PHY6246, Section 3916, Fall 2017, Homework 11

Due at the start of class on Monday, November 13.

Answer all questions. Please write neatly and include your name on the front page of your answers. To gain maximum credit you should explain your reasoning and show all working.

1. A single particle moves in space under a conservative potential.

   a) Set up the Hamilton-Jacobi equation in ellipsoidal coordinates u, v, ϕ defined in terms of the usual cylindrical coordinates r, z, ϕ by the equations:

   $$r = a \sinh v \sin u \quad \text{and} \quad z = a \cosh v \cos u.$$

   For what forms of $V(u, v, \phi)$ is the equation separable?

   b) Use the results of part a) to reduce to quadratures the problem of a point particle of mass m moving in the gravitational field of two unequal mass points fixed on the z-axis a distance $2a$ apart.

2. One end of a uniform rod of length $2l$ and mass m rests against a smooth horizontal floor and the other end against a smooth vertical surface.

   a) Assuming that the rod is confined to move under gravity with its ends always in contact with the surfaces, use the Hamilton-Jacobi equations to reduce the solution of the problem to quadratures.

3. A particle of mass m is constrained on a curve in the vertical plane defined by:

   $$y = l(1 - \cos 2\phi), \quad \text{and}$$
   $$x = l(2\phi + \sin 2\phi).$$

There is the usual constant gravitational force acting in the vertical direction.

   a) By the method of action-angle variables, find the frequency of oscillation for all initial conditions such that the maximum of ϕ is less than or equal to $\pi/4$.

4. This problem concerns the motion of a point projectile moving in the vertical plane.

   a) Using the Hamilton-Jacobi method, find both the equation for the trajectory and the dependence of the coordinates on time. Assume the projectile is fired off at time $t = 0$ from the origin with a velocity v_0, making an angle α with the horizontal.

5. A three dimensional harmonic oscillator has the force constant k_1 in the $x-$ and $y-$directions, and k_3 in the $z-$direction.

a) Using cylindrical coordinates (with the axis of the cylinder in the $z-$direction), describe the motion in terms of the corresponding action-angle variables, showing how the frequencies can be obtained. Transform to the "proper" action-angle variables to eliminate degenerate frequencies.