PHY 6246 Graduate Classical Mechanics

Final Exam

December, 2011

Two hours Duration

Read and answer all questions carefully. All questions are of equal weight. Good luck

1 A circle lies in a vertical plane, touching the x-axis from above at the origin. A particle moves around the circle, subject to an attractive central force directed towards the coordinate origin.

a) First, find an equation for the orbit, and then show that the force must vary as the inverse fifth power of the distance.

b) Show that for this orbit of the particle, the energy must be zero.

c) Find the period of the motion.

d) Find $\dot{x}, \dot{y}$ and v as functions of angle around the circle, and show that all three quantities are infinite as the particle goes through the force center.

2 Consider the problem of a point projectile moving in a vertical plane under the influence of a uniform gravitational field. Assume the projectile starts at the origin at $t=0$, with velocity v, and that it is launched at an angle α to the horizontal.

a) Write down a Lagrangian describing the motion, and find the Hamiltonian.

b) Write down and solve the Hamilton-Jacobi equation.

c) Hence find, fully, the equation for the trajectory.

d) Similarly, find the dependence of the coordinates on time during the flight of the projectile.

3 Consider the problem of a point particle moving in a quadratic potential subject to a small quartic perturbation.

a) Write down a Lagrangian describing the motion, and find the Hamiltonian.

b) For the unperturbed Hamiltonian, express the canonical coordinates (q and p) and the Hamiltonian in terms of action-angle variables.

c) For the perturbed Hamiltonian, write down the perturbation in action-angle variables.

d) Write down and solve the equations of motion, to first order in the perturbation. (Hint: use time-dependent perturbation theory).

e) Comment on the effects of the perturbation upon the periodicity of the motion for this system.

4 The Lagrangian density for a charged scalar meson field can be written as:

$$\mathcal{L} = \dot{\phi}\dot{\phi}^* - c^2\nabla\phi\nabla\phi^* - \mu_0^2 c^2 \phi\phi^*,$$

in which ϕ and ϕ^* are to be taken as two independent field variables.

a) Write out the Euler-Lagrange equations for this system, and obtain the equation of motion for ϕ.

b) Find the canonical momenta, and obtain the complete Hamiltonian density.

c) What physical dimension does μ_0 have? What physical (quantum ?) characteristic of the meson field might it represent?

d) Write out an expression for the conserved current j_μ, and explain why it might be a valid current to include as a source term in Maxwell's Equations.

e) What property of the original Lagrangian density allows this current to be a conserved quantity?