Test II, PHY4905, Chapters 6-10

Spring, 2016

3 significant figures unless otherwise stated on all answers please

Constants: N (Avogadro’s number)=6.022 1023; ħ (Planck’s constant)=1.055 10-34 J-s

1 J=0.6242 1019 eV; m(electron)=9.11 10-31 kg; kB=0.86167 10-4 eV/K = 1.38044 10-23J/K

1. An metallic element has a density of 3.51 g/cm3 and a molar volume of 39.1 cm3. The element is divalent, i. e. each atom of the element contributes two electrons to the Fermi sea.

a.(1 pt) What is the mass per mole, in g (4 sig figs)?

 137.2

 b. (2 pts) What is the density of electrons, N/V, per unit volume, in units of

 electrons/cm3?

A mole of the material takes up 39.1 cm3, so N/V=2\*6.022 1023 /39.1 cm3 = 3.08 1022 e/cm3

 c. (4 pts) Using the free electron model, calculate the specific heat gamma (γ) in

 units of mJ/molK2.

Kittel, eqn. 34, Cel=1/3 π2D(εF)kB2T where, eqn. 20, D(εF)=[V/(2π2)](2m/ћ2)3/2εF1/2 and eqn. 17, εF=(ћ2/2m)(3π2N/V)2/3

so γ=1/3 π2kB2[V/(2π2)](2m/ћ2)3/2(ћ2/2m)1/2 (3π2N/V)1/3

 =1/6 kB2 V (2m/ћ2) (3π2N/V)1/3 =

1/6 (1.38 10-23 J/K)2 39.1 cm3/mole (2x9.11 10-31 kg/1.0552 10-68 J2s2)(3π23.08 1022e/cm3)1/3

203.15 \* (912)1/3 10-46 1037 107 J2/K2 cm3/mole kg J-2 s-2 cm-1 =

1970.1 10-2 kgcm2/s2K2 = 1.97 mJ/molK2

Many of you used instead Kittel eq. 36 γ=1/2 π2 NkB2/εF, and then

εF = (ћ2/2m)(3π2N/V)2/3 eq. 17 (used in solution above). That would be fine. About 4 of you got εF right, and then plugged into eq. 36. \*But\*, although you used the correct N/V from part b.) where there are two electrons per atom and therefore N=2\*Avogadro’s number, when you got to eq. 36 and plugged your correct εF  in, you reverted to using N as Avogadro’s number. It’s still the number of electrons, which is 2\*Avogadro’s #. So you were a factor of 2 too small. This mysterious mistake on the part of a number of you took a while to figure out. Just for fun, the correct γ is 2.7 mJ/molK2, so the free electron model is not so bad.

2.) In the homework, we did the empty lattice bands for simple cubic in the [111] direction of k-space. Here we do the empty lattice bands for simple cubic in the [110] direction, called the M direction. The graph at left is to scale.

a.) (2 pts) Write the general expression for the energy of the bands ­involving the kx, ky, and kz as well as **Gx, Gy, and Gz.**

ε=(kx + Gx)2 + (ky + Gy)2 + (Gz)2

b.) (2 pts) What is the energy, using ћ2/2m=1, for band 1 at the M point? (Express in terms of (π/a)2)

Band 1: G=000; kx2 and ky2 at the zone boundary are each (π/a)2 so band 1 rises from ε=0 to 2(π/a)2 at the zone boundary.

c.) (4 pts) Use the general expression from part a.) for the energy of band 8. Which values of Gx, Gy, and Gz are degenerate for this band (i. e. give the same energy)? What is the energy (Express in terms of (π/a)2) for band 8 at the Γ point, and at the M point, using ћ2/2m=1?

From looking at the graph, Band 5 starts at 4(π/a)2 at the Γ point and Band 8 starts at 8(π/a)2 and ends at the M point at 10(π/a)2. So, need an even multiple of (π/a)2 (specifically 8 and 10), so we could have G= -1, 1, 0 or G= 1, -1, 0 to give at zone center:

ε=(Gx)2 + (Gy)2 =(2π/a)2 + (2π/a)2 = 8(π/a)2  and at the M point for -1,1,0:

ε=(kx + Gx)2+(ky+Gy)2=(π/a - 2π/a)2+(π/a + 2π/a)2=(1+9) (π/a)2

and for 1,-1,0 similarly (π/a + 2π/a)2+(π/a - 2π/a)2 = 10 (π/a)2

3.) The electron carrier concentration at room temperature in intrinsic Ge is 7 1019/m3, and the electron mobility is 0.36 m2/V-s (both numbers out of Kittel). The hole mobility is 0.18 m2/V-s and, in intrinsic material, the hole carrier concentration is the same as for the electrons, 7 1019/m3.

a.) (2 pts) What is the electrical resistivity, in units of Ω-m?

ρ=1/(neμe + peμh) = 1/(7x1019 1.6 10-19 0.36 C/(V-s-m) + 7 1019 1.6 10-19 0.18 /Ω-m)

Since C/s = Amp and Amp/V = 1/Ω

ρ = 1/(4.03 /Ω-m + 2.016 /Ω-m) = (1/6.046 )Ω-m = 0.165 Ω-m

b.) (2 pts) Considering now just the electron charge carriers, if melectron in Ge is 0.5 times the rest mass of the electron, what is τ, the collision time, in units of seconds?

μe=qτe/me so τe = [0.36 m2/Vs] \* 0.5\* 9.11 10-31 kg/1.602 10-19 C = 1.02 10-12 s

4.) In 1911 the group of Kammerlingh Onnes discovered superconductivity in Hg, with an onset transition temperature, Tc, of 4.15 K. It was found that magnetic field suppressed Tc, with 0.030 T suppressing Tc to 2.2 K.

(2 pts) If the critical field as a function of temperature, Hc(T), of a superconductor behaves as

Hc(T) = Hc(0)[1-(T/Tc)2]

what is the value (remember, 3 sig figs) of the critical field Hc at T=0, Hc(0), for mercury, in Tesla?

Hc (2.2 K)=0.030 T = Hc(0)[1-(2.2/4.15)2], so Hc(0) =0.0417 T

5.) The condensation energy of a superconductor is defined as the integral from 0 to Tc of the difference between the normal state, Sn, and superconducting state, Ssc, entropies:

U = ∫0Tc (Sn – Ssc)dT

Since the phonon contribution to Sn and Ssc is the same, one can also just consider the electronic parts of the entropies, i. e. Snelec and Sscelec, in the integral.

One way to arrive at the entropy is via S(T) = ∫0T (C/T’)dT’, where C is the specific heat. Consider the figure below (Fig. 6 from Kittel, Chap. 10) showing the normal and superconducting state entropies of superconducting Al (the entropies are the electronic ones), Tc=1.18 K



The entropy at Tc is 1.6 mJ/molK.

a.) (2 pts) Considering the normal state electronic entropy in the figure, what is the electronic specific heat γ of Al?

S(T) = ∫0T (C/T’)dT’, where Celec/T = γ, so Snelec (Tc) = 1.6 mJ/molK = γTc, so γ =1.6mJ/molK/1.18 K = 1.36 mJ/molK2 (correct literature value)

b.) (2 pts) We have discussed in class that, before the BCS theory, the superconducting electronic specific heat, C, was approximated as const.\*T3. Plug this into the integral for Sscelec(Tc), what is the const.?

1.6 mJ/molK = ∫0Tc (C/T’)dT’ = const.\*Tc3/3 => const.= 3\*(1.6 mJ/molK)/1.183K3

=2.92 mJ/molK4

c.) (3 pts) What is the condensation energy of Al (keep 5 sig figs until the answer, in 3 sig figs)?

Sscelec = γT ; Snelec = 2.92 mJ/molK4 T3/3

U = ∫0Tc [1.36\*T mJ/molK2 – (2.92/3) mJ/molK4(T3)]dT = 1.36 \*1.182/2 –[2.92/3]\*(1.18)4/4 in units of mJ/mol = 0.94683 – 0.47177 = 0.475 mJ/mol (vs literature value of 0.44)