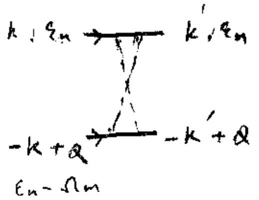
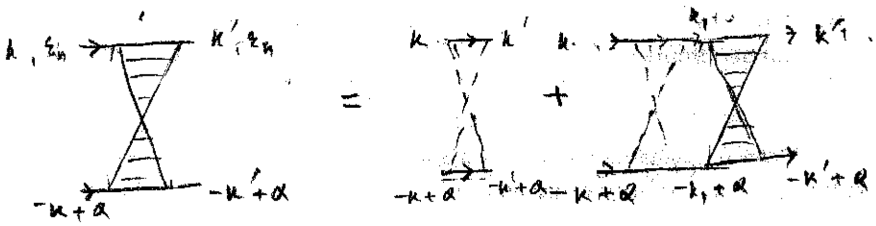
7.1 : Particle-particle propagator

As in p-h case, we define p-p scattering amplitude



$$\bar{t}^{\,p,ss'}_{kk',\sigma} = \frac{n_{imp}}{(\pi N_\sigma)^2}\, \bar{f}^{\,s}_{kk',\sigma}(i\varepsilon_n)\, \bar{f}^{\,s'}_{-k+Q,\,-k'+Q,\sigma}(i\varepsilon_n - i\Omega_m)$$

and the corresponding p-p (Cooperon) propagator



$$\bar{C}_{kk'}(Q, i\varepsilon_n, i\Omega_m) = t^{\,p}_{kk'}(Q, i\varepsilon_n, i\Omega_m) + \frac{1}{2\pi N_0 \tau_\sigma} \sum_{k_1} \bar{t}^{\,p}_{kk_1}(Q; i\varepsilon_n, i\Omega_m)$$
$$\times\, G_{k_1\sigma}(i\varepsilon_n)\, G_{-k_1+Q,\sigma}(i\varepsilon_n - i\Omega_m)\, \bar{C}_{k_1 k'}(Q; i\varepsilon_n, i\Omega_m)$$

The difference with p-h propagator $\bar{\Gamma}_{kk'}$ is that the $m = m' = 0$ component $\bar{C}_{00}$ is no longer singular in the presence of phase-relaxation processes giving finite relaxation time τ_φ!

$$\bar{C}_{kk'} = \frac{1}{\tau} \frac{\gamma^{p}_{k} \tilde{\gamma}^{p}_{k'}}{|\Omega_m| + D^p Q^2 + \tau_\varphi^{-1}} + \sum \tilde{\lambda}^{p}_{m} \chi_m(\hat{k}) \chi^*_m(\hat{k}')$$

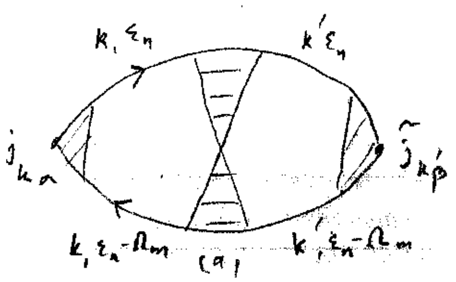
with $\tilde{\lambda}^{p}_{m} = \lambda^{p}_{m} / (1 - \lambda^{p}_{m})$, $D^p = D_0 \left[1 + \frac{1}{2}(\tilde{\lambda}^{p}_{1} + \tilde{\lambda}^{p}_{-1})\right] \neq D_0$

$$\gamma^{p}_{k} = 1 - i\tau \sum_{m=\pm 1} \tilde{\lambda}^{p}_{m} \chi_m(\hat{k}) \langle Q \cdot v_{k'} \chi^*_m(\hat{k}') \rangle_{k'}$$

$$\tilde{\gamma}^{p}_{k} = 1 - i\tau s \sum_{m=\pm 1} \tilde{\lambda}^{p}_{m} \chi^*_m(\hat{k}) \langle Q \cdot v_{k'} \chi_m(\hat{k}') \rangle_{k'}$$

<u>Note</u>: The "protected" singularity in the limit $q, \omega \to 0$ in the p-h propagator comes from particle-number conservation, and is absent in p-p propagator

7.2: Weak localization correction to conductivity: SS



(a)

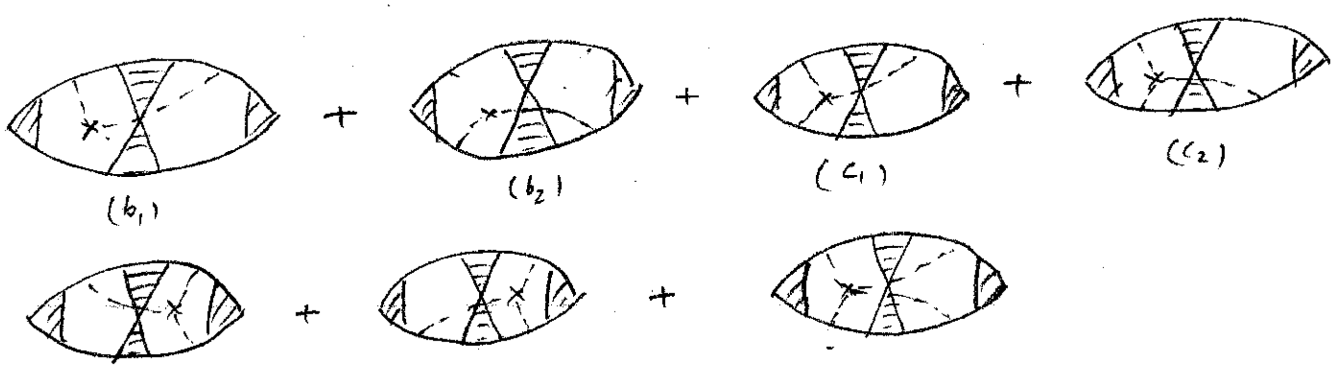
$$L^{(a)}_{\alpha\beta} = \sum_\sigma T \sum_{\varepsilon_n} \sum_{kk'Q} G_{k\sigma}(i\varepsilon_n)\, G_{k\sigma}(i\varepsilon_n - i\Omega_m)\, G_{k'\sigma}(i\varepsilon_n) \times G_{k'\sigma}(i\varepsilon_n - i\Omega_m)\, j^\sigma_{k\alpha}\, \tilde{j}^\sigma_{k'\beta}\, \frac{1}{2\pi N_\sigma \tau_\sigma} \cdot \bar{C}_{kk'}(Q; i\varepsilon_n, i\Omega_m)$$

cooperon $\longrightarrow$ $Q = \vec{k} + \vec{k}' \to 0 \Rightarrow k' \approx -k$ ($\tilde{j}_{k'\beta} \approx -\tilde{j}_{k\beta}$

Then $$L^{ss}_{\alpha\beta} = -\frac{\Omega_m}{2\pi} \sum_\sigma \left(4\pi N_\sigma \tau_\sigma^3\right)\left(2\pi N_\sigma \tau_\sigma\right)^{-1} \langle j_{k\alpha}\, \tilde{j}_{k\beta} \rangle_k \sum_Q \bar{C}_{k,-k}(Q)$$

The cooperon contribution is

$$\Phi = \sum_Q \bar{C}_{k,-k}(Q) = \int_0^{Q_c} \frac{Q\,dQ}{2\pi} \frac{1/\tau}{|\Omega_m| + D^\rho Q^2 + 1/\tau_\varphi} = \left(4\pi \tau_\sigma D^\rho\right)^{-1} \ln \frac{\tau_\varphi}{\tau_\sigma}$$



(b₁) (b₂) (c₁) (c₂)

$$L^{WL;ss}_{\alpha\beta} = -\frac{\Omega_m}{4\pi^2} \sum_\sigma \underbrace{\left(D_0 / D^\rho\right)}\, J^{\alpha\beta} \ln \tau_\varphi/\tau_\sigma$$

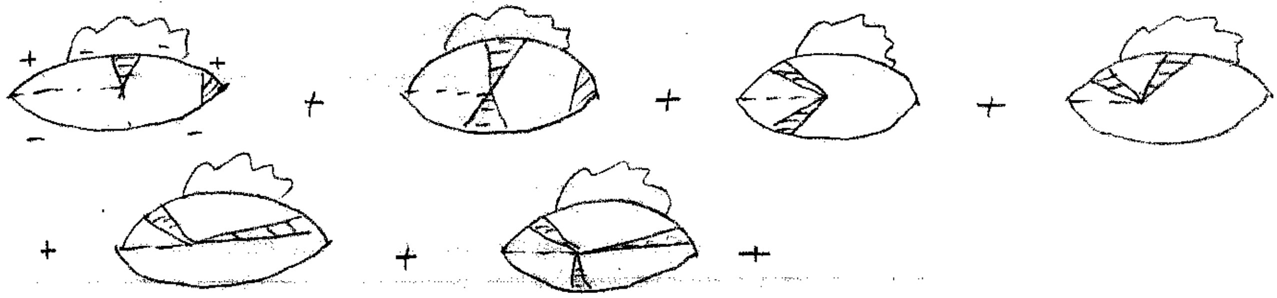
$J^{xx} = \mathrm{Re}\,\Lambda$, $J^{xy} = \mathrm{Im}\,\Lambda$, $\Lambda \equiv \frac{1}{1-\lambda_1}$ for short range (only V_0^{ns}, $V_{\pm 1}^{ss}$)

Note: $\Lambda = 1 + \frac{\lambda_1}{1-\lambda_1} = 1 + \tilde{\lambda}_1$, $D = D_0(1 + \mathrm{Re}\,\tilde{\lambda}_1) = D_0\, \mathrm{Re}\,\Lambda$

$$\Rightarrow \boxed{\delta\sigma^{WL,ss}_{xx} = -\frac{e^2}{2\pi^2} \ln \tau_\varphi/\tau}$$

$$\boxed{\delta\sigma^{WL,ss}_{xy} = \delta\sigma^{WL}_{xx} \cdot \frac{\mathrm{Im}\,\lambda_1}{1 - \mathrm{Re}\,\lambda_1}} \Leftarrow \delta\sigma^{WL}_{xx} \cdot \frac{\mathrm{Im}\,\Lambda}{\mathrm{Re}\,\Lambda}$$

7.3 : Interaction and WL corrections : SJ



$$L^{sj,I}_{\alpha\beta} = n_{imp} \sum G^+_{k'} G^-_{k'} (G^-_k)^2 G^+_k \left(-ig\frac{v_0^2}{\epsilon_F}\right)\left[\vec{\tau} \times \frac{\vec{k}-\vec{k}'}{2m}\right]_\alpha \bar{f}_{k'k} \tilde{J}_{k\beta} \gamma_k \gamma_{k'}$$

$+ \; \ldots\ldots$

$\Rightarrow \delta\sigma^{sj} \propto n_{imp}\, g v_0 \ln T/T_0 \ll \delta\sigma^{ss,I}_{\alpha\beta}$

Similarly WL contribution $\delta\sigma^{sj,WL}_{\alpha\beta} \ll \delta\sigma^{ss,WL}_{\alpha\beta}$

7.4 : Phase relaxation rate :

Cooperon contribution $\sum \bar{C}_{k,-k}(Q) = \int \frac{1/C}{|\Omega_m| + D^p Q^2 + \tau_\varphi^{-1}} \frac{Q\,dQ}{2\pi}$

$= (4\pi\tau_0 D^p)^{-1} \ln \tau_\varphi/\tau_0$

This gives $\ln T$ only if $\tau_\varphi^{-1} \propto T$.

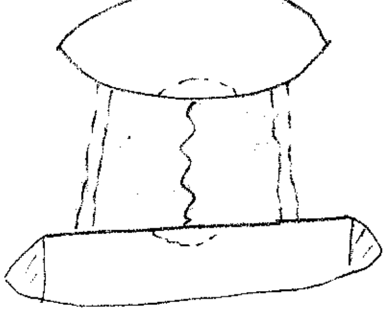
In general, $\tau_\varphi^{-1}(e\text{-}e) \sim T$ in 2d, but τ_φ due to spin flip, spin-orbit, or magnetic field may dominate and destroy the $\ln T$ dependence.

Note : contribution from e-e is given by

$$\frac{1}{\tau_\varphi} = \frac{T}{\epsilon_F \tau_{tr}} \ln \frac{\epsilon_F \tau_{tr}}{2}$$

Need $\max\left(\frac{1}{\tau_s}, \frac{1}{\tau_{so}}, \omega_H\right) \ll \frac{1}{\tau_\varphi} \ll \frac{1}{\tau_{tr}}$ for $\ln T$ correction from WL. $\left[\omega_H \sim (4\epsilon_F\tau_{tr})\cdot\left(\frac{e}{m^* c} B_{\perp}\right)\right]$

7.5 : Hartree contributions



Complicated, weak scattering result :

$$\delta\sigma_{xx}^{H,I} = \frac{e^2}{2\pi^2} \cdot \frac{3}{4}\tilde{F}_1 \ln\frac{\omega_c}{T}$$

where $\tilde{F} = 8(1+F/2)\ln(1+F/2) - 4$

and $F = \frac{1}{v(q=0)}\int \frac{d\theta}{2\pi}\, v(q = 2k_F \sin\theta/2)$

7.6 : Berry phase :

Bloch states : $u_{nk}(\vec{r})$

$$\vec{X}(\vec{k}) = \int_{cell} d^2r\, u^*_{nk}(\vec{r})\, i\nabla_k u_{nk}(\vec{r})$$: Berry vector potential

→ non-zero in the presence of spin-orbit interaction and ferromagnetic polarization.

Effective Hamiltonian : ← Bloch energies including s-o & polarization

$$H_k = V(r)(i\nabla_k + X_k) + \varepsilon_{n\sigma}(k)$$

↳ applied : $\nabla V = -eE$

$\Rightarrow \dot{r} = \nabla_k \varepsilon_{nk} - eE\times\Omega$ where $\Omega(k) = \vec{\nabla}_k \times \vec{X}(k)$

The additional term in $\dot{r}$, just as in side-jump, gives rise to a Hall current :

$$j_H = -e^2 n\langle\Omega\rangle \times \vec{E} \Rightarrow \sigma_{xy}^{B} = e^2 n\langle\Omega_z\rangle$$

where $\langle\Omega\rangle = n^{-1}\sum_{k\sigma}\Omega_z(k)\, f(\varepsilon_{k\sigma})$ is the average over all occupied states in k-space & $n = \sum n_\sigma$.

Note : This ave. is zero unless time-reversal symmetry is broken, as in a ferromagnet.