PHY6246, Section 3916, Fall 2017, Homework 1

Due at the start of class on Wednesday, September 6.

Answer all questions. Please write neatly and include your name on the front page of your answers. To gain maximum credit you should explain your reasoning and show all working.

1. Two points of mass m are joined by a rigid weightless rod of length l, the center of which is constrained to move on a circle of radius a. Express the kinetic energy in generalized coordinates.

2. A particle of mass m moves in one dimension such that it has the Lagrangian:

$$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 V(x) - V^2(x).$$

where V is some differentiable function of x. Find the equation of motion for $x(t)$ and describe the physical nature of the system on the basis of this equation.

3. The term *generalized mechanics* has come to designate a variety of classical mechanics in which the Lagrangian contains time derivatives of the q_i higher than the first. Problems for which $\dddot{x} = f(x, \dot{x}, \ddot{x}, t)$ have been referred to as "jerky" mechanics. Such equations of motion have interesting applications in chaos theory.

a) By applying the method of the calculus of variations, show that if there is a Lagrangian of the form $L(q_i, \dot{q}_i, \ddot{q}_i, t)$, and Hamilton's principle holds with the zero variation of both q_i and $\dot{q}_i$ at the endpoints, then the corresponding Euler-Lagrange equations are

$$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial q_i} = 0, \quad i = 1, 2, \ldots n.$$

b) Apply this result to the Lagrangian:

$$L = -\frac{m}{2} q\ddot{q} - \frac{k}{2} q^2.$$

c) Do you recognize the equations of motion?

4. In certain situations, particularly in one dimensional situations, it is possible to incorporate frictional effects without introducing the dissipation function.

a) As an example, find the equation of motion for the Lagrangian:

$$L = e^{\gamma t}\left(\frac{m}{2}\dot{q}^2 - \frac{k}{2}q^2\right).$$

b) How would you describe the system?

c) Are there any constants of the motion?

d) Suppose a point transformation is made of the form:

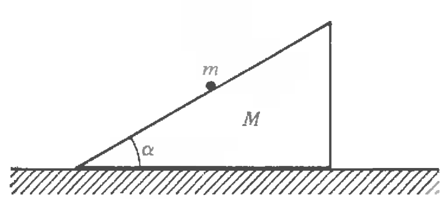
$$s = e^{\gamma t/2} q.$$

What is the effective Lagrangian in terms of s?

e) Find the equation of motion for s.

f) What do these results say about the conserved quantities for the system?

5. A particular mass m slides without friction on a wedge of angle α and mass M that can move without friction on a smooth horizontal surface, as shown in the figure.



a) Treating the constraint of the particle on the wedge by the method of a Lagrange multiplier, find the equations of motion for the particle and the wedge.

b) Obtain an expression for the force of constraint.

c) Calculate the work done in time t by the forces of constraint acting on the particle and on the wedge.

d) What are the constants of motion for the system?

e) Contrast the results you have found with the situation when the wedge is fixed. [*Suggestion*: For the particle you may either use a Cartesian coordinate system with y vertical, or one with y normal to the wedge or, even more instructively, do it in both systems.]