Lecture 4:

In order to understand the UF experiments, we need to consider ferromagnetic metals, in 2d. We will consider conduction electrons with Fermi-energy $\epsilon_{F\sigma}$ depending on the spin index $\sigma = \uparrow, \downarrow$, and with strong spin-orbit coupling. Coulomb interaction will be considered later, as a weak perturbation.

4.1 / The Hamiltonian

The simplest model is

$$H_1 = \left[-\frac{\nabla^2}{2m} + V_{dis}(r) \right] \delta_{\sigma\sigma'} - M \tau^z_{\sigma\sigma'} - i \frac{\lambda_c^2}{(4\pi)^2} \left[\hat{\tau}_{\sigma\sigma'} \cdot \nabla V_{dis} \times \nabla \right]$$

where $\lambda_c = \frac{2\pi}{mc}$ is the compton wavelength, M is the Zeeman energy splitting caused by ferromagnetic polarization. $\hat{\tau}$ is the vector of Pauli matrices. The disorder potential is

$$V_{dis}(\vec{r}) = \sum_j v(\vec{r} - \vec{R}_j)$$

with random scattering centers at $\vec{R}_j$.

In terms of electron creation & annihilation operators, we write

$$H = \sum_{\vec{k}\sigma} (\epsilon_k - M\sigma) c^+_{k\sigma} c_{k\sigma} + \sum_{kk'\sigma\sigma'} \sum_j V(\vec{k} - \vec{k}') e^{i(\vec{k}-\vec{k}')\cdot\vec{R}_j}$$

$$\left[\delta_{\sigma\sigma'} - i\,\bar{g}\, \hat{\tau}_{\sigma\sigma'} \cdot (\hat{k} \times \hat{k}') \right] c^+_{k'\sigma'} c_{k\sigma}$$

Note: $\bar{g} = \frac{\lambda_c^2 k_F^2}{(4\pi)^2} \sim 10^{-4}$. However, band-mixing results in a renormalized $\bar{g}$ which is of order unity. We will consider $\bar{g}$ as a phenomenological coupling parameter.

4.2: Impurity scattering:

We will keep the discussion as general as possible, with strong, finite range impurity scattering in 2d. The impurity potential at $\vec{R}=0$ is

$$V_{kk',\sigma} = V(\vec{k}-\vec{k}')\,\Big[\underbrace{1}_{\text{normal scatt (ns)}} - \underbrace{i g_\sigma \tau^z_{\sigma\sigma}\cdot(\hat{k}\times\hat{k}')}_{\text{"skew" scatt (ss)}}\Big]$$

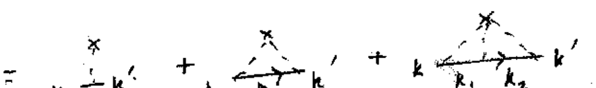
For isotropic systems, $V(k-k')$ depends only on the angle θ between $\hat{k}$ & $\hat{k}'$. In 2d, we can then expand V in terms of eigenfuctions $\chi_m(\hat{k}) \equiv e^{im\phi}$, where ϕ is the polar angle of the vector $\hat{k}$:

$$V_{kk',\sigma} = \sum_m V_{m\sigma}\,\chi_m(\hat{k})\,\chi_m(\hat{k}') \quad ; \quad V_{m\sigma} = V^{ns}_{m\sigma} + V^{ss}_{m\sigma}$$

Note : $V^{ns}_m = (V^{ns}_m)^* = V^{ns}_m$ [from TRI & rotational sym.]

Prob 4.1: Show that $V^{ss}_{m\sigma} = \frac{1}{2}\, g_\sigma\, \tau^z_{\sigma\sigma}\,(V^{ns}_{m-1} - V^{ns}_{m+1})$

4.2.1: Scattering amplitude:

$f^s_{kk',\sigma}$ = 

$+ \;\cdots\cdots$

$$= V_{kk'\sigma} + \sum_{k_1} G^0_{k_1\sigma}(i\omega_n)\, V_{kk_1,\sigma}\, f^s_{k_1k',\sigma} \quad ; \quad s \equiv \text{sign}(\omega_n)$$

$$= V_{kk'\sigma} - is\pi N_\sigma \left\langle V_{kk_1\sigma}\, f^s_{k_1k'\sigma}\right\rangle_{k_1} \quad ; \quad N_\sigma : \text{density of states}$$

Note : $\sum_{k_1} \to N_\sigma \int d\varepsilon_{k_1} \int d\Omega_k$ & $\int d\varepsilon_{k_1}\, G^0_{k_1\sigma} = -is\pi$

and $\langle\ \rangle_{k_1}$ is averaging over directions of $\hat{k}_1$.

Prob 4.2: Show that $\bar{f}^s_{m\sigma} = \dfrac{\bar{V}_{m\sigma}}{1+is\bar{V}_{m\sigma}}$, where $\bar{V}_{m\sigma} \equiv \pi N_\sigma V_{m\sigma}$

and $\bar{f}_{kk'\sigma} \equiv \pi N_\sigma f_{kk'\sigma} \equiv \sum \bar{f}_{m\sigma}\,\chi_m(\hat{k})\,\chi_m(\hat{k}')$

4.2.2: Single particle relaxation rate:

By definition, imaginary part of the self-energy $\Sigma_{k\sigma}$ is related to the single-particle relaxation rate τ_σ by

$$\frac{1}{2\tau_\sigma} = -s\,\mathrm{Im}\left(\Sigma_{k\sigma}(i\omega_n)\right) = -s\, n_{imp}\,\mathrm{Im}(f^s_{kk,\sigma})$$

Prob 4.3: Show that $\dfrac{1}{2\tau_\sigma} = \dfrac{n_{imp}}{\pi N_\sigma}\gamma_\sigma$, where

$$\gamma_\sigma \equiv -s\sum_m \mathrm{Im}(\bar f_{m\sigma}) = \sum_m \frac{\bar V^2_{m\sigma}}{1+\bar V^2_{m\sigma}}.$$

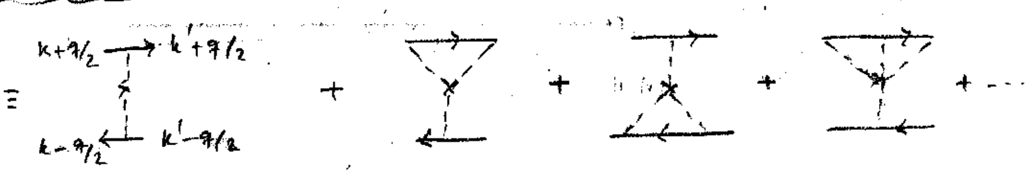
Note: Single particle (impurity) Green's function:

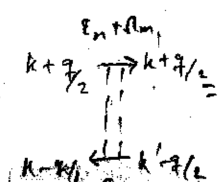
$$G_{k\sigma}(i\omega_n) \equiv \left[i\omega_n - \varepsilon_{k\sigma} - \Sigma_{k\sigma}(\omega_n)\right]^{-1}; \qquad \omega_n \equiv \pi T(2n+1)$$

$$= \left[i\omega_n - \varepsilon_{k\sigma} + is/2\tau_\sigma\right]^{-1}$$

where a shift in real part of Σ has been ignored.

4.3: Particle-hole scattering amplitude:

$t^{ss'}_{kk'}(q; i\varepsilon_n, i\Omega_m) \equiv$  $+ \cdots$



$$= \frac{n_{imp}}{(\pi N_\sigma)^2}\,\bar f^{s}_{k+q/2,\,k'+q/2,\,\sigma}(i\varepsilon_n)\,\bar f^{s'}_{k'-q/2,\,k-q/2,\,\sigma}(i\varepsilon_n - i\Omega_m)$$

$$= t^{ss'}_{kk'}(q=0) + \underbrace{\Delta t^{ss'}_{kk'}(q)}_{\text{small}} \quad \text{for } q \ll k_F$$

Note: $\bar t^{-+}_{kk'}(q=0) = \left[\bar t^{+-}_{k'k}(q=0)\right]^*$ $\qquad \bar t \equiv 2\pi N_\sigma \tau_\sigma t$

Prob 4.4: show that $\bar t^{ss'}_{kk'\sigma}(q=0) = \sum_m \bar t^{ss'}_{m\sigma} Y_m(\hat k) Y_m(\hat k')$,

where $\bar t^{ss'}_{m\sigma} = \dfrac{1}{\gamma_\sigma}\sum_{m'} \bar f^{s}_{m'\sigma}\bar f^{s'}_{m'-m,\sigma}$. Define $\bar t^{+-}_{m\sigma} \equiv \lambda_m$. For short range scatt: where $\bar V^{ns}_m = 0$ for all $m \neq 0$, show that $\lambda_0 = 1$, $\lambda_1 = 2\tilde V_0 \tilde V_1 (\tilde V_0 + 2\tilde V_1)^{-1}(1 + i/V_1)$, where $\tilde V_m \equiv V_m^2/(1+V_m^2)$