Midterm, Fall 2011, Graduate Classical Mechanics

1. A particle moves in a quadratic potential in one dimension. It is perturbed by a small, third order contribution to the potential: $\delta V_3 = \epsilon * x^3/3$.

   a) Starting from a Lagrangian, find the equation of motion for the particle. (*4 points*)

   b) Solve the equation of motion for the particle assuming $\epsilon = 0$. (*5 points*)

   c) Using your solution as an initial approximation, find the solution to the perturbed equation to first order in ϵ. (*7 points*)

   d) State whether you expect that this solution should be periodic, and give a reason for your answer. (*3 points*)

   e) If the perturbing potential were actually quartic ($\delta V_4 = \gamma * x^4/4$), part of the first order perturbed solution can be shown to have an amplitude which grows linearly with time. Give an explanation of why this would be so. (*4 points*)

   f) State whether you would expect that the exact solution for this quartic perturbation should be periodic, and give a reason (may differ from d)) for your answer. (*3 points*)

2. A rigid body rotates in free fall. Consider it's motion in terms of it's principal moments of inertia, all of which are assumed different.

   a) Write down Euler's equations for the rotational motion about the center of mass of the body, described with respect to the principal axes fixed in the body. (*4 points*)

   b) Show that rotation about some principle axes results in stable motion. (*6 points*)

   c) Show that there is one axis (find it) about which the motion is unstable. (*6 points*)

   d) Now suppose that two of the principle moments are actually equal. Solve the non-trivial equations of motion. (*5 points*)

   e) Identify the precession frequency in terms of your chosen variables. (*3 points*)