

PHY6246, Section 3916, Fall 2016, Homework 8

Due at the start of class on Monday, October 24.

Answer all questions. Please write neatly and include your name on the front page of your answers. To gain maximum credit you should explain your reasoning and show all working.

1. A particle at rest decays into two identical particles which fly away with non-zero speed.
 - a) If the speed of each fragment is $0.8c$, how much of the original particle's mass is in the rest mass of each fragment?
 - b) If 96% of the initial mass is shared equally as the rest mass of each fragment, at what speed are the fragments moving after the decay occurs? Be sure to show all your working for this calculation.
 - c) Suppose instead that three identical fragments are formed which all move off at the same speed. If each one at rest would have 32% of the original rest mass, with what speed do they move away after the decay occurs?
2.
 - a) For any two objects a and b , show that:

$$P_{(a)}^\mu P_{(b)}^\nu g_{\mu\nu} = -m_{(a)}E_{(b)} = -m_{(b)}E_{(a)} = -m_{(a)}m_{(b)}c^2\gamma(v_{rel})$$

where $m_{(a)}$ is the mass of a , etc., $E_{(b)}$ is the energy of b in a 's rest frame, and vice versa, and v_{rel} is the speed of b in the rest frame of a (or vice versa).

- b) The relativistic kinetic energy of a particle is $T = (\gamma - 1)mc^2$. Use the binomial series to express T as a series in powers of $\beta = v/c$, and verify that the first term is just the non-relativistic kinetic energy.
 - c) Show that to lowest order in β the difference between the relativistic and non-relativistic kinetic energies is $3\beta^4 mc^2/8$.
 - d) Use this result to find the maximum speed at which the non-relativistic value is within 1% of the correct relativistic value.
3. An electron and a positron are traveling towards each other.
 - a) In the center of mass frame, each has $|\mathbf{v}| = 0.75c$. What is their relative velocity toward each other.
 - b) What is the energy of each photon emitted in the CM frame?
 - c) In the lab frame the electron is at rest. What Lorentz transformation is required to bring the electron to rest?

- d) In the lab frame, what is the energy of each of the emitted photons?
4. The Lagrangian for a relativistic charged particle of mass m and charge q moving in an external electromagnetic field, can be in a reparameterization invariant form. By choosing the parameter to be coordinate time, *i.e.*, $d/d\lambda = d/dt = \dot{}$, the Lagrangian is given by:

$$L(\mathbf{r}, \dot{\mathbf{r}}, t) = -mc\sqrt{c^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2} + q(-V + \dot{x}A_x + \dot{y}A_y + \dot{z}A_z),$$

where the scalar V and the components A_i of the vector \mathbf{A} are all functions of (x, y, z, t) .

- a) Write down the Euler-Lagrange equations for each of the three coordinates.
- b) The equation of motion for the particle can be written as $\dot{\mathbf{p}} = q(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$. By suitable identification of terms, express \mathbf{p} in terms of m , γ and $\dot{\mathbf{r}}$, and both \mathbf{E} and \mathbf{B} in terms of V and \mathbf{A} .
5. Consider two arbitrary (non-diagonal) points on a sphere, given by (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.
- a) From purely geometric considerations, write down the equations for a point (x, y, z) which lies on the great circle between them.
- b) By converting to polar coordinates throughout, rewrite this result as a single equation and, by suitable substitutions, express it in terms of the extremal values of θ_{\max} or θ_{\min} and an angle ϕ_0 at which the great circle crosses the equator.
- c) Write down an extremal principle for the geodesic (shortest path) on a sphere and the Euler equation(s) which follow from it.
- d) Solve the Euler equation(s) completely and show that your solution is consistent with your result in part b). *Hint:* You may find it helpful to make several trigonometric substitutions. If all else fails, show that your answer in part b) satisfies the Euler equation(s) you obtained in part c).