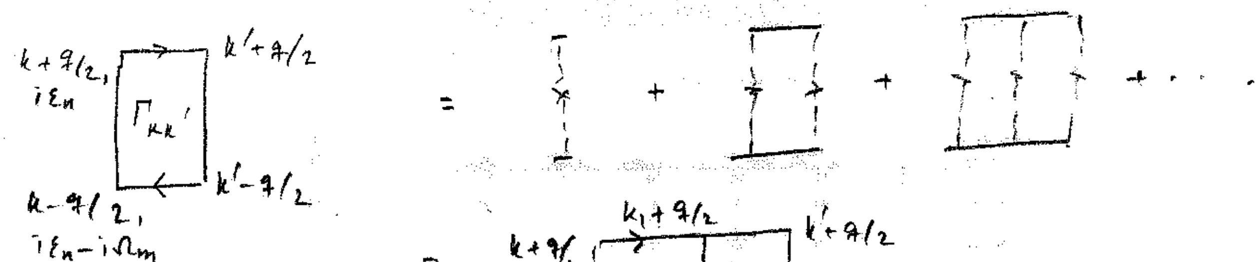
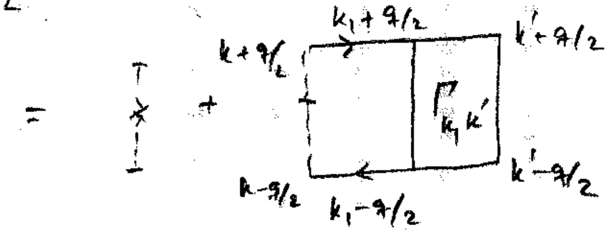
Lecture 3:

In order to calculate quantum corrections, we will need to sum diagrams representing repeated scattering from same impurity. There are two such dominant cases:

3.1 : Particle-hole propagator:





$$\Gamma_{kk'}(q,\varepsilon,\Omega) = V_{kk'}^2 + \sum_{k_1} V_{kk_1}^2 \, G^+_{k_1+q/2}(\varepsilon+\Omega/2)\, G^-_{k_1-q/2}(\varepsilon-\Omega/2)\, \Gamma_{k_1k'}(q,\varepsilon,\Omega)$$

we will be interested in small q, Ω limit where this is large, so we expand G in this limit:

$$G^+_{k_1+q/2}(i\varepsilon_n) = \frac{1}{i\varepsilon_n - \xi_{k_1+q/2} + i/2\tau} \approx \frac{1}{i\varepsilon_n - \xi_{k_1} + \frac{\vec q}{2}\cdot\vec v_{k_1} - \frac{q^2}{8m} + i/2\tau}$$

$$\approx G^+_{k_1}(i\varepsilon_n) + \left(\tfrac{\vec q}{2}\cdot\vec v_{k_1}\right)(G^+_{k_1})^2 + \left(\tfrac{\vec q}{2}\cdot\vec v_k\right)^2 (G^+_k)^3 + \dots$$

Prob 3.1 Show that for small $qv_F\tau \ll 1$, & $\Omega\tau \ll 1$

$$N_0\int d\xi_{k_1}\, G^+_{k_1+q/2}(i\varepsilon_n)\, G^-_{k_1-q/2}(i\varepsilon_n - i\Omega_m) \approx 2\pi N_0\tau\left[1 + i\tau\left(i\Omega_m - \vec q\cdot\vec v_{k_1}\right) - \tau^2(\vec q\cdot\vec v_{k_1})^2\right]$$

Then averaging over directions of $\hat k_1$, show that

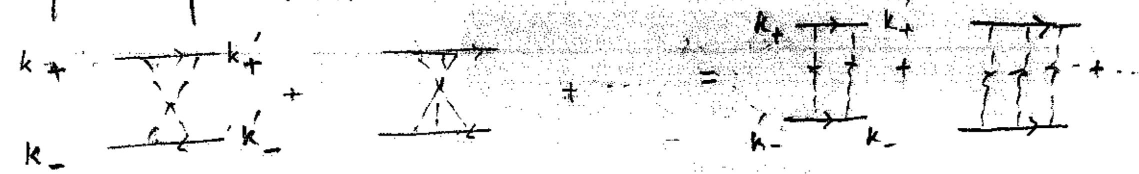
$$\bar\Gamma(q,\Omega_m) = \frac{1}{\tau\left(|\Omega_m| + Dq^2\right)}$$

where $\bar\Gamma = 2\pi N_0\tau\,\Gamma$ and $D = \frac{1}{3}v_F^2\tau$

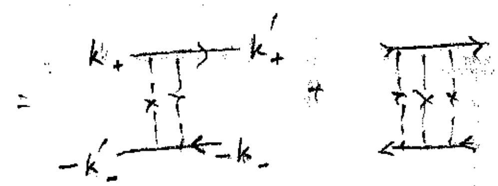
Hint: Use $2\pi N_0 \tau V^2 = 1$

3.2 : Particle-particle propagator :

Redraw the "maximally crossed" particle-hole diagrams as particle-particle ladder diagrams using time-reversal symmetry ($G^R_{pp}(\omega) = G^R(-p', -p, \omega)$) :



$$k_\pm = k \pm q/2$$



$$k_+ - (-k_-') = k + q/2 + k' - q/2$$

$$= k + k' \equiv Q, \quad k \approx -k'$$

$$\Rightarrow Q \text{ small}$$

$$C_{kk'}(Q; i\epsilon_n, i\Omega_m) = V^2_{kk'} + \sum V^2_{kk_1} G^+_{k_1}(i\epsilon_n) G^-_{-k_1+Q}(i\epsilon_n - i\Omega_m) C_{k_1k'}$$

As in the p-h propagator case, this eventually gives

$$C(Q, \Omega_m) = \frac{1}{\tau} \cdot \frac{1}{|\Omega_m| + DQ^2}$$

However, if time reversal symmetry is broken, eg by a magnetic field, then the "pole" is cut-off :

$$C(Q, \Omega) = \frac{1}{\tau} \frac{1}{|\Omega_m| + DQ^2 + \frac{1}{\tau_\varphi}}$$

where τ_φ^{-1} is the phase-breaking relaxation rate.

Note1: A similar cut-off does not exist for the p-h propagator, because of particle-number conservation.

Note2: Note that the momentum q in p-h propagator is the "difference" momentum, where as the momentum Q is the p-p propagator is the "sum" momentum

3.3 : Quantum corrections to conductivity.

3.3.1 : Weak localization corrections

The p-h ladder contributes to the conductivity and gives rise to an effective "transport" scattering rate. However, the p-p ladder gives rise to

$$\delta\sigma = -\frac{2De^2}{\pi}\int\frac{d^dQ}{-i\Omega + DQ^2}$$

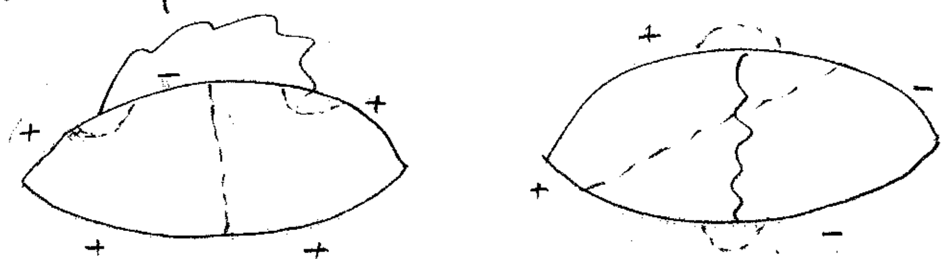
In 2d, this is logarithmically divergent, and must be cut off at the phase relaxation scale:

$$\delta\sigma^{2d} = -\frac{e^2}{2\pi^2\hbar}\ln\frac{\tau_\varphi}{\tau}$$

This is the "weak-localization" correction. For $\tau_\varphi^{-1} \sim T$, this leads to a $\ln T$ correction to σ.

3.3.2 : Interaction corrections:

The p-h ladders do become important in the presence of interactions:



Each broken line represents a p-h ladder.
Note the G^+G^- combination in each.
In the end, this also gives rise to a $\ln T$ correction to σ in 2d, indistinguishabe from WL corrections.