Lecture 8:

8.1: Summary of strong scattering model:

$$\sigma_{xx} = \sigma_{xx}^{ss} + \cancel{\sigma_{xx}^{sj}} \nearrow 0 \approx \sum \tfrac{1}{2} v_{F\sigma}^2 N_\sigma \tau_{tr}$$

$$\sigma_{xy} = \sigma_{xy}^{ss} + \sigma_{xy}^{sj} = \sigma_{xx}^{ss} \frac{\mathrm{Im}\lambda_1}{1-\mathrm{Re}\lambda_1} + L_{00}\,\pi \sum \tau_{\sigma\sigma}^{z} g_\sigma \frac{1-\mathrm{Re}\lambda_1}{|1-\lambda_1|^2}$$

Quantum corrections, (only ss important)

$$\delta\sigma_{xx}^{I} = L_{00}\, h_{xx} \ln(T\tau) \; ; \; \delta\sigma_{xx}^{WL} = L_{00} \ln(T\tau)$$

$$\delta\sigma_{xy}^{I} = 0 \; , \quad \delta\sigma_{xy}^{WL} = \delta\sigma_{xx}^{WL} \frac{\mathrm{Im}\lambda_1}{1-\mathrm{Re}\lambda_1}$$

8.2: UF expts on Fe-film; $R_0 < 3\,k\Omega$

$$\Delta^N\sigma_{xx} = \frac{\sigma_{xx}^{ss}}{L_{00}} \frac{\delta\sigma_{xx}^{ss,I} + \delta\sigma_{xx}^{ss,WL}}{\sigma_{xx}^{ss}} = (\underbrace{1}_{WL} + \underbrace{h_{xx}}_{\text{exchange plus Hartree}}) \ln(T\tau)$$

$$\Delta^N\sigma_{xy} = \frac{\sigma_{xx}^{ss}}{L_{00}} \cdot \frac{\delta\sigma_{xy}^{ss,WL}}{\sigma_{xy}^{ss} + \sigma_{xy}^{sj}} = \frac{1}{1+r_{xy}} \ln(T\tau)$$

where $r_{xy} \equiv \dfrac{\sigma_{xy}^{sj}}{\sigma_{xy}^{ss}} \rightarrow$ a non universal quantity

Compare expt:

$$\Delta^N\sigma_{xx} = A'_R \ln\frac{T}{T_0}$$

$$\Delta^N\sigma_{xy} = (2A_R - A_{AH}) \ln\frac{T}{T_0}$$

$$\Rightarrow A_R = 1 + h_{xx} \; , \quad 2A_R - A_{AH} = \frac{1}{1+r_{xy}}$$

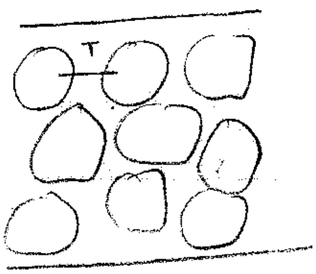
glass substrate : $h_{xx} \to 0$, $r_{xy} \to 0$

sapphire / BY : $h_{xx} \to 0$, $r_{xy} \gg 1$

Note: Contradicts BY explanation that $\ln T$ is interaction correction as opposed to WL correction.

8.3 : Fe-film : $R_0 >> 3k\Omega$

Experimental observation → film becomes granular



R_{xx} dominated by tunneling : $R_{xx} \approx R_{xx}^T$

No current along $y \Rightarrow$ grains coupled as capacitors

$\Rightarrow R_{xy} \approx R_{xy}^g$ of a grain independent of R_{xx}^T

ie $R = \begin{pmatrix} R_{xx}^T & -R_{xy}^g \\ R_{xy}^g & R_{xx}^T \end{pmatrix} \Rightarrow \dfrac{\delta R_{xy}}{R_{xy}} \approx \dfrac{\delta R_{xy}^g}{R_{xy}^g}$

→ independent of $R_{xx}^T \approx R_0$

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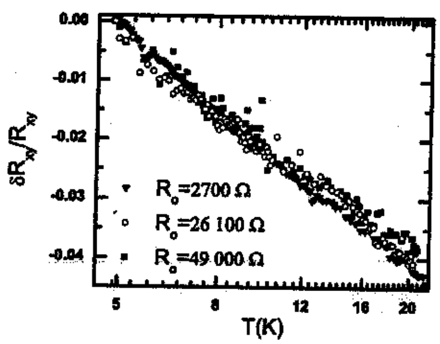

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FIG. 4 (color online). ln(T) dependence of relative changes in the AH resistance for type A films with three different R_0.

Prob : Show that in this granular model

$$\frac{\delta\sigma_{xy}}{\sigma_{xy}} = \frac{\delta\sigma_{xy}^g}{\sigma_{xy}^g} + 2\,\frac{\delta\sigma_{xx}^T}{\sigma_{xx}^T} - 2\,\frac{\delta\sigma_{xx}^g}{\sigma_{xx}^g}$$

Note : AHE calculations remain valid within single grains.

$\Rightarrow A_R^g = 1 , \quad A_{AH}^g = 1$

Using $\dfrac{\delta\sigma_{xx}^T}{\sigma_{xx}^T} = L_{00} R_0^T A_R \ln(T/T_0)$, this gives

$$\Delta^N(\rho_{xy}^{WL}) = \left(2A_R - \frac{R_0^{\,g}}{R_0^T}\right) \ln T/T_0$$

Compare : $A_{AH} = R_0^g / R_0^T \sim \frac{1}{R_0}$

· consistent with expt.

Also, A_R involves only tunneling resistances, while $A_{AH} \sim R_0^g/R_0^T \ll 1$, so expect $A_{AH} < A_R$.

again consistent with expt.

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8.4 : Phase relaxation rates in Fe and spin waves :

In order to have WL corrections in Fe, we need large τ_φ^{-1} s.t.

$$\max\left(\frac{1}{\tau_s}, \frac{1}{\tau_{so}}, \omega_H\right) \ll \frac{1}{\tau_\varphi} \ll \frac{1}{\tau_{tr}}.$$

where $\omega_H = 4\epsilon_F\tau_{tr} \cdot \frac{eB_{in}}{m^* c}$, B_{in} is the internal field in Tesla

It turns out that $\frac{1}{\tau_\varphi} = \frac{T}{\epsilon_F\tau_{tr}} \cdot \ln\frac{\epsilon_F\tau_{tr}}{2}$ is too small, and $\ln T$ from WL should not be observable.

However a much larger contribution comes from spin waves :

$$\frac{1}{\tau_\varphi} \simeq 4\pi \frac{J^2}{\epsilon_F\Delta_g} T ,$$

$J = 160K$: exchange energy of s-electrons

$\Delta_g \simeq 1K \left(\frac{m}{m^*}\right) B_{in}$: spin wave gap

For $150\Omega < R_0 < 3 k\Omega$, $\epsilon_F\tau_{tr} < 10$

$\therefore$ $\omega_H\tau_\varphi < 1$ can be satisfied down to $5K$.

Lecture 9

9.1 : UF Expts on Gd

In order to understand the linear T-dependence of σ_{xx}, we consider the spin-wave contributions, because Gd has a much smaller Curie temp. (293K) compared to Fe (~1000°K) and also a small spin-wave gap ($\Delta_g \sim 30\,mK$)

[Ref : R. Misra et al, e-print cond-mat arXiv: 0808.4103]

9.2 : Model Hamiltonian

$$H = \sum_{k\sigma} \left(\epsilon_k - \tfrac{1}{2}\sigma B \right) c^\dagger_{k\sigma} c_{k\sigma} + \sum_{kk'\sigma j} V(\bar{k}-\bar{k}')\, e^{i(\bar{k}-\bar{k}')\cdot \bar{R}_j} c^\dagger_{k\sigma} c_{k\sigma}$$

$$+ \sum_q \omega_q a^\dagger_q a_q + J \sum_{q,k} \left[a^\dagger_q c^\dagger_{k+q\downarrow} c_{k\uparrow} + h.c. \right]$$

where $c_k, c^\dagger_k$ are electron field operators and $a_q, a^\dagger_q$ are the spin-wave operators. J is the effective spin-exchange interaction, and the spin-wave is characterized by $\omega_q = \Delta_g + Aq^2$, where $\Delta_g = \mu_B B_{ext} \approx 1K/\text{Tesla}$ is the spin wave gap and $A \approx J/k_F^2$ is the spin stiffness.

$B \approx J k_F^2$ is the exchange splitting.

Note 1 : $\Delta_g < T$ & we can set $\Delta_g = 0$

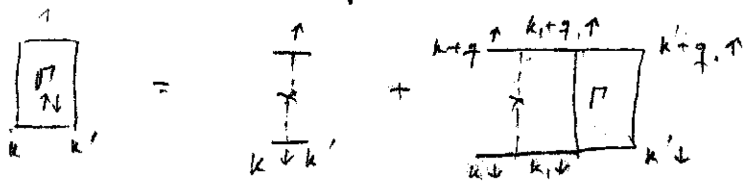
Note 2 : $B = 700\, meV$ at $20K$ and $\epsilon_F = 3.4\, eV$

$\therefore$ $B\tau \gg 1$ but $B/\epsilon_F \ll 1$

9.3 : Spin-wave propagators :

$$S_{\uparrow\downarrow}(q, \omega_n) = \frac{1}{i\omega_n - a\omega_q} = \left[S_{\downarrow\uparrow} \right]^*$$

Spin p-h propagator :



$$\Gamma_{\uparrow\downarrow} = \frac{1}{2\pi N_0 \tau} + \frac{1}{2\pi N_0 \tau} \sum_{k_1} G_{k_1\downarrow}(\varepsilon_n)\, G_{k_1+q\uparrow}(\varepsilon_n+\omega_m)\, \Gamma_{\uparrow\downarrow}$$

Define $\quad \sum_k G_{k\downarrow}(\varepsilon_n)\, G_{k+q\uparrow}(\varepsilon_n+\omega_m) \equiv X_{\uparrow\downarrow}(q, \varepsilon_n, \omega_m)$

$$\Gamma\left(1 - \frac{X}{2\pi N_0 \tau}\right) = \frac{1}{2\pi N_0 \tau}$$

$$\Rightarrow \quad \Gamma = \frac{1/2\pi N_0\tau}{1 - X/2\pi N_0 \tau} \quad \text{and} \quad 1 + \Gamma X = \frac{1}{2\pi N_0 \tau}\Gamma$$

It is easy to show that for $q=0$, $X_{\uparrow\downarrow}^{+-}$ is given by

$$\sum_k G_{k\uparrow}^{+}(\varepsilon_n+\omega_m)\, G_{k\downarrow}^{-}(\varepsilon_n) = 2\pi N_0 \hat{\tau}, \quad \text{where}$$

$$\frac{1}{\hat{\tau}} \equiv \frac{1}{\tau} + \omega_m - iB$$

where we have used $\quad \xi_{k\uparrow} = \xi_k - \frac{B}{2}, \quad \xi_{k\downarrow} = \xi_k + \frac{B}{2}$

Given the result for $q=0$, we expand for small q :

$$X_{\uparrow\downarrow}^{+-} = \sum G_{k\uparrow}^{+}(\varepsilon_n+\omega_m)\left[1 + \underbrace{(\vec{q}\cdot\vec{v}_F)\, G_{k\uparrow}^{+}}_{\int d\theta \to 0} + \underbrace{(\vec{q}\cdot\vec{v}_F)^2}_{\int d\theta \to \frac{1}{2}} G_{k\uparrow}^{3} + \cdots\right] G_{k\downarrow}(\varepsilon_n)$$

$$= 2\pi N_0 \hat{\tau}\left(1 - \hat{D} q^2 \tau\right)$$

where $\quad \hat{D} \equiv \frac{1}{2} v_F^2 \frac{\hat{\tau}^2}{\tau} = D\left(\hat{\tau}/\tau\right)^2$

This leads to the p-h propagator

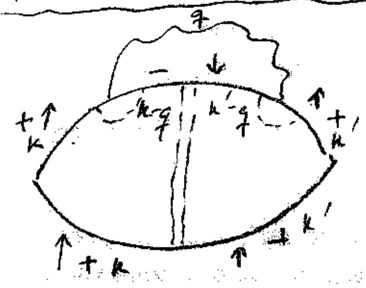
$$\Gamma_{\uparrow\downarrow}(q, \omega_m) = \frac{1}{2\pi N_0 \tau}\, \frac{1}{\hat{\tau}}\, \frac{1}{\omega_m - iB + \hat{D} q^2}$$

Similarly →

$$\Gamma^{\downarrow\uparrow}(q,\omega_m) = \frac{1}{2\pi N_0 \tau}\,\frac{1}{\hat{\hat{\tau}}}\,\frac{1}{\omega_m + iB + \hat{D} q^2}, \quad \text{where}$$

$$\hat{\hat{D}} = \frac{1}{2} v_F^2 \frac{\hat{\hat{\tau}}^2}{\tau}, \qquad \frac{1}{\hat{\hat{\tau}}} = \frac{1}{\tau} + \omega_m + iB$$

9.4 : Correction to conductivity

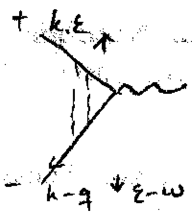


$$= \sum G^{+}_{k\uparrow}(\varepsilon)\, G^{-}_{k-q\downarrow}(\varepsilon_n-\omega_m)\, G^{-}_{k'-q\downarrow}(\varepsilon_n-\omega_m)\, G^{+}_{k'\uparrow}(\varepsilon_n)$$

$$\Gamma_{\uparrow\downarrow}(q,\omega)\, G^{+}_{k'\uparrow}(\varepsilon_n)\, G^{+}_{k\uparrow}(\varepsilon_n)\, S^{-+}_{\uparrow\downarrow}(q,\omega)$$

$$\left[T_{\uparrow\downarrow}(q,\omega)\right]^2$$

where $T_{\uparrow\downarrow}$ =



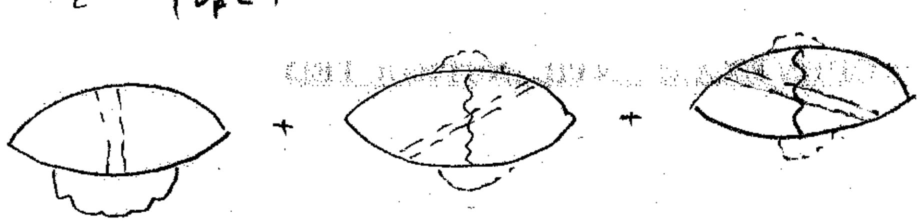
$= 1 + \Gamma^{-+}_{\uparrow\downarrow} X_{\uparrow\downarrow} = 2\pi N_0 \tau\, \Gamma_{\uparrow\downarrow}$

$$\rightarrow q^2 v_F^2 \left(2\pi N_0 \tau \hat{\tau}^2\right)^2 \left(\Gamma_{\uparrow\downarrow}\right)^3 S_{\uparrow\downarrow} \left(2\pi N_0 \tau\right)^2$$

$$\delta\sigma \sim T \sum_{\omega_m} \sum_q q^2 \hat{\tau} \frac{1}{(\omega_m - iB + \hat{D} q^2)^3} \frac{1}{i\omega_m - b q^2} + (\uparrow\downarrow \to \downarrow\uparrow)$$

$$= \int_0^{q_c} \frac{q^2 dq^2}{(2\pi)^2} \int_{2\pi T}^{\omega_c} d\omega \frac{\tau}{1+(\omega - iB)\tau} \frac{1}{(\omega - iB + \hat{D} q^2)^3} \frac{1}{i\omega - b q^2} + (\ )$$

$q_c = \frac{1}{|v_F \hat{\tau}|}$, $\omega_c = 1/\tau$, $B\tau \gg 1$



$$\Rightarrow \frac{\delta\sigma}{\sigma_{00}} \approx 4 N_0 J \frac{nJ}{B} \frac{\varepsilon_F}{B} (\varepsilon_F \tau) \frac{T}{A k_F^2}$$