Lecture 9

9.1 : UF Expts on Gd

In order to understand the linear T-dependence of σ_{xx}, we consider the spin-wave contributions, because Gd has a much smaller Curie temp. (293K) compared to Fe (~1000°K) and also a small spin-wave gap ($\Delta_g \sim 30$ mK)

[Ref : R. Misra et al, e-print cond-mat arXiv: 0808.4103]

9.2 : Model Hamiltonian

$$H = \sum_{k\sigma} \left(\epsilon_k - \tfrac{1}{2}\sigma B\right) c^{+}_{k\sigma} c_{k\sigma} + \sum_{kk'\sigma j} V(\bar{k}-\bar{k}')\, e^{i(\bar{k}-\bar{k}')\cdot \bar{R}_j} c^{+}_{k\sigma} c_{k\sigma}$$

$$+ \sum_{q} \omega_q a^{+}_q a_q + J \sum_{q,k} \left[a^{+}_q c^{+}_{k+q\downarrow} c_{k\uparrow} + h.c.\right]$$

where c_k, c_k^{+} are electron field operators and a_q, a_q^{+} are the spin-wave operators. J is the effective spin-exchange interaction, and the spin-wave is characterized by $\omega_q = \Delta_g + Aq^2$, where $\Delta_g = \mu_B B_{ext} \approx 1\,K/\text{Tesla}$ is the spin wave gap and $A \approx J/k_F^2$ is the spin stiffness. $B \approx J k_F^2$ is the exchange splitting.

Note 1 : $\Delta_g < T$ & we can set $\Delta_g = 0$

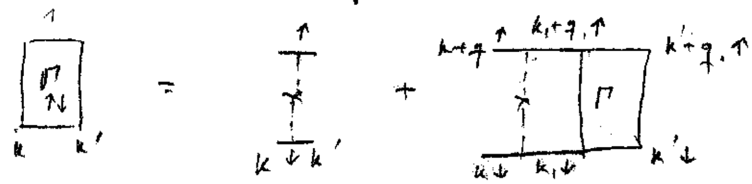
Note 2 : $B = 700$ meV at 20K and $\epsilon_F = 3.4$ eV

$\therefore$ $B\tau \gg 1$ but $B/\epsilon_F \ll 1$

9.3 : Spin-wave propagators :

$$S_{\uparrow\downarrow}(q, \omega_n) = \frac{1}{i\omega_n - a\omega_q} = \left[S_{\downarrow\uparrow}\right]^{*}$$

Spin p-h propagator :



$$\Gamma_{\uparrow\downarrow} = \frac{1}{2\pi N_0 \tau} + \frac{1}{2\pi N_0 \tau} \sum_{k_1} G_{k_1\downarrow}(\varepsilon_n)\, G_{k_1+q\uparrow}(\varepsilon_n+\omega_m)\, \Gamma_{\uparrow\downarrow}$$

Define $\sum_k G_{k\downarrow}(\varepsilon_n)\, G_{k+q\uparrow}(\varepsilon_n+\omega_m) \equiv X_{\uparrow\downarrow}(q, \varepsilon_n, \omega_m)$

$$\Gamma\left(1 - \frac{X}{2\pi N_0 \tau}\right) = \frac{1}{2\pi N_0 \tau}$$

$$\Rightarrow \quad \Gamma = \frac{1/2\pi N_0 \tau}{1 - X/2\pi N_0 \tau} \quad \text{and} \quad 1 + \Gamma X = \frac{1}{2\pi N_0 \tau}\Gamma$$

It is easy to show that for $q=0$, $X^{+-}_{\uparrow\downarrow}$ is given by

$$\sum_k G^{+}_{k\uparrow}(\varepsilon_n+\omega_m)\, G^{-}_{k\downarrow}(\varepsilon_n) = 2\pi N_0 \hat{\tau}, \quad \text{where}$$

$$\frac{1}{\hat{\tau}} \equiv \frac{1}{\tau} + \omega_m - iB$$

where we have used $\xi_{k\uparrow} = \xi_k - \frac{B}{2}$, $\xi_{k\downarrow} = \xi_k + \frac{B}{2}$

Given the result for $q=0$, we expand for small q :

$$X^{+-}_{\uparrow\downarrow} = \sum G^{+}_{k\uparrow}(\varepsilon_n+\omega_m)\left[1 + \underbrace{(\vec{q}\cdot\vec{v}_F)\, G^{+}_{k\uparrow}}_{\int d\theta \to 0} + \underbrace{(\vec{q}\cdot\vec{v}_F)^2}_{\int d\theta \to \frac{1}{2}} G^{3}_{k\uparrow} + \cdots\right] G_{k\downarrow}(\varepsilon_n)$$

$$= 2\pi N_0 \hat{\tau}\left(1 - \hat{D} q^2 \tau\right)$$

where $\hat{D} \equiv \frac{1}{2} v_F^2 \frac{\hat{\tau}^2}{\tau} = D(\hat{\tau}/\tau)^2$

This leads to the p-h propagator

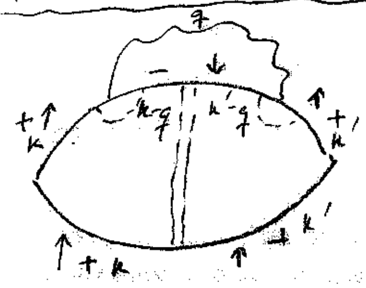
$$\Gamma_{\uparrow\downarrow}(q, \omega_m) = \frac{1}{2\pi N_0 \tau}\, \frac{1}{\hat{\tau}}\, \frac{1}{\omega_m - iB + \hat{D} q^2}$$

Similarly $\rightarrow$

$$\Gamma^{\downarrow\uparrow}(q,\omega_m) = \frac{1}{2\pi N_0 \tau} \; \frac{1}{\hat{\hat{\tau}}} \; \frac{1}{\omega_m + iB + \hat{D} q^2} \,, \quad \text{where}$$

$$\hat{\hat{D}} = \frac{1}{2} v_F^2 \frac{\hat{\hat{\tau}}^2}{\tau} \,, \qquad \frac{1}{\hat{\hat{\tau}}} = \frac{1}{\tau} + \omega_m + iB$$

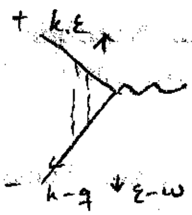
9.4 : Correction to conductivity



$$= \sum G^{+}_{k\uparrow}(\varepsilon)\, G^{-}_{k-q\downarrow}(\varepsilon_n - \omega_m)\, G^{-}_{k'-q\downarrow}(\varepsilon_n-\omega_m)\, G^{+}_{k'\uparrow}(\varepsilon_n)$$

$$\Gamma_{\uparrow\downarrow}(q,\omega)\, G^{+}_{k'\uparrow}(\varepsilon_n)\, G^{+}_{k\uparrow}(\varepsilon_n)\, S^{-+}_{\uparrow\downarrow}(q,\omega)$$

$$\left[T_{\uparrow\downarrow}(q,\omega) \right]^2$$

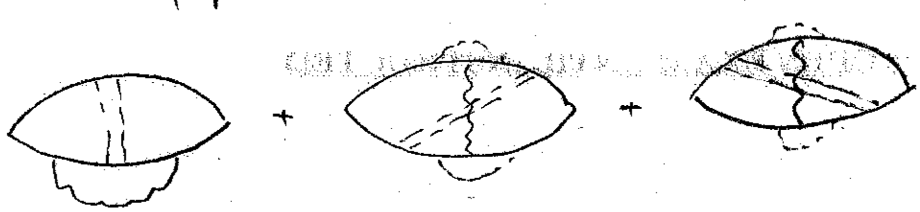
where $T_{\uparrow\downarrow} =$  $= 1 + \Gamma^{-+}_{\uparrow\downarrow} X_{\uparrow\downarrow} = 2\pi N_0 \tau \, \Gamma_{\uparrow\downarrow}$

$$\rightarrow q^2 v_F^2 \left(2\pi N_0 \tau \hat{\tau}^2\right)^2 \left(\Gamma_{\uparrow\downarrow}\right)^3 S_{\uparrow\downarrow} \left(2\pi N_0 \tau\right)^2$$

$$\delta\sigma \sim T \sum_{\omega_m} \sum_q q^2 \hat{\tau} \frac{1}{(\omega_m - iB + \hat{D} q^2)^3} \frac{1}{i\omega_m - b q^2} + (\uparrow\downarrow \rightarrow \downarrow\uparrow)$$

$$= \int_0^{q_c} \frac{q^2 dq^2}{(2\pi)^2} \int_{2\pi T}^{\omega_c} d\omega \frac{\tau}{1 + (\omega - iB)\tau} \frac{1}{(\omega - iB + \hat{D} q^2)^3} \frac{1}{i\omega - b q^2} + (\;\;)$$

$$q_c = \frac{1}{|v_F \hat{\tau}|} \qquad \omega_c = 1/\tau \,, \qquad B\tau \gg 1$$



$$\Rightarrow \frac{\delta\sigma}{\sigma_{00}} \approx 4 N_0 J \frac{nJ}{B} \frac{\varepsilon_F}{B} (\varepsilon_F \tau) \frac{T}{A k_F^2}$$