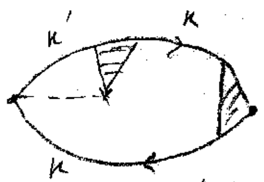
Lecture 6:

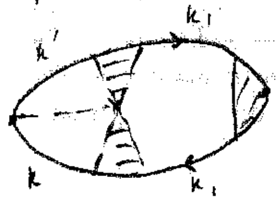
6.1: conductivity: side jump

$$\vec{v} = \frac{d\vec{r}}{dt} = -i\,[r, H] = \frac{\vec{p}}{m} + \text{spin-orbit dependent term}$$

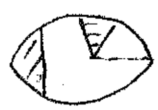
$$\langle k'\sigma' | \vec{v} | k\sigma \rangle = \frac{\vec{k}}{m}\,\delta_{kk'}\,\delta_{\sigma\sigma'} - i\,\frac{g_\sigma}{2m\epsilon_F} \sum_j V(\vec{k}-\vec{k}')\, e^{i(\vec{k}-\vec{k}')\cdot\vec{R}_j} \left[\hat{\tau}_{\sigma\sigma'} \times (\vec{k}-\vec{k}')\right]$$



$$L_{\alpha\beta} = -i n_{imp}\,\frac{g}{\epsilon_F}\, T \sum_{kk'} V^2 G_k G_{k'} G_k \left[\tau \times \frac{k-k'}{2m}\right]_\alpha f^+_{k'k}\, \tilde{J}_{k\beta}$$



$$L_{\alpha\beta} = -i n_{imp}\,\frac{g}{\epsilon_F}\, T \sum_{kk'} V^2 G_{k'} G_{k_1} G_{k_1} G_k \left[\tau \times \frac{k-k'}{2m}\right]_\alpha f^+_{k'k_1} f^-_{k_1 k}\, \tilde{J}_{k_1\beta}$$

similarly for  +  +  + 

$$\Rightarrow \sigma^{sj}_{xy} \approx \frac{e^2}{2\pi} \sum_\sigma \tau^z_{\sigma\sigma}\, g_\sigma\, \frac{1 - \mathrm{Re}\,\lambda_1}{|1-\lambda_1|^2}$$

Note: $\sigma^{sj}_{xx} \ll \sigma^{ss}_{xx}$

6.2: Quantum corrections due to interactions:

To first order in the screened Coulomb interactions,

$$\delta\sigma^I = T \sum_{\omega_n} \int dq^2\, K(q, i\omega_n)\, V_c(q, i\omega_n)$$

Note: $K(q, i\omega_n) \xrightarrow[q\to 0]{} 0$: gauge invariance. ($V(r) \to V(r) + c \Rightarrow V(q) \to V(q) + c\,\delta(q) \to$ must not change)

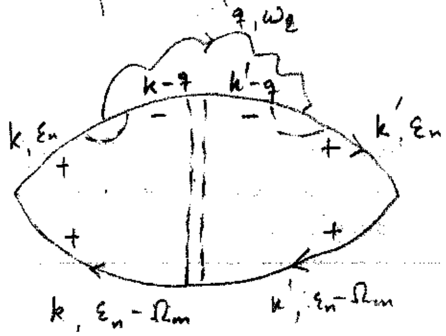
$$V_c(q, i\omega_n) = \frac{V_b(q)}{1 + V_b(q)\,\Pi(q,\omega)} \qquad V_b(q): \text{bare int.} = \frac{2\pi e^2}{q} \text{ in 2d}$$

polarization: $$\Pi(q, i\omega_n) = \frac{dn}{d\mu}\,\frac{Dq^2}{|\omega_n| + Dq^2}$$

$$\Rightarrow V_c(q, i\omega_n) \xrightarrow[q,\,\omega_n\to 0]{} \left(\frac{dn}{d\mu}\right)^{-1}$$

6.2.1: Singular diagrams

Example of a most singular contribution:



$$L_{\alpha\beta} = -T \sum_{\varepsilon_n} T \sum_{\omega_\ell} \sum_{kk',q} G_k(\varepsilon_n) G_k(\varepsilon_n - \Omega_m)$$

$$V(q,\omega_n) G_{k-q}(\varepsilon_n - \omega_\ell) G_{k'-q}(\varepsilon_n - \omega_\ell) G_{k'}(\varepsilon_n) G_{k'}(\varepsilon_n - \Omega_m)$$

$$\Big[\Theta(\varepsilon_n)\Theta(\Omega_n)\Theta(\omega_\ell - \varepsilon_n) T_k^{+-}(q,\omega_\ell) T_{k'}^{-+}(-q,-\omega_\ell) \Gamma_{k'k}^{+-}(q,\omega_\ell - \Omega_m)$$

$$+ \Theta(-\varepsilon_n)\Theta(\Omega_m - \varepsilon_n)\Theta(\varepsilon_n - \omega_\ell) T^{-+}(q,-\omega_\ell) T_{k'}^{+-}(-q,-\omega_\ell) \Gamma_{kk'}^{+-}(-q,\omega_\ell + \Omega_m)$$

$$\times\ v_{k\alpha} v_{k'\beta}$$

where the renormalized density

$$T_k(q,\omega_\ell) \equiv 1 + \langle \bar{\Gamma}_{kk'} \rangle_{k'} = 1 + \frac{1/\tau}{|\omega_\ell| + Dq^2} \gamma_k$$

Using only the singular contributions of Γ & T:

$$L_{\alpha\beta} = \sum_\sigma \left(-2\pi i N_0 \tau^2\right)^2 \sum_q \Big[T \sum_{\omega_\ell > \Omega_m} (\omega_\ell - \Omega_m) \left\langle v_{k\alpha} \gamma_k(q) \tilde{\gamma}_k(q) \xi_k(q) \right\rangle_k$$

$$\times \left\langle v_{k'\beta} \tilde{\gamma}_{k'}(q) \gamma_{k'}(q) \xi_{k'}(q) \right\rangle_{k'}$$

$$+ T \sum_{\omega_\ell < 0} |\omega_\ell| \left\langle v_{k\alpha} \tilde{\gamma}_k(-q) \gamma_k(-q) \xi_k^*(q) \right\rangle \left\langle v_{k'\beta} \gamma_{k'}(-q) \tilde{\gamma}_{k'}(-q) \xi_{k'}^*(-q) \right\rangle_{k'} \Big]$$

$$\times \frac{1}{2\pi} \mathcal{D}_q(\omega_\ell, \Omega_m)$$

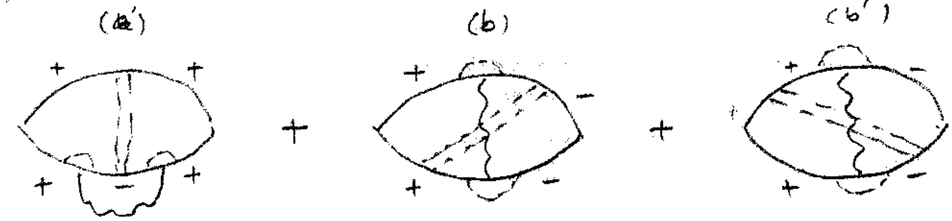
where $\mathcal{D}_q(\omega_\ell, \Omega_m) \equiv \dfrac{V(q,\omega_\ell)}{(|\omega_\ell| + Dq^2)^2 (|\omega_\ell - \Omega_m| + Dq^2)}$

and $\xi_k \equiv 1 - 2i\tau(\vec{q} \cdot \vec{v}_k)$

Prob 6.1: Show that upto linear in q,

$$\left\langle v_{k\alpha} \gamma_k(q) \tilde{\gamma}_k(q) \xi_k(q) \right\rangle_k = -i v_F^2 \tau q_\alpha (1 + \mathrm{Re}\,\tilde{\lambda}_1)$$

6.2.2: Corrections to longitudinal conductivity:



$$L_{xx}^{a+a'+b+b'} = \frac{1}{2\pi}\sum_\sigma \left(2\pi N_\sigma \tau^2\right)^2 \left(v_F^2 \tau\right)^2 \left(1+\mathrm{Re}\,\tilde{\lambda}_1\right)^2 \sum_q q^2\, \Psi(q,\Omega_m)$$

where $\Psi(q,\Omega_m) = T\sum_{\omega_\ell>0} \omega_\ell \left[\mathcal{D}(-\omega_\ell,\Omega_m) - \mathcal{D}(-\omega_\ell-\Omega_m,\Omega_m)\right]$

such that $\displaystyle\sum_q q^2 \Psi(q,\Omega_m) = \frac{1}{4\pi}\frac{e^2}{D^2 \kappa}\,\Omega\left(1+\ln\frac{\omega_c}{2\pi T}\right)$

where $\kappa = 2\pi e^2 \sum_\sigma N_\sigma$ is the screening length & $D = D_0\left(1+\mathrm{Re}\,\tilde{\lambda}_1\right)$.

Therefore, the exchange interaction correction to the longitudinal conductivity is

$$\boxed{\delta\sigma_{xx}^{I,ss} = \frac{e^2}{\Omega_m} L_{xx} = -\frac{e^2}{2\pi^2}\ln\frac{\omega_c}{T}}$$

6.3: Corrections to AH conductivity

We already noted the $K_\ell(q) \sim q^2$ due to gauge inv.

In addition, $\sigma_{xy}(B) = -\sigma_{xy}(-B)$ from symmetry of Hamiltonian

and $\sigma_{xy}(B;x) = \sigma_{xy}(-B,-x) = -\sigma_{xy}(B,-x)$

which means that K_ℓ must be proportional to $q_x q_y$ to preserve the "mirror symmetry". Averaging over the angles would then give zero.

$$\Rightarrow \quad \boxed{\delta\sigma_{xy}^{I} = 0} \quad \text{from symmetry arguments.}$$

$\rightarrow$ true for both ss & sj.