Homework H

Instructor: Yoonseok Lee

Submit only HW's. EX's are additional problems that I encourage you to work on.

(**a**.**b**) means problem number **b** of chapter **a** in *Introduction to Solid State Physics* (8th ed.) by Kittel.

Use SI unit.

Due April 19

HW 1: (10.1) (10 pt)

HW 2: **Cooper pair problem:** Answer the following questions after reading L.N. Cooper, Phys. Rev. **104**, 1189 (1956).

In Cooper's model, the two electrons are added right outside of the Fermi surface at $T = 0$ in an L^3 box. The interactions between the electrons in the Fermi sphere as well as the interactions between the additional electrons with with Fermi sea are ignored. Only the spin-independent (weak attractive) interaction between the additional electrons $V(\vec{r}_1, \vec{r}_2)$ is considered. Then, the two-body Schrödinger equation can be written as:

$$-\frac{\hbar^2}{2m}\left(\nabla^2_{r_1} + \nabla^2_{r_2}\right)\psi_{\alpha,\beta}(\vec{r}_1, \vec{r}_2) + V(\vec{r}_1, \vec{r}_2)\psi_{\alpha,\beta}(\vec{r}_1, \vec{r}_2) = (\epsilon + 2E_F)\psi_{\alpha,\beta}(\vec{r}_1, \vec{r}_2),$$

where α and β are spin indices. Recognizing translational invariance, $V(\vec{r}_1, \vec{r}_2) = V(\vec{r}_1 - \vec{r}_2)$. In general, the two body wavefunction is given by

$$\psi_{\alpha,\beta}(\vec{r}_1, \vec{r}_2) = e^{i\vec{K}\cdot(\vec{r}_1+\vec{r}_2)/2}\phi(\vec{r}_1 - \vec{r}_2)\chi(\alpha, \beta).$$

Here, $\vec{K}$ is the total momentum and $\chi(\alpha, \beta)$ is the spin part of the wavefunction.

(a) Using $\phi(\vec{r}_1 - \vec{r}_2) = \frac{1}{V}\sum_{k>k_F} g(\vec{k})e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}$, show that

$$\frac{\hbar^2}{2m}\left\{(\vec{k} + \vec{K}/2)^2 + (-\vec{k} + \vec{K}/2)^2\right\} g(\vec{k}) + \sum_{k'>k_F} g(\vec{k'})V_{\vec{k},\vec{k'}} = (\epsilon + 2E_F)g(\vec{k}),$$

where $V_{\vec{k},\vec{k'}} = \frac{1}{V}\int V(\vec{\rho})e^{-i(\vec{k}-\vec{k'})\cdot\vec{\rho}}d^3\rho$. (10 pt)

(b) Show that the kinetic energy of the two electrons is minimum for $\vec{K} = 0$. (5 pt)

(c) Therefore, one can choose $\vec{K} = 0$. For

$$V_{\vec{k},\vec{k'}} = -V_o \quad \text{only for} \quad E_F < \frac{\hbar^2 k^2}{2m}, \frac{\hbar^2 k'^2}{2m} < E_F + \hbar\omega_D,$$

derive the following relation:

$$1 = V_o N(E_F) \int_{E_F}^{E_F+\hbar\omega_D} \frac{dE}{2E - \epsilon - 2E_F}. (10pt)$$

(d) As discussed in class, the above result leads to the existence of a bound state for an arbitrarily weak attractive interaction, which is directly related to the presence of the Fermi surface. Discuss the consequence of allowing the summation for all $\vec{k}$, not limited to $\vec{k} > \vec{k_F}$. (10 pt)

HW 3: Read Kittel Ch.10. Starting from Eq. (19) follow the calculations to Eq. (27) quantization of fluxoid. (10 pt) You may find the following paper interesting: R.D. Parks and W.A. Little, "Fluxoid Quantization in a Multiply-Connected Superconductors, Phys. Rev. **133**, A97 (1964).