio used to determine the thermometer’s resistance.

**Data acquisition and analysis**

All data acquisition and analysis programs are in the Tuning Fork folder in the PHY4803L folder on the desktop. To use the frequency scanning program requires enabling the GPIB...
(IEEE-488) communications on the DS340. It must be enabled on the DS340 every time it is powered up (shift then 1 key then up arrow). The SR830 powers on with the interface already enabled. Furthermore, once the computer sends a command to the DS340 or to the SR830, the instrument goes into remote command mode and disables the front panel controls. If the software leaves the instrument in remote mode, you will have to manually return it to local (front panel) control mode—shift then 3 key for the DS340, the Local button on the SR830 front panel.

**Frequency scan and Analyze resonance program**

Frequency scans are performed with the Frequency scan program. In it you will find controls for the starting and ending frequency, the frequency step size, the time to wait after each frequency change before reading the lock-in, and whether to do a forward scan, a reverse scan, or both. The program displays the predicted time for the scan to complete after any change to these parameters.

The resulting data set is the in-phase $V_x$ and quadrature $V_y$ values at each frequency. The program also averages temperature readings during the wait at each frequency and so has a temperature for each frequency.

When complete, the program saves the data to the file specified at launch time.

Equations 74 and 42 give the lock-in phasor

$$v_d = \frac{\epsilon_s V_0 \cdot 10^4 \Omega}{R} \cdot \left(j\omega RC_p + \frac{1}{1 + j(\omega^2 - \omega_0^2)/\omega\gamma}\right).$$

Analysis of frequency scans over a resonance is performed by the Analyze Resonance program.

To take into account that the lock-in phase adjustment may be off by a small angle $\phi$, an overall phase factor $e^{j\phi}$ should multiply the prediction of Eq. 76. In addition, small offsets $C_x$ and $C_y$ are expected in the lock-in’s $x$- and $y$-outputs arising from d.c. errors in their amplifiers. Adding these two effects give the final prediction for the output phasor from the lock-in.

$$v_d = Ae^{j\phi} \left(1 + j(\omega^2 - \omega_0^2)/\omega\gamma\right) + C_x + D_x\omega + j(C_y + D_y\omega)$$

where

$$A = \frac{\epsilon_s V_0 \cdot 10^4 \Omega}{R}$$

$$D_x = -ARC_p \sin \phi$$

$$D_y = ARC_p \cos \phi$$

To describe a few unique details associated with fits involving complex variables, it will be useful to distinguish the measured lock-in phasor $v_m$ from the prediction $v_d$ of Eq. 77. The measured data are the signed scalars for the in-phase $V_{mx}$ and quadrature $V_{my}$ lock-in outputs as the frequency is varied in $N$ steps through the resonance. The corresponding predictions are the real and imaginary parts of Eq. 77 for each frequency.

Assuming both lock-in outputs have equal uncertainties $\sigma_v$, the reduced chi-square $\chi^2 = \frac{\sum \left(\frac{v_m - v_d}{\sigma_v}\right)^2}{N-2}$.
$s^2_v/\sigma^2_v$ is proportional to the sample variance $s^2_v$ taken as

$$s^2_v = \frac{1}{2N-M} \sum_{i=1}^{N} \left[ (V_{mx}(\omega_i) - V_{dx}(\omega_i))^2 
+ (V_{my}(\omega_i) - V_{dy}(\omega_i))^2 \right]$$  \hspace{1cm} (81)

where $M$ is the number of fitting parameters. While there are only $N$ independent variables (scan points $\omega_i$), there are two measurements ($V_{mx}$ and $V_{my}$) for each of them and hence the number of degrees of freedom is $2N-M$.

The fit minimizes the sample variance using a standard nonlinear fitting algorithm available in LabVIEW and reports the resulting sample standard deviation $s_v$. It also provides graphs of the $V_x$- and $V_y$-deviations.

The parameters $\omega_0$ and $\gamma$ are both scaled down by $2\pi$ and are labeled $f_0$ and $\Delta f$ in the program. This is a program feature, not a bug. It is designed to make it easier to estimate and compare parameters with standard data plots in which the independent variable is the frequency $f$ rather than the angular frequency $\omega$.

To decrease the covariance between the $C$ and $D$ parameters and improve the program’s performance, both the $x$- and $y$-terms in Eq. 77 of the form $C + D\omega$ are replaced with the equivalent terms

$$C' + D(\omega - \omega_0) = C + D\omega$$  \hspace{1cm} (82)

Thus, for both the $x$- and $y$-terms,

$$C = C' - D\omega_0$$  \hspace{1cm} (83)

Thus $C'$ is the offset voltage near resonance.

Resonance curve fits may fail if the initial guesses for the parameters, particularly $f_0$, are not close enough to the correct values. Play with them a bit before hitting the Do Fit button. Also keep an eye on the plots of the $V_x$- and $V_y$-deviations. For a good fit, these should be random and should not show any systematic dependence on frequency.

The Save button on the front panel writes a single row of data containing the Run #, the average temperature for the run, its rms deviation over the run, the $y$-scale factor (typically $10^{-3}$) and all fitting parameters and their sample standard deviations. Supply a new file name for the first data set and you can repeatedly save to it; the program will append one new row each time. The file must not be open in another program when you try to write a new row.

**Acquire and analyze transient program**

Transient solutions or “ring downs” will be measured and analyzed using the Acquire and Analyze Transient program.

During ring downs, a computer-controlled reed relay quickly disconnects the function generator and reconnects this point to ground as shown in Fig. 6. Any initial charge on the tuning fork’s parallel capacitance $C_p$ will decay away on a time scale around $R_d C_p$ (where $R_d$ is the transimpedance amplifier’s input impedance) that is quite short compared to the decay time for the current in the mechanical arm. The current through the transimpedance amplifier’s virtual ground input should then be given by Eq. 32 with Eq. 21 for $x$. The transimpedance amplifier output voltage $v$ will then be that current times the $10^4 \Omega$ feedback resistance

$$v = 10^4 \Omega \kappa \frac{d}{dt} \left\{ A e^{-\gamma t/2} \cos(\omega_0 t + \delta_h) \right\}$$  \hspace{1cm} (84)

**Exercise 7** Show that Eq. 84 gives

$$v = A_v e^{-\gamma t/2} \cos(\omega_0 t + \delta_v)$$  \hspace{1cm} (85)

where

$$A_v = 10^4 \Omega \kappa \omega_0 A$$  \hspace{1cm} (86)
\[ \delta_v = \delta_h + \tan^{-1} \frac{2\omega_0'}{-\gamma} \]  

(87)

Hint: Show that Eq. 21 can be expressed
\[ x_h = \Re \{ Ae^{j\delta_h} e^{(-\gamma/2+j\omega_0')t} \} \], then show that the order of differentiation and taking the real part can be exchanged and perform the calculation in that order.

This exercise shows that the voltage measured in a free oscillation decay has the same frequency and decay constant as that of the displacement oscillations.

The output of the transimpedance amplifier is wired to channel 1 of the DAQ board and to a bnc connector on the top of the interface box for connection to the lock-in. The lock-in is used to monitor vibration amplitudes when you “ring up,” or excite, the tuning fork.

To see a ring down, the tuning fork must already be oscillating with appreciable amplitude. To get it oscillating, you will initiate a ring up. The data acquisition program has a toggle button that will send a signal to the relay to connect one side of the tuning fork to the drive voltage for a ring up or connect it to ground for a ring down.

The function generator frequency must be set near the resonant frequency to get any appreciable amplitude on a ring up. The lock-in is needed for this step. With the relay in the ring up position, adjust the function generator frequency for maximum amplitude on the lock-in. But keep an eye on the lock-in amplitude. If it goes above 10 mV lower the function generator drive voltage before continuing. If it goes above 100 mV, the tuning fork motion is getting large enough for it to shatter. Continue to adjust the frequency to get near the resonance and the drive amplitude to get a lock-in signal around 10 mV. You do not have to be right on resonance before initiating a ring down. You just need a lock-in amplitude around 10 mV.

In the data acquisition program’s Time domain|Acquire tab you will find controls for setting the ADC sampling rate, the number of samples to acquire, and the ADC range. The ADC range should always be set to the most sensitive 50 mV range; our tuning fork signal should never go higher than about 20 mV. Our ADC runs at a top speed of 500,000 sample per second. Use this speed whenever possible and adjust the number of samples so that an entire ring down is acquired and the signal has decayed well into the noise. Because temperature monitoring also uses this ADC, it is shut down during the short intervals needed to record ring downs, but temperature readings are made just before and just after these transient measurements and are displayed on the front panel.

As you change the helium temperature, the resonance frequency and damping will vary. Manually adjust both the frequency and amplitude on the function generator after a ring up so it is running near the resonance frequency and the lock-in amplitude is around 10 mV.

The Save button in the Acquire tab saves the ring down data to a file that can be read by a spreadsheet. The first two numbers in the file are the before and after measured temperatures, then the \( y \)-scale factor (typically \( 10^{-3} \)), then the time interval then the array of scaled ADC readings. This data file can also be reread by the program by hitting the Read button.

Fitting is performed in the Time domain|Analyze tab. The program has built in delays so that the ADC will start acquiring readings about 50 ms before the relay switches. The program will fit the data between the two cursors on the graph, so find where the relay switched and set the starting cursor right after the oscillations begin to decay. Take a look in this region with an
expanded time scale so you can better see the start of the decay. Generally, the ending cursor should be set so the fitting region includes all of the freely decaying oscillations, but does not include too many points after the decay is complete. However, there is a limit of around 700,000 for the number of points LabVIEW will allow in this fit. If the entire decay has more points than that, use a smaller fitting region or lower the acquisition rate. The rate is divided down from a 20 MHz clock and so a divisor of 40 gives the recommended and maximum 500k samples per second rate. Other reasonable rates to try are 400k (divisor of 50), 250k (divisor of 80) 200k (divisor of 100). Keep in mind that at 200k samples per second there are only about 6 measured data points on each cycle of the 32 kHz oscillations.

The Do Fit button then initiates a nonlinear fit of the data between the cursors to the form of Eq. 85 plus a constant to take into account any offset in the transimpedance amplifier and/or the ADC. The program assumes \( t = 0 \) at the starting cursor, and returns the oscillation amplitude at that point. It also returns the resonant frequency and damping constant scaled by \( 2\pi \), i.e., it returns \( f_0' = \omega_0/2\pi \) and \( \Delta f = \gamma/2\pi \). Check the graph of residuals to make sure the fit was successful. If it was not, the starting guesses for the fit parameters may need to be closer to correct. The time scale must be expanded considerably to see the fit.

The Save button on this tabbed page writes a single row of data containing the Run #, an Excel time stamp giving the date and time right after the ring down, the temperature readings before and after the ring down, the y-scale factor, and all fitting parameters and their sample standard deviations. Supply a new file name for the first set of results and you can repeatedly save to it; the program will append one new row each time. The file must not be open in another program when you try to write a new row.

The Acquire and Analyze Transients program can also perform a Fourier transform of the ring up or ring down and, for ring downs only, can perform fits to expectations for these transforms. This kind of analysis greatly reduces the number of points needed in the fit at the expense of the extra step to compute the transform. The instructor can show you these features if you are interested and an addendum on the subject is on the web site.

**Apparatus**

Figure 9 (at the end of the write-up) is a schematic drawing of all relevant cryogenic components. It is not to scale and does not include all gauges in the gas handling manifold. Refer to it for valve and other component locations.

**The Suck Stick Cryostat**

The suck stick is inserted into the neck of the liquid helium dewar as shown in Fig. 9. It is designed to hold a small volume of liquid helium that can then be brought under vacuum conditions. An insert inside the suck stick holds the tuning fork and thermometer at the bottom with wiring to electrical feedthroughs at the top. The suck stick and insert comprise the cryostat.

The suck stick is an invention of low temperature researchers here at UF. The design principle is simple. Insert the stick in a liquid helium dewar, pump on the volume inside the stick and the pressure difference will suck liquid helium from the dewar through the capillary and into the volume. The length and diameter of the capillary are chosen to give a mass flow conductance that is neither too high...
nor too low. The flow rate of liquid helium entering the volume should be just about equal to the evaporation rate from the thermal load on the volume.

If the flow rate is too low, the volume will never fill. If it is too high, the volume will overfill and the incoming liquid will be at a higher temperature—closer to the ambient 4.2 K temperature in the dewar than the lower temperature in the volume. When the conductance is just right, the liquid flowing into the volume will just make up for the amount of helium gas being pumped away. Moreover, the capillary will have a temperature gradient such that the incoming liquid helium will be at the temperature inside the volume.

Various low temperature techniques are used to keep the heat load low. Most importantly, there is a vacuum jacket around the volume to insulate it from the 4.2 K environment inside the dewar. A few torr of helium gas can be let into the volume to increase the heat conductance, but only when the apparatus is being cooled down from or warmed up to room temperature. The helium gas must be pumped out of the jacket once the apparatus is cold so as to insulate the experimental volume from the 4.2 K liquid in the dewar. Adding helium gas to the vacuum jacket and then removing it does not save much time, and so we simply keep the vacuum jacket evacuated throughout the experiment.

There are radiation baffles along the inner volume to minimize radiative energy barreling down from the top of the suck stick where the temperature is near ambient. In addition, the materials used, such as stainless steel, polycarbonate and phosphor-bronze wiring and coax are chosen for their low heat conductance or low heat capacity.

When the liquid helium first enters the volume, it quickly evaporates—cooling the contents until they are below 4.2 K, at which point entering liquid begins to pool inside the volume. Above the pooling liquid is helium in the gaseous state. As the gas is pumped away and the pressure above the liquid decreases, the liquid cools further. The gas reaches a steady state pressure that depends on the pumping speed and the heat load. The liquid will ultimately reach a steady state temperature for that pressure as determined by the temperature dependence of the condensation and evaporation rates. The relationship between the equilibrium vapor pressure and the liquid helium temperature is shown in Fig. 10.

Thus the temperature of the liquid helium can be adjusted by changing the vacuum pumping speed. The pumping speed is changed by partially opening or closing valves 6 and 7 in the plumbing lines from the vacuum pump to the experimental volume.

**Pressure Meters**

The three main pressure meters all use different units and none are the SI unit of pascal (Pa) for which 1 standard atmosphere (atm) is 101325 Pa.

The main vacuum meter for the experimental volume is the Bourdon-type Matheson gauge which works off the pressure difference.
inside and outside a spiral-shaped tube. It reads in torr (1 atm is 760 torr) and can be calibrated with a two point procedure. First, make a reading \( P_{\text{atm}} \) with the inlet opened to the room. This reading should be about 760 torr and would be independent of the actual room pressure. The actual atmospheric pressure \( P_{\text{atm}} \) can be obtained from the physics department weather station web site where it is labeled inHg (inches of mercury). The conversion factor is a 25.4 torr/inHg. Next, pump the air out of the Matheson gauge until the thermocouple gauge bottoms out. The true pressure \( P_0 \) and the thermocouple reading should be well below 0.3 torr and, if so, \( P_0 = 0 \) should be an accurate approximation. Record the Matheson reading \( P_{\text{meas}}^0 \) at this pressure. The actual pressure \( P \) in terms of the gauge reading \( P_{\text{meas}} \) is then

\[
P = \frac{P_{\text{atm}} - P_0}{P_{\text{meas}} - P_{\text{meas}}^0} (P_{\text{meas}} - P_{\text{meas}}^0) + P_0 \quad (88)
\]

The thermocouple gauges read in torr up to a maximum of 2 torr. The meter reading may go above 2 torr at higher pressures, but these readings are very inaccurate and essentially useless. A thermocouple gauge depends on the thermal conductivity of the residual gas and reads differently for different gases at the same pressure. It is calibrated for air, but don’t try to make corrections for helium when recording readings. All values given in the instructions are raw readings.

The diaphragm-type Magnehelic gauge on the helium dewar reads the amount the dewar pressure is above atmospheric pressure and is in inches of water (1 atm is about 407 inches of water).

**Initial observations**

There are a lot of ways to explore the apparatus and the physics of the tuning fork to be sure everything is working properly and well understood. The following sections describe one regimen that should help you fulfill these goals. It starts with measurements that do not require liquid helium.

1. Check how the DS340 function generator works. Set the DS340 for “High-Z” mode (Shift then 6 key). Set it for 10 kHz sine wave with an output amplitude of 0.2 V. (Remember to set the amplitude in rms volts. Check the indicator to be sure.) Simultaneously look at the function generator waveform output and sync output on a two-channel oscilloscope. Trigger on the rising edge of the sync. What is the rough phase difference of the waveform’s rising zero-crossing relative to the rising edge of the sync? Express it in degrees and note whether it leads (occurs before the sync crossing) or lags (occurs after the crossing). Does the phase difference change as you change the frequency to 1 or 100 kHz?

2. Set the frequency to 33 kHz. This is near the frequency needed to measure the tuning fork response.

3. Check out what the lock-in does. Set the lock-in time constant to 1 s with a 24 db/octave slope. Set the sensitivity to 0.5 V with no line filters in and set the Reserve to Normal. Set the input for A, DC Coupling and Ground. Connect the DS340 sync signal to the lock-in reference input and its output to the lock-in A input. Set the reference channel for rising edge and set the lock-in phase offset to 0°. Set the front panel displays to \( x \) and \( y \) (\( V_x \) and \( V_y \)). Record the \( V_x \) and \( V_y \) and then change the display to \( R \) and \( \theta \) (\( V \) and \( \phi \)) record these values. In particular, note the sign of the \( \theta \) in comparison to whether the input led or lagged the sync.
4. Hit the **Auto Phase** button. This button adjusts the phase offset to make the input signal in phase with the reference (after the reference has been shifted by the phase offset). Record the new phase offset and values for $x$, $y$, $R$, $\theta$.

5. Change the DS340 amplitude to 0.1 V and record how long it takes the lock-in to settle to the new correct amplitude. Set the lock-in time constant to 1 ms and change the DS340 amplitude back to 0.2 V to see how long it takes now to react to a quick change in amplitude. Set the lock-in time constant back to 1 s.

6. Check how the shunt resistor affects the function generator output. The shunt resistors are located in the interface box. Connect the output of the function generator to the bnc labeled **Signal Input** on the interface box. The function generator output with the effect of the shunt resistor is then also available at the **Input Signal Monitor** bnc on the interface box. Connect it to the lock-in input. The **Signal Input/Input Signal Monitor** bncs are connected to the rotary switch labeled **Input Signal Attenuation**. In the $\times 1$ position there is no shunt, $\times 0.1$ puts in a $5.6 \, \Omega$ shunt, and $\times 0.01$ puts in a $0.5 \, \Omega$ shunt. Set the function generator amplitude to 1 V. Predict the lock-in outputs for each shunt position, adjust the lock-in sensitivity and record the results.

7. Study how the transimpedance amplifier works. Leave the function generator connected to **Signal Input**, but move the lock-in input so it measures the output of the transimpedance amplifier—the bnc connector labeled **Output Signal Monitor**. Adjust the **Device Selector** switch for the 100 k$\Omega$ resistor. This will connect a 100 $\Omega$ resistor in place of the tuning fork in the circuit diagram of Fig. 6. Predict the lock-in outputs, adjust the lock-in sensitivity and record the results. Is the current in-phase with the drive voltage? Should it be? Why?

8. Hit the lock-in **Auto Phase** button, record the new lock-in outputs and phase offset. How does the phase offset change? Why?

9. Switch the **Device Selector** to the 220 pf capacitor. Do not adjust the phase offset. Predict the lock-in $x$ and $y$ outputs, adjust the lock-in sensitivity and record the results. Does the capacitor current lead or lag the drive voltage?

### Frequency scans

10. Switch the **Device Selector** to the tuning fork still sealed in its canister (labeled **Vacuum** on the **Device Selector**).

11. Set the lock-in sensitivity to 20 mV and the time constant to 10 ms. The lock-in time constant is being kept short to isolate the effects of the tuning fork time constant.

12. Set the DS340 drive amplitude to 1 V. Be sure the 0.5 $\Omega$ shunt ($\times 0.01$ on the **Attenuator** switch) is connected so the actual drive level is now around 10 mV. Find the resonance by manually adjusting the DS340 frequency around 32760-32770 Hz, further homing in on the frequency where the lock-in $R$ maximizes; the amplitude should be on the order of 5 mV on resonance.

13. Change the drive amplitude to 2 V and record how long it takes for the lock-in to settle at the new equilibrium value.
14. Set the DS340 for remote mode so it can communicate with the computer via the GPIB interface. To do so, press the **SHIFT** key then the **GPIB** (1 key) and then set it to **on** with the arrow keys. (The lock-in powers up ready for GPIB communications.)

15. Launch the Frequency Scan program. Set the frequency start and stop so that the sweep covers a range of ±3 Hz around the resonance. Set the sweep direction for forward. Set the frequency step to 0.05 Hz and set the program’s step time (between setting the frequency and measuring the lock-in values) to $\Delta t = 1$ s. Repeat for a reverse frequency scan. Use the Compare Runs program to see forward and reverse scans on one graph.

16. Repeat forward and reverse scans with both a smaller $\Delta t = 0.2$ s (that’s about the shortest $\Delta t$ possible with this program) and another with a longer $\Delta t = 8$ s.

17. Decide which of these six runs should agree with predictions and analyze them. (See Data acquisition and analysis section for instructions.)

18. Repeat some or all of the measurements above with the unsealed tuning fork opened to the atmosphere (labeled Air on the Device Selector switch). But first make some predictions. Will the resonance frequency increase or decrease? Will the damping constant increase or decrease? Would you expect to be able to scan faster or slower? Will the FWHM of the resonance increase or decrease? Find the new frequency and then adjust the drive amplitude to get a lock-in amplitude around 5 mV. How should you adjust the frequency step size and range to get a data set with similar number of points in the resonance peak? Make some measurements as you try to roughly determine the shortest $\Delta t$ that gives accurate steady state results. Also determine a reasonable frequency range and step size. Get a good data set and analyze it to get the fitting parameters.

Aesthetics and tradition dictate that an independent variable be scanned in steps that are a member of the 1, 2, 5 sequence. For example, a frequency scan might have steps of 0.02, 0.1, 0.5 Hz, but not 0.04 or 0.25 Hz.

Keep in mind that using a larger lock-in time constant reduces noise. However, the lock-in time constant also affects how long it takes for the output phasor to be an accurate representation of the input voltage. Typically, a wait of around 7-10 lock-in time constants is recommended. Generally, try to use the longest possible lock-in time constant that is consistent with this rule. For example, if your time step is 1 s, use a lock-in time constant of 0.1 s. Also do forward and reverse scans when in doubt about the scan parameters and/or as a general check on the scan conditions.

**Ring downs**

19. Set the lock-in to the 50 mV range. Switch to the 5.6 $\Omega$ shunt resistor ($\times 0.1$ on the Attenuation switch. Perform and analyze ring down measurements on the sealed and unsealed tuning forks attached to the selector switch. When setting up, adjust the frequency close to resonance (maximum amplitude on the lock-in) and then adjust the drive amplitude to get around a 10 mV amplitude on the lock-in. For the tuning fork in vacuum, this should be around 0.2 V and it should be around 3 V for the tuning fork in air. When adjusting the ring up frequency or ampli-
Quartz Crystal Tuning Fork in Superfluid Helium

C.Q. 1 Compare the results of the frequency scans and ring downs and discuss their pros and cons for determining $\omega_0$ and $\gamma$. Describe how to determine the motional resistance $R$ and do so for both tuning forks in the interface box.

You will find ring downs are the best way to obtain $\omega_0$ and $\gamma$. The big advantage is that data for a ring down takes only a few seconds to collect, while frequency scans can require several minutes. When working with a tuning fork in liquid helium, keeping the temperature stable during a resonance scan is not always easy with our apparatus. The amplitude of the motion needed for measuring a ring down will be moderately higher than for frequency scans, but the small amplitude approximations assumed in the analysis still appear to be satisfied.

CHECKPOINT: Frequency scans and ring downs should be measured and analyzed for the two tuning forks in the interface box. You should understand how to adjust data acquisition parameters depending on damping constants and how to adjust the lock-in phase setting so that circuit currents that are in phase with the drive voltage show up in the lock-in’s in-phase or $x$-output.

Cryogenic Measurements

Precautions

Safely working with liquid helium requires patience and some experience. Consequently, this experiment requires an instructor’s presence any time the cryostat is in use.

Frostbite is, of course, one danger. So take care not to play with liquid helium or touch objects recently chilled in it.

Liquid helium evaporates quickly and can build up to dangerous pressures when confined in a sealed volume. There are relief valves set at 1 or 2 psi on all components so that should a volume become sealed off, as soon as the pressure begins to build, the relief valve will open and prevent an over-pressurization that might lead to an explosion. Nonetheless, report any escaping gas sounds to the instructor immediately. More importantly, be sure you understand the plumbing and valves used for vacuum handling and helium recovery.

Be sure to explain any odd or confusing occurrences to the instructor. Consult with the instructor if vacuum pressures do not reach requested values—leaks in the vacuum system might be the problem and can lead to additional problems if left unrepaired.

Cool down and warm up procedures referred to in this section are described later.

20. If the experimental insert is inside the suck stick, lay the suck stick horizontally on the lab bench, remove the sealing clamp, and take care sliding the insert straight out so as not to bend the thin-walled stainless steel tubing or break any of the delicate wiring inside. Examine the insert checking that the tuning fork and thermometer are properly mounted on the polycarbonate mounting platform. Check that their electrical connections have not come loose.

21. Give the gasket a very light film of vacuum grease, carefully reassemble the cryostat insert into the suck stick, reclamp it and leave it horizontal on the lab bench.

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22. Find the resonance frequency and perform a ring down on the tuning fork in the cryostat to check it is operating properly. Record the resonance frequency and width parameters.

23. Run the cool down procedure (see separate section) to Step 46. While waiting with the cryostat just inside the dewar neck, measure the resonance frequency and damping constant as a function of helium pressure. Try to get around 10-15 points from 0-760 torr, with three or four from 0-100 torr. Close values 6 and 7 and then slowly open valve 8 to fill the cryostat to around 760 torr. Take a measurement there and then take additional measurements as you slightly open valve 7 and then close it each time the pressure drops to the desired point. You can write the pressure into the Run # control so it will be written to the file row containing the fitting parameters.

24. Plot $\omega_0$ and $\gamma$ vs. pressure to check the data.

25. Go back to the cool down procedure: Step 47 and begin lowering the cryostat as described there. Continue pumping and while the cryostat cools from 300 K, monitor the resonance frequency. You can do this manually by adjusting the frequency for a maximum on the lock-in. You may need to adjust the amplitude as well. Acquire and analyze ring downs as the temperature dependence lowers the resonance to about 32710 Hz at liquid helium temperatures. The temperature monitor program must be shut down in order to run the acquire and analyze transient program. Don’t be concerned that the temperature is falling during the measurements. Take particular care to get measurements at several temperatures from 2-10 K.

26. The temperature goes below 2 K when the first liquid helium enters the experimental volume and evaporates. About 5-10 minutes later, enough will have accumulated to submerge the tuning fork. Before this happens, be sure to acquire and analyze a few ring downs under vacuum conditions at the lowest temperatures. These measurements will be used to get $\omega_{00}$ and $\gamma_0$.

27. Once the liquid hits the tuning fork, the resonance around 32710 Hz will no longer be achievable. About 5-10 minutes later, the liquid helium should have risen enough to submerge the tuning fork and the temperature should be close to the minimum achievable—around 1.6 K. The apparatus is now ready to make measurements.

**Varying the temperature**

Molecules in the gas phase continuously condense on the liquid surface and molecules in the liquid continuously evaporate into the gas phase. The rates at which molecules evaporate and condense will only be equal at one particular pressure, called the vapor pressure of the liquid. If the gas pressure is below the vapor pressure, the evaporation rate will be higher and the liquid will cool. If the gas pressure is above the vapor pressure, the condensation rate will be higher and the liquid will warm. In these two cases, the liquid is not in equilibrium. Only if the gas pressure equals the vapor pressure will the rates be in equilibrium with no net transfer of molecules or energy into or out of the liquid.

Vapor pressure increases with increasing temperature, and for liquid helium, it varies...
from a few Torr at 1.5 K to 760 Torr at 4.2 K. The temperature is adjusted by partially opening or closing the gas-handling valves to change the pumping speed of the vacuum system and thus the helium gas pressure above the column.

The experimenter must be wary because there can be a distribution of temperatures within the column of liquid helium with the top and bottom at different temperatures. The length of the column, whether one is increasing or decreasing the temperature, and how long one waits after making changes play a role in the distribution. The thermometer is near the tuning fork, but the assumption that they are at the same temperature may not always be accurate. Thus it is generally wise to make tuning fork measurements only after waiting long enough for a stable thermometer reading. This should be the case if the vapor pressure for that temperature is equal to the actual gas pressure. The actual pressure can be read from the Matheson gauge and should be compared with the vapor pressure calculated by the data acquisition program based on the thermometer temperature. Be sure they agree before taking data.

The heat capacity and thermal conductivity of liquid helium are temperature dependent over the 1.5-4.2 K range explored in this experiment and affect how the liquid will cool down or warm up. Below 2.18 K, the superfluid helium is basically an ideal conductor and a uniform temperature distribution is just about guaranteed. You can raise or lower the pressure and be fairly certain that the thermometer, tuning fork, and liquid all come to the same temperature within a few minutes. Above 2.18 K, liquid helium is a relatively poor conductor and changing the pressure on the column will change the temperature of the liquid at the top, but it can take some time for heat to conduct up or down the column and get the whole column at the equilibrium temperature.

Above 2.18 K, if the gas pressure is below the current vapor pressure, the liquid can boil. Boiling will not occur below the transition temperature. Boiling mixes the fluid and helps achieve a uniform temperature distribution, but will only occur when the gas pressure is being lowered. When the gas pressure is being increased to raise the liquid helium temperature, boiling will not occur and the heater may be needed to get to equilibrium in a reasonable time. The heater is particularly needed when raising the temperature significantly and more so when the liquid column is higher.

28. With valves 6 and 7 fully open, the temperature should be near the minimum possible—around 1.6 K. Take a ring down there and then decrease the pumping speed to raise the temperature. Start by closing valve 6 and see what temperature that gets you to. Take a measurement there.

29. Decrease the pumping speed in increments by partially closing valve 7. Each partial closure decreases the pumping speed, which should cause the temperature to rise. Wait for the new temperature/pressure settings to stabilize before making a measurement. Try to get around 10-15 measurements in the range 1.6-2.18 K, with at least three between 2.1 and 2.18.

30. **Caution.** Be careful not to close valve 7 all the way while valve 6 (and 8) are also closed. Closing valves 6, 7, and 8, isolates the experimental volume and as the helium evaporates, the pressure will build until the pop-off valve activates. More importantly, the Matheson gauge will also
be over-pressurized and could get damaged.

31. Of course, measurements must not be made while the liquid helium is boiling. Experience with the apparatus has demonstrated that the best way to make measurements in the 2.2-4.2 K range is to start at 2.2 K and go monotonically higher. Since the temperature and pressure are always rising, no boiling should occur. For each desired temperature use the data acquisition program to determine the vapor pressure there. Then slowly decrease the pumping speed by continuing to incrementally close valve 7 to get the Matheson gauge pressure equal to that vapor pressure. These adjustments take patience and practice. Don’t try to rush them. Either wait for the temperature to rise to equilibrium or help it along with the heater taking care not to overshoot.

If the temperature does overshoot, one can simply keep the pressure at the desired set point and wait for the temperature to come back down into equilibrium. However, boiling may occur during the wait and it not obvious when the boiling is over and it is safe to make a measurement. To be safe, one can purposely lower the pressure significantly below the desired value by opening valve 6. Boiling will occur, but once the temperature has gone far enough below the desired set point, close valve 6 and readjust valve 7 to achieve the desired pressure. Again, wait for the temperature to stabilize or help it along with the heater.

32. Try to get at least 15 measurements between 4.2 and 2.18 K with at least three in the range between 2.18 and 2.3 K.

Analysis

You have two data sets of values for $\gamma$ and $\omega_0$: one with the fork in helium gas nearer room temperature as a function of pressure; the other in liquid helium as a function of temperature. Make plots of this data and of the corresponding $F$ and $G$.

Look up the viscosity of gaseous helium and learn about why it is independent of pressure. Use the ideal gas law to express the mass density in terms of the pressure and rewrite the predictions in terms of the pressure $P$ in the form

$$F = \alpha_1 P + \alpha_2 \sqrt{P/\omega_0} \quad (89)$$

and

$$G = \alpha_3 \sqrt{P\omega_0} \quad (90)$$

and give expressions for each $\alpha$. Use linear regression to determine the $\alpha$’s and use the $\alpha$’s to determine the near-unity integration parameters $\beta$, $B$, and $C$.

Analyze the data set in liquid according to the tuning fork/liquid helium model. There is a Liquid Helium.xls Excel spreadsheet with functions for the total density—lhedensity(T), superfluid density—shedensity(T), and viscosity—lheviscosity(T) already written and available to create the necessary columns for a linear regression to the $F$ and $G$ functions as described in the theory sections. The columns you should fit to are $\rho$ and $\sqrt{\eta\rho/\omega_0}$ for $F$ and $\sqrt{\eta\rho_0\omega_0}$ for $G$. Use the fitting coefficients to again determine the near-unity integration parameters $\beta$, $B$, and $C$.

Compare the two sets of values for $\beta$, $B$, and $C$. 

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Procedures

Cooling down

The following steps demonstrate how to bring the cryostat from room temperature to liquid helium temperatures. This procedure should only be attempted with an instructor present, or with the instructor’s permission. If, during any of the steps, something doesn’t look or sound correct, ask the instructor for help, for example, if you see a pressure rising or falling unexpectedly, or if you hear the hissing sound of escaping gas.

Do not perform this procedure until the appropriate time. It is possible that some of the steps may already be completed. Also keep in mind there may be measurements that need to be recorded or other experimental procedures that need to be performed at particular steps in this procedure.

33. When the dewar is brought up from the cryo-shop, connect the dewar’s venting hose to the helium recovery line and open valve 9. This connects the dewar’s helium volume to the recovery line. Valve 9 should stay open at all times unless you are trying to pressurize the dewar. Helium evaporates vigorously inside the dewar when inserting the room-temperature suck stick and without the outlet path to helium recovery, the dewar would overpressurize very quickly. There is a relief valve that opens when the dewar pressure rises above a few psi and should protect the dewar from overpressurization, but it should not open unless a mistake has been made.

34. Install and tighten the large hose clamp around the suck stick about 15 cm up from the bottom, a centimeter or two above the “fully raised” line drawn around the suck stick. Rub a very small amount of vacuum grease on the o-ring in the brass dewar flange and slide the flange (one piece at a time is easiest) up the suck stick until it butts up against the hose clamp. Tighten it to increase the friction at the o-ring and check that it will not slide off when the suck stick is vertical.

35. Mount the suck stick/cryostat insert on the stand near the dewar. Hook up the 5/8 inch vacuum hose between valve 3 on the manifold and valve 4 on the suck stick’s insulating vacuum space. Hook up the 1 3/8 inch vacuum hose from the 3/4 inch copper pipe on the suck stick to the gas handling manifold inlet as shown in Fig. 9.

36. Close valves 2 through 8. Make sure valve 1 is set to the room vent position so the vacuum pump exhausts into the room.

37. Turn on the vacuum pump. It should start gurgling and quiet down in a few seconds.

38. Open the main roughing valve 2. Again, the pump should gurgle and quiet down within a few seconds. Check that the thermocouple gauge gets below 100 mtorr. Record its value.

39. Open valves 3 and 4 to pump out the insulating vacuum space. Record the final thermocouple gauge reading and check that it gets below 100 mtorr. Close valve 4, then valve 3. The vacuum jacket is now evacuated and will be left that way until it is time to take the cryostat out of the dewar.

40. Open valve 6 to pump out the experimental volume. Record the final thermocouple gauge reading and check that it gets below 200 mtorr.

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41. In this step the remaining air in the experimental volume is removed by displacing it with helium and then pumping it away. Close valve 6 to stop the pumping and then slowly open valve 8 to let helium from the recovery line into the experimental volume. Watch the Matheson gauge and close valve 8 when the pressure gets in the range 600-760 torr. Slowly open valve 6 to pump out the helium.

42. Continue pumping while you install the suck stick into the helium dewar as follows. Be sure to have two people for the task. While one person lifts up and holds the suck stick ready for insertion near the dewar opening, the other person should quickly open the dewar cap, place the cap and sealing clamp on the table, and make sure the rubber sealing gasket stays properly positioned on the dewar neck. Then that person should immediately lend a hand positioning the suck stick into the dewar opening, making sure the rubber gasket stays properly positioned and tightening the clamp ring to seal the brass flange to the dewar neck. The suck stick should be stable but still just inside the dewar neck extending about four feet above it.

43. Repeat step 41 now with the suck stick in the dewar.

44. The experimental volume and the insulating vacuum space should still be under vacuum conditions with valve 6 still open. Check and record that the thermocouple pressure is below 200 mtorr.

45. Move valve 1 to the helium recovery position. Now all gas exhausted from the vacuum pump will be going back to the helium liquefier facility.

46. Wait in this configuration for at least 10 minutes to be sure any air in the capillary is expelled. Air in the capillary could freeze solid and prevent the flow of helium. This is a good time to make measurements near room temperature as a function of helium pressure.

47. Close valve 2 and record the time for the thermocouple gauge reading to go off-scale (over 2000 mtorr). This should take about 30-50 seconds. If the pressure rises too slowly, it could be an indication that the capillary is clogged. If it rises too quickly, there may be a leak in the system. Check with the instructor if you think there may be a problem. Open valve 2.

48. Launch the Temperature Monitor program.

49. The suck stick can be sucked down quickly into the dewar due to the vacuum conditions and its own weight. So hold it tightly whenever it is free to move (as when lowering it) and be sure to tighten the hose clamp properly so it can’t fall on its own. Holding the suck stick tightly, loosen the hose clamp around the suck stick. Slightly loosen the brass flange to adjust the o-ring friction. It should be hand-tightened once the suck stick is in position, but during the height adjustment it should be slightly loosened. Not enough to let helium blow by, but just enough to make it easier to move. Slowly lower the suck stick into the dewar until the Magnehelic pressure meter on the dewar just starts to rise. Tighten the hose clamp. Keep this dewar pressure below 20 inches of water as you continue lowering the suck stick about 10 cm at a time. The pressure will rise as the warm suck stick evaporates helium in the dewar and
it will fall as the suck stick cools down. Raising the suck stick should also lower the dewar pressure. Raise it if necessary to keep the pressure below 20 inches of water. (It is OK if it goes a bit above.)

50. Continue lowering the suck stick while maintaining a dewar pressure below 20 inches of water until it is fully inserted with about 10-15 cm between the top of the brass flange and the first vacuum fitting. Remember to tighten the brass flange and the hose clamp.

51. Open valve 7 for the tiny additional pumping you will get though that valve. The thermometer should be around 250-300 K and slowly dropping. Check that valves 2, 6 and 7 are open.

52. Check the cryoshop web site at www.phys.ufl.edu/~cryogenics/cryonet.htm and click on the UGLab helium gas meter. Make sure the helium gas meter reading is rising. The counter reading is in units of about 0.01 liquid liters. It take 2-3 liters to cool down the suck stick and about 0.1 liter per hour to maintaining the temperature.

The liquid helium coming in through the capillary will quickly evaporate and bring down the temperature. It takes about an hour for this spray to cool the insides below 2 K. After this point, the liquid will begin pooling in the experimental volume.

The Temperature Monitor program not only gives the thermometer reading, it also shows the predicted pressure based on equilibrium with the liquid. The measured pressure on the Matheson gauge should be in close agreement if liquid is present and the liquid column is only a few millimeters thick or it is in equilibrium throughout its length.

The helium level should rise in the experimental volume at a rate of a few cm/hour. It could take half an hour or more before the tuning fork is submerged. This is a good time to make measurements of $\omega_0$ and $\gamma_0$.

**Stand by, Warm up**

The experimental volume will probably contain liquid helium at this point. If it is allowed to warm up too quickly, the evaporating helium could cause an overpressurization. Keep an eye on the Magnehelic and Matheson gauges and take special care to have the valves correctly opened or closed during the procedure.

**Stand-by:** This step is for when you want to leave the suck stick cold, overnight inside the dewar.

53. Close valves 6 and 7 to isolate the inlet side of the vacuum pump, and then move valve 1 to the room vent position to isolate the outlet side from helium recovery.

54. Slowly open valve 8 to vent the experimental volume to the helium recovery line.

55. Turn off the vacuum pump and open valve 5 to vent the pump to room air. Close valve 5.

56. To restart from stand-by conditions, first turn on the pump and wait for the thermocouple gauge to go below 100 mtorr, then switch valve 1 to the helium recovery position.

**Warm up:** The warm up procedure is for removing a cold suck stick from the dewar and returning the dewar to the cryogenics shop.

57. Close valves 6 and 7 and slowly open valve 8 to vent the experimental volume to the recovery line and leave it open.

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58. Slowly raise the cryostat about 10 cm/minute until it is all the way up with only 15-25 cm still inside the Dewar neck. Keep an eye on the Matheson and Magnehelic gauges for overpressurization. Lower the suck stick if the Matheson gauge goes over 770 torr and then continue raising it after the pressure reduces.

59. Leave the suck stick, fully raised, a few hours to warm up.

60. You can decrease the warm up wait to about an hour by adding a little helium gas to the insulating volume. Check with the instructor before trying this. Make sure that the cryostat has no liquid helium by waiting until the temperature is above 10 K before adding the gas. You can use the internal heater to speed this part up, but turn it off once the temperature goes above 4.3 K. Valves 6 and 7 should be closed and valve 8 should be open. Open valve 3 to evacuate the blue vacuum hose and wait until the thermocouple reads below 200 torr. Power up the thermocouple gauge on the insulating volume and keep an eye on it. Close valve 2 to isolate the pump, crack open valve 7 and then close it to let a small amount of gas into the hose, then close valve 3 to isolate the hose volume. Open valve 2 so the pump is ready for use. Then briefly open and close valve 4 to add a few torr of helium gas to the vacuum jacket. The outside surface of the cryostat should quickly frost over because of the increased heat conduction between the experimental volume and the room. Open valve 3. As the cryostat warms up, the pressure in the insulating volume will rise. Keep the reading on the thermocouple gauge below 100 torr by opening and closing valve 4 as needed. You can further decrease the warm up time by blowing hot air around the brass flange and dewar neck while monitoring the temperature.

61. The thermometer is not accurate near room temperature, but it will continue to give readings. When it reads above 290 K, the cryostat is warm enough to remove from the dewar.

62. Removing the cryostat from the dewar is another two-person job. Make sure the brass flange is tightened around the suck stick. As one person holds the suck stick, the other should release the clamp around the dewar neck and be ready with the dewar cap. As the suck stick and brass flange are lifted away from the dewar, be sure the rubber gasket stays on the dewar neck and replace the cap and clamp it in place to close off the dewar. Carefully position the suck stick on the holding stand and clamp it in place.

63. If you only wanted to get the suck stick out of the dewar, you are done. Leave the dewar connected to the recovery line. If you want to return the dewar to the cryogenics shop, disconnect the dewar hose from the recovery line. You can leave valve 9 open. The hose self seals when it is disconnected. But now the dewar is sealed and isolated. The helium inside will pressurize the dewar as it evaporates. So get it down to the shop immediately and reconnect it to a recovery line down there. Make sure valve 9 is open.

Comprehension Questions

1. Summarize the data and explain what you learned in the initial observations.
2. Make plots of \( f_0 \) and \( \Delta f \) vs. \( T \) and of \( \mathcal{F} \) and \( \mathcal{G} \) vs. \( T \) and fit the latter two to the theory. Are the deviations reasonable? Do you see any evidence of systematic differences between the theory and the data?

3. How does a liquid at the bottom of a long column of liquid helium cool down when the pressure at the top is reduced? How does the liquid at the bottom warm up when the pressure is increased?
Figure 9: Vacuum, gas handling, dewar and cryostat.