PHY 6645 - Quantum Mechanics I - Fall 2011
Homework set # 4, due September 21

1. Show that

$$e^{\Omega} = \lim_{n\to\infty}\left(1+\frac{1}{n}\Omega\right)^n \tag{0.1}$$

where Ω is a linear operator.

2. Show that if U is a linear operator on a vector space $\mathcal{V}$ and if for all vectors $V \in \mathcal{V}$

$$< V'|V' >=< V|V > \tag{0.2}$$

where $|V' >= U|V >$, then U must be a unitary operator on $\mathcal{V}$.

3. If X and P are canonically conjugate observables, we have $X = X^\dagger$, $P = P^\dagger$ and $[X, P] = \hbar i$. Using these equations, show that

a. $[X, F(P)] = \hbar i \frac{dF}{dP}$, where $F(P)$ is any function of the operator P.

b. $< x|P^n|\Psi >= \left(\frac{\hbar}{i}\right)^n \frac{d^n}{dx^n} < x|\Psi >$ where the $|x >$ are the eigenstates of X.

c. $< x|p >= Ne^{ixp/\hbar}$ where the $|p >$ are the eigenstates of P and N is a normalization constant.

4. Problem 4.2.1 in Shankar's book.