Quantum kinetic equations
Lecture #2

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Outline

Kadanoff-Baym/Keldysh equations ▸ more

Kadanoff-Baym/Keldysh equations (KBE)—Review

$$\left(i\frac{\partial}{\partial t_1} - H^1(1)\right) G(1,\bar{1}) = \delta_{\mathscr{C}'}(1-\bar{1}) \pm i\int_{\mathscr{C}'} \mathrm{d}2\, W(1-2)\, G_{12}(1,2;\bar{1},2^+)$$

$$\left(-i\frac{\partial}{\partial t_{\bar{1}}} - H^1(\bar{1})\right) G(1,\bar{1}) = \delta_{\mathscr{C}'}(1-\bar{1}) \pm i\int_{\mathscr{C}'} \mathrm{d}2\, W(\bar{1}-2)\, G_{12}(1,2;\bar{1},2^+)$$

- $H^1(1) = -\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_1} + V(1) - \mu\hat{N}$ denotes the single-particle Hamiltonian
- The upper sign refers to bosons the lower one refers to fermions, and the temporal integrations are taken over the Schwinger/Keldysh time contour $\mathscr{C}'$
- KBE not closed but coupled to higher orders via the 2-particle Green's function $G_{12}(1,2;\bar{1},\bar{2})$
- *Martin-Schwinger (MS) hierarchy*: The n-particle Green's function generally requires information from the $(n \pm 1)$-particle Green's function

MS Hierarchy decoupling

Idea: Express 2-particle Green's function G_{12} in terms of products of 1-particle Green's functions

$$G_{12}(1,2;\bar{1},\bar{2}) \quad = \quad [G(\cdot,\cdot)G(\cdot,\cdot)](1,2;\bar{1},\bar{2}) + \Delta_{\mathrm{corr}}(1,2;\bar{1},\bar{2})$$

with corrections $\Delta_{\mathrm{corr}}(1,2;\bar{1},\bar{2})$ ($\Rightarrow$ correlation contributions)

- Simplest example: *Hartree-Fock* approximation (first order)

Fock (exchange)

$$G_{12}(1,2;\bar{1},\bar{2}) \quad \approx \quad G(1,\bar{1})\,G(2,\bar{2}) \pm G(1,\bar{2})\,G(2,\bar{1})$$

Hartree (classical mean-field)

Formal MS hierarchy decoupling

Introduce self-energy Σ:

$$\pm i \int_{\mathscr{C}'} \mathrm{d}2 \; W(1-2)\, G_{12}(1,2;\bar{1},2^+) \quad \overset{!}{=} \quad \int_{\mathscr{C}'} \mathrm{d}2 \; \Sigma[G,W](1,2)\, G(2,\bar{1})$$

Remarks:

- All interaction effects are now contained in the self-energy $\Sigma[G,W](1,\bar{1})$
- The self-energy appears as a functional of the interaction potential $W(1,\bar{1})$ and the 1-particle Green's function $G(1,\bar{1})$, and thus varies on the Schwinger/Keldysh contour $\mathscr{C}' \longrightarrow \Sigma^{\gtrless}, \Sigma^{\rceil/\lceil}$, and Σ^{M}.
- In general, the pair interaction enters Σ as infinite summations
- A diagrammatic expansion of Σ (known from ground state theory) can be extended to finite temperature and nonequilibrium situations

Conserving approximations

Definition: Approximations for G_{12} which conserve total energy, momentum and angular momentum [when calculated from $G(1,\bar{1})$] are denoted as *conserving approximations*.

How can one guarantee that a self-energy gives a conserving approximation for the Green's function?

- Conservation of total energy and momentum if
  - (A) $G(1,\bar{1})$ satisfies the general equations of motions (KBE),
  - (B) the approximation for G_{12} satisfies the symmetry condition

$$G_{12}(1,2;1^+,2^+) \quad = \quad G_{12}(2,1;2^+,1^+)\,.$$

[G. Baym, and L.P. Kadanoff, Phys. Rev **124**, 287 (1961)]

- Conclusion: Φ-derivable self-energy more

$$\Sigma(1,\bar{1}) \quad = \quad \frac{\delta\Phi[G]}{\delta G(\bar{1},1)}\,.$$

How Φ can be constructed? [J.M. Luttinger, and J.C. Ward Phys. Rev. **118**, 1417 (1960)]

Lowest-order Σ-diagrams

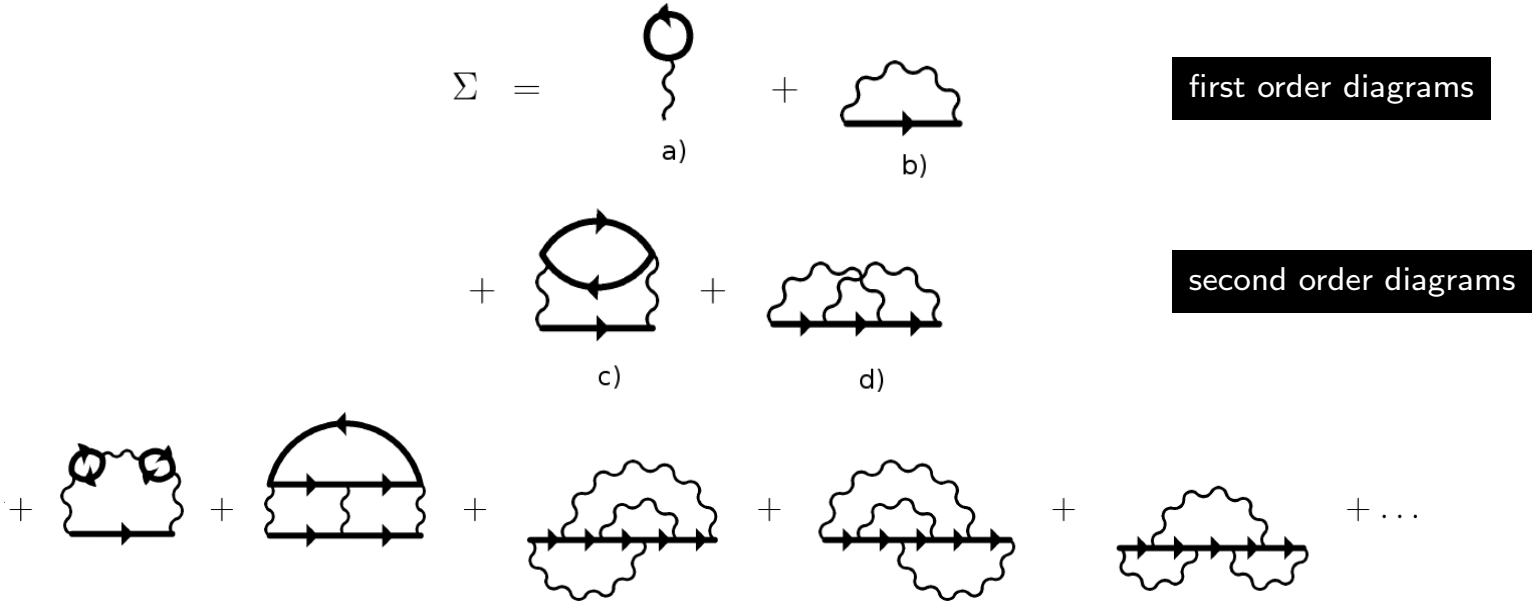

Figure: Self-energy (Feynman) diagrams. First order in the interaction $W(1,\bar{1})$: a) Hartree, b) Fock. Second order in $W(1,\bar{1})$: c) & d) second Born terms.

- Waved lines ($1 \sim\sim \bar{1}$) indicate the interaction $W(1-\bar{1})$, whereas solid lines ($1 \rightarrow \bar{1}$) denote the *full* nonequilibrium Green's function $G(1,\bar{1})$
- All self-energy terms contain an infinite summation of diagrams.

Interpretation of Σ-diagrams

Contributions of *first order* in the interaction W :

Figure: First order self-energies: Hartree (class. mean-field) term [left], Fock (exchange) term [right].

Hartree-Fock self-energy $\Sigma^{\mathrm{HF}} = \Sigma^{\mathrm{H}} + \Sigma^{\mathrm{F}}$

$$\Sigma^{\mathrm{H}}(1,\bar{1}) = \pm i\,\delta_{\mathscr{C}}(1-\bar{1}) \int_{\mathscr{C}} \mathrm{d}2\, W(1-2)\, G(2,2^{+})$$

$$\Sigma^{\mathrm{F}}(1,\bar{1}) = i\, G(1,\bar{1})\, W(1^{+}-\bar{1})$$

- Note the time-ordering and recall $1^{+} = t_1 \longrightarrow t_1 + \epsilon_{>0}$
- Contributions of order n take a sign of $(-i)^n$, the direct (Hartree) term has an additional minus sign only for fermions

Interpretation of Σ-diagrams

Contributions of *second order* in the interaction W :

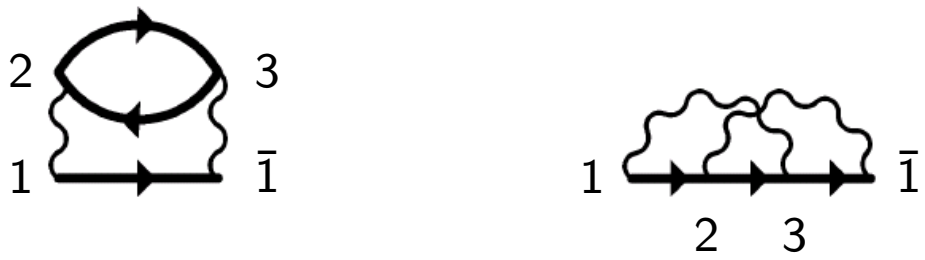

Figure: Second (order) Born diagrams, composed of two interaction lines and three Green's functions. Direct term [left], exchange term [right].

Second Born contributions $\Sigma^{\rm 2ndB} = \Sigma_1^{\rm 2ndB} + \Sigma_2^{\rm 2ndB}$

$$\Sigma_2^{\rm 2ndB}(1,\bar{1}) = \pm(-i)^2 \int_{\mathscr{C}} {\rm d}2\,{\rm d}3\; G(1,\bar{1})\, W(1^+ - 2)\, W(\bar{1} - 3)\, G(3,2)\, G(2,3^+)$$

$$\Sigma_1^{\rm 2ndB}(1,\bar{1}) = (-i)^2 \int_{\mathscr{C}} {\rm d}2\,{\rm d}3\; G(1,2)\, W(1 - 3^+)\, G(2,3)\, G(3,\bar{1})\, W(2^+ - \bar{1})$$

- $\Sigma_1^{\rm 2ndB}$ and $\Sigma_2^{\rm 2ndB}$ are to be added to the Hartree-Fock self-energy $\Sigma^{\rm HF}$

Equation of motion for $G^*(1,\bar{1})$, $* \in \{M, >, <, \rceil, \lceil\}$

Keldysh/Kadanoff-Baym equations:

$$\left(i\frac{\partial}{\partial t_1} - H^1(1)\right) G(1,\bar{1}) \;=\; \delta_{\mathscr{C}'}(1-\bar{1}) + \int_{\mathscr{C}'} \mathrm{d}2\, \Sigma(1,2)\, G(2,\bar{1})$$

together with its adjoint equation including the derivative $\partial_{\bar{1}}$

- $G(1,\bar{1})$ is a 3×3-matrix of the form

$$\left(\begin{array}{cc|c} G^{++} & G^{+-} & G^{+\rceil} \\ G^{-+} & G^{--} & G^{-\rceil} \\ \hline G^{\rceil +} & G^{\rceil -} & G^{\rceil\rceil} \end{array}\right) = \left(\begin{array}{cc|c} G^{c} & G^{<} & G^{\rceil} \\ G^{>} & G^{a} & G^{\rceil} \\ \hline G^{\lceil} & G^{\lceil} & G^{M} \end{array}\right)$$

- Need equations for the elements of the Keldysh matrix

$$G^{\gtrless}(1,\bar{1})\,, \qquad G^{M}(1,\bar{1})\,, \qquad G^{\lceil/\rceil}(1,\bar{1})$$

- **Question**: How to evaluate the integrals $\int_{\mathscr{C}'} \mathrm{d}2\, \Sigma(1,2)\, G(2,\bar{1})$ for the matrix elements of a product or convolution? $\Rightarrow$ "Langreth rules"

Langreth rules

Results for Convolution and product of two (Keldysh) functions defined on $\mathscr{C}'$

quantity[1]	convolution	product
	$c(z,z') = \int_{\mathscr{C}'} d\bar{t}\, a(t,\bar{t})\, b(\bar{t},t')$	$c(z,z') = a(t,t')\, b(t',t)$
$c^{>}(t,t')$	$a^{>}\circ b^{A} + a^{R}\circ b^{>} + a^{\rceil}\star b^{\lceil}$	$a^{>}\, b^{<}$
$c^{<}(t,t')$	$a^{<}\circ b^{A} + a^{R}\circ b^{<} + a^{\rceil}\star b^{\lceil}$	$a^{<}\, b^{>}$
$c^{\rceil}(\tau,t)$	$a^{R}\circ b^{\rceil} + a^{\rceil}\star b^{M}$	$a^{\rceil}\, b^{\lceil}$
$c^{\lceil}(t,\tau)$	$a^{\lceil}\circ b^{A} + a^{M}\star b^{\lceil}$	$a^{\lceil}\, b^{\rceil}$
$c^{M}(\tau,\tau')$	$a^{M}\star b^{M}$	$a^{M}\, b^{M}$
$c^{R}(t,t')$	$a^{R}\circ b^{R}$	$a^{R}\, b^{\gtrless} + a^{\gtrless}\, b^{A}$
$c^{A}(t,t')$	$a^{A}\circ b^{A}$	$a^{A}\, b^{\gtrless} + a^{\gtrless}\, b^{R}$
	$[a\circ b](t,t') = \int_{t_0}^{\infty} \mathrm{d}\bar{t}\;\; a(t,\bar{t})\, b(\bar{t},t')$	
	$[a\star b](t,t') = -i\int_{t_0}^{t_0-i\beta} \mathrm{d}\bar{\tau}\;\; a(t,\bar{\tau})\, b(\bar{\tau},t')$	$a^{R/A}(t,t') = a^{\gtrless}(t,t') - a^{\lessgtr}(t,t')$

[1] Space variables are dropped for simplicity

Equation of motion for the correlation functions $G^{\gtrless}$

Summary:

Write ∂_t instead of $\frac{\partial}{\partial_t}$

$$\left(i\,\partial_{t_1} - H^1(1)\right) G^{\gtrless}(1,\bar{1}) = \int_{t_0}^{t_1} \mathrm{d}2\; \Sigma^R(1,2)\, G^{\gtrless}(2,\bar{1}) + \int_{t_0}^{t_{\bar{1}}} \mathrm{d}2\; \Sigma^{\gtrless}(1,2)\, G^A(2,\bar{1})$$

1-p. energy — memory kernel — memory kernel

$$-i\int_0^{\beta} \mathrm{d}2\; \Sigma^{\rceil}(1,2)\, G^{\lceil}(2,1)$$

initial correlations

$$\left(-i\,\partial_{t_{\bar{1}}} - H^1(\bar{1})\right) G^{\gtrless}(1,\bar{1}) = \int_{t_0}^{t_1} \mathrm{d}2\; G^R(1,2)\,\Sigma^{\gtrless}(2,\bar{1}) + \int_{t_0}^{t_{\bar{1}}} \mathrm{d}2\; G^{\gtrless}(1,2)\,\Sigma^A(2,\bar{1})$$

$$-i\int_0^{\beta} \mathrm{d}2\; G^{\rceil}(1,2)\,\Sigma^{\lceil}(2,1)$$

with definitions of retarded (R) and advanced (A) quantities

$$G^{R/A}(1,\bar{1}) = G^{\gtrless}(1,\bar{1}) - G^{\lessgtr}(1,\bar{1})\,, \qquad \Sigma^{R/A}(1,\bar{1}) = \Sigma^{\gtrless}(1,\bar{1}) - \Sigma^{\lessgtr}(1,\bar{1})$$

Memory kernels: Memory and dissipation, decay of the Green's function. Initial correlations enter via the Matsubara Green's function $G^M \Rightarrow G^{\gtrless}(\mathbf{r}_1 t_0, \mathbf{r}_{\bar{1}} t_0)$ and $G^{\rceil}(\mathbf{r}_1 t_0, \mathbf{r}_{\bar{1}} t_0 - i\tau)$, $G^{\lceil}(\mathbf{r}_1 t_0 - i\tau, \mathbf{r}_{\bar{1}} t_0)$.

Derivation of single-time kinetic equations

Single-time kinetic equations can be obtained in the limit of slowly varying disturbances.

Motivation:

- Avoid to deal with two-time quantities, or obtain only statistical information (fully included in the Wigner function—the time-diagonal part of $G^<$)
- Relevant for studying *simple* transport processes

Basic assumptions:

- Consider the case where the single-particle energy $H^1(\mathbf{r}_1, t_1)$ is slowly varying in time and space
- Consequence of a scale separation: $G^{\gtrless}(1, \bar{1})$ are slowly varying functions of the coordinates

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_{\bar{1}}}{2} , \qquad \mathbf{T} = \frac{t_1 + t_{\bar{1}}}{2} ,$$

but are sharply peaked about zero values of

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_{\bar{1}} , \qquad \mathbf{t} = t_1 - t_{\bar{1}} .$$

$\Rightarrow$ Consider $G^{\gtrless}(\mathbf{r}t, \mathbf{R}T)$ and its Fourier transform in $\mathbf{r}$ and t ▸ remind yourself

Derivation of the Boltzmann equation I

Recipe: [see L.P. Kadanoff, and G. Baym, *Quantum Statistical Mechanics* (Benjamin, Inc., New York, 1962)]

1. Substract adjoint KBE (with time arguments t_1 and $t_{\bar{1}}$ interchanged) from the KBE for $G^<(1,\bar{1})$, and express the space time variables of G and Σ in terms of relative and center of mass coordinates.
2. Neglect the third collision integral on the r.h.s. accounting for initial correlations—instead integrate over $\int_{-\infty}^{t_1} \mathrm{d}\bar{t}\ \ldots$ and $\int_{-\infty}^{t_{\bar{1}}} \mathrm{d}\bar{t}\ \ldots$
3. Expand the difference of

$$H^1\left(\mathbf{r}_1 = \mathbf{R} - \frac{\mathbf{r}}{2}, t_1 = T - \frac{t}{2}\right) - H^1\left(\mathbf{r}_{\bar{1}} = \mathbf{R} + \frac{\mathbf{r}}{2}, t_{\bar{1}} = T - \frac{t}{2}\right)$$

   in powers of $\mathbf{r}$ and t, and retain only lowest-order terms ($\mathbf{r}$ and t small)
4. Similarly, proceed with the relative variables on the r.h.s. (collision integrals) of the KBE—fully neglect the small quantities added to $\mathbf{R}$ and T.

Typical integrals:

$$\int_{\mathscr{C}'} \mathrm{d}^3\bar{r}\,\mathrm{d}\bar{t}\ G(\bar{\mathbf{r}}\,\bar{t};\mathbf{R}\,T)\,\Sigma(\mathbf{r}-\bar{\mathbf{r}}\,t-\bar{t};\mathbf{R}\,T)$$

convolution in Fourier space

Derivation of the Boltzmann equation II

5. Fourier transformation with respect to variables $\mathbf{r}$ and t lead to approximate equations for $G^{\gtrless}(\mathbf{p}\omega, \mathbf{R}T)$

   Intermediate result: $H^1 = \frac{-\nabla^2}{2m} + V(1)$ $\qquad \hbar = 1$

$$\left[\partial_T + \frac{\mathbf{p}\,\nabla_{\mathbf{R}}}{m} - \nabla_{\mathbf{R}} V(\mathbf{R}, T)\nabla_{\mathbf{p}} + \partial_T V(\mathbf{R}, T)\,\partial_\omega\right] G^{<}(\mathbf{p}\omega, \mathbf{R}T)$$

$$= -G^{<}(\mathbf{p}\omega, \mathbf{R}T)\,\Sigma^{>}(\mathbf{p}\omega, \mathbf{R}T) + G^{>}(\mathbf{p}\omega, \mathbf{R}T)\,\Sigma^{<}(\mathbf{p}\omega, \mathbf{R}T)$$

6. Insert second Born expressions for the self-energy $\Sigma^{\gtrless}$ (take all second spatial arguments in G to be $\mathbf{R}$—the disturbance varies very little within a distance of the order of the potential range)

7. Fourier transform in $\mathbf{r}$, t: $\Sigma^{\gtrless}(\mathbf{r}t, \mathbf{R}T) \to \Sigma^{\gtrless}(\mathbf{p}\omega, \mathbf{R}T)$

Derivation of the Boltzmann equation III

Self-energy (second order):

$$\begin{aligned}\Sigma^{\gtrless}(\mathbf{p}\omega,\mathbf{R}T) \approx & \frac{1}{2}\int \mathrm{d}^3\mathbf{p}'\mathrm{d}\omega'\,\mathrm{d}\bar{\mathbf{p}}\,\mathrm{d}\bar{\omega}\,\mathrm{d}^3\bar{\mathbf{p}}'\,\mathrm{d}\omega' \\ & \times \delta(\mathbf{p}+\mathbf{p}'-\bar{\mathbf{p}}-\bar{\mathbf{p}}')\,\delta(\omega+\omega'-\bar{\omega}-\bar{\omega}') \\ & \times [w(\mathbf{p}-\bar{\mathbf{p}}) \pm w(\mathbf{p}-\bar{\mathbf{p}}')]^2 \\ & \times G^{\lessgtr}(\mathbf{p}'\omega',\mathbf{R}T)\,G^{\gtrless}(\bar{\mathbf{p}}\bar{\omega},\mathbf{R}T)\,G^{\gtrless}(\bar{\mathbf{p}}'\bar{\omega}',\mathbf{R}T)\end{aligned}$$

8. Reconstruction problem: Assume that the Green's functions can be expressed via the Wigner distribution function $f(\mathbf{p},\mathbf{R},T)$ and the spectral function $A(\mathbf{p}\omega,\mathbf{R}T) = 2\pi\delta[\omega - E(\mathbf{p},\mathbf{R},T)]$

$$\begin{aligned}G^{<}(\mathbf{p}\omega,\mathbf{R}T) &= A(\mathbf{p}\omega,\mathbf{R}T)\,f(\mathbf{p},\mathbf{R},T) \\ G^{>}(\mathbf{p}\omega,\mathbf{R}T) &= A(\mathbf{p}\omega,\mathbf{R}T)\,[1 \pm f(\mathbf{p},\mathbf{R},T)]\end{aligned}$$

9. Some differential algebra ...

The Boltzmann (Landau) equation

Result: Boltzmann equation with Born approximation collision cross section:

$$\left[\partial_T + \frac{\mathbf{p}\,\nabla_{\mathbf{R}}}{m} - \nabla_{\mathbf{R}}\, V(\mathbf{R},T)\nabla_{\mathbf{p}}\right] f(\mathbf{p},\mathbf{R},T) = I(\mathbf{p},\mathbf{R},T)$$

collision integral

$$= -\frac{1}{2}\int \mathrm{d}^3\mathbf{p}'\,\mathrm{d}\bar{\mathbf{p}}\,\mathrm{d}^3\bar{\mathbf{p}}'\;\delta(\mathbf{p}+\mathbf{p}'-\bar{\mathbf{p}}-\bar{\mathbf{p}}')\,\delta\big(E(\mathbf{p})+E(\mathbf{p}')-E(\bar{\mathbf{p}})-E(\bar{\mathbf{p}}')\big)$$

$$\times\,[w(\mathbf{p}-\bar{\mathbf{p}}) \mp w(\mathbf{p}-\bar{\mathbf{p}}')]^2$$

$$\times\,\left[f\,f'\,(1\pm\bar{f})\,(1\pm\bar{f}') - (1\pm f)\,(1\pm f')\,\bar{f}\,\bar{f}'\right]$$

- $f(\mathbf{p},\mathbf{R},T)$ is the Wigner distribution function, $E(p)=\frac{p^2}{2m}$ denotes the quasiparticle energy. $f = f(\mathbf{p},\mathbf{R},t)$, $f' = f(\mathbf{p}',\mathbf{R},t)$, etc.
- Kinetic energy and momentum are preserved due to the δ-functions
- Limited to times larger than the correlation time, $T \gg \tau_{\mathrm{corr}}$

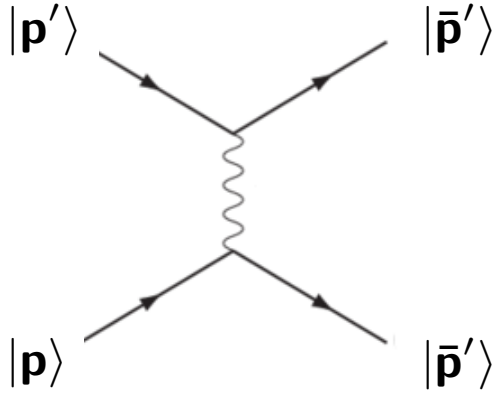

Figure: Feynman diagram

Conclusion I

Boltzmann equation

- Single-time equation for the Wigner function $f(\mathbf{p}, \mathbf{R}, T)$
- **Stationary solutions**: Maxwell, Fermi or Bose distributions
- $I(\mathbf{p}, \mathbf{R}, T) \longrightarrow 0$ leads to thermodynamic properties of the *ideal gas*, where correlations are neglected
- **Reason**: local approximation (neglect of memory)

  $\Rightarrow$ Memory is fully contained in the two-time Keldysh/Kadanoff-Baym equations for $G(1, \bar{1})$

Conclusion II

- MS hierachy formally decoupled by introducing the self-energy $\Sigma[G, W]$
  $\Rightarrow$ Describe binary interactions as an infinite summation of diagrams
- Conserving approximations allow for a systematic truncation of the hierarchy.
  Examples: Hartree-Fock, second Born, GW ▸ GW

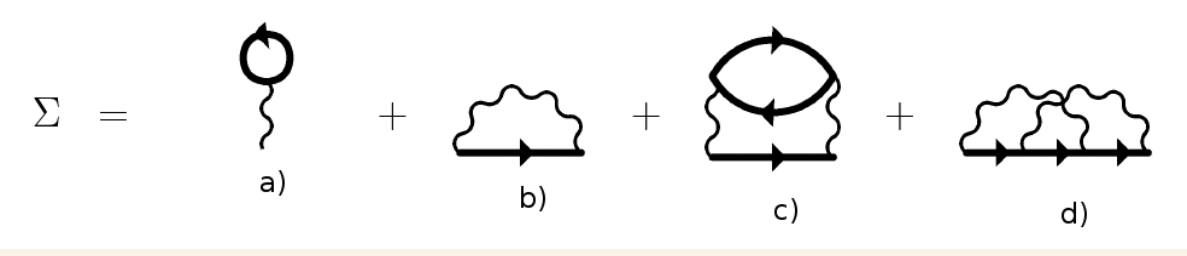

Figure: Self-energy diagrams. First order in the interaction $W(1, \bar{1})$: a) Hartree, b) Fock. Second order in $W(1, \bar{1})$: c) & d) second Born terms.

- Structure/identification and meaning of the appearing terms in the equation:

  memory kernels & inclusion of initial correlations

Fin

Thanks for your attention!

Next lecture: (i) Numerical technique solving the KBE
(ii) Applications to semiconductors (homogeneous systems) as well as quantum dots, atoms and molecules (inhomogeneous systems)

Structure of the nonequilibrium Green's function

back

Indeed, the nonequilibrium Green's function (NEGF) has the structure of a usual (mathematical) Green's function in the following sense:

(a) When neclecting all collision integrals on the r.h.s. of the KBE $\longrightarrow$ effective single-particle or ideal quantum system described by $G_0(1,\bar{1})$ which obeys

$$\left[i\frac{\partial}{\partial t_1} - H^1(1)\right] G_0(1,\bar{1}) \quad = \quad \delta_{\mathscr{C}'}(1-\bar{1}) \tag{3}$$

Eq. (3) then gives rise to an inverse Green's function

$$G_0^{-1}(1,\bar{1}) \quad = \quad \left[i\frac{\partial}{\partial t_1} - H^1(1)\right] \delta_{\mathscr{C}'}(1-\bar{1})$$

with matrix equation $G_0^{-1} G_0 = 1$.

(b) generaliztaion to case with correlations: define

$$G^{-1}(1,\bar{1}) \quad = \quad G_0^{-1}(1,\bar{1}) - \Sigma(1,\bar{1})$$

such that

$$\int_{\mathscr{C}'} \mathrm{d}2 \; G^{-1}(1,2)\, G(2,\bar{1}) \quad = \quad \delta_{\mathscr{C}'}(1-\bar{1})$$

is equivalent to the Kadanoff-Baym equations.

Lowest-order Φ-diagrams

back

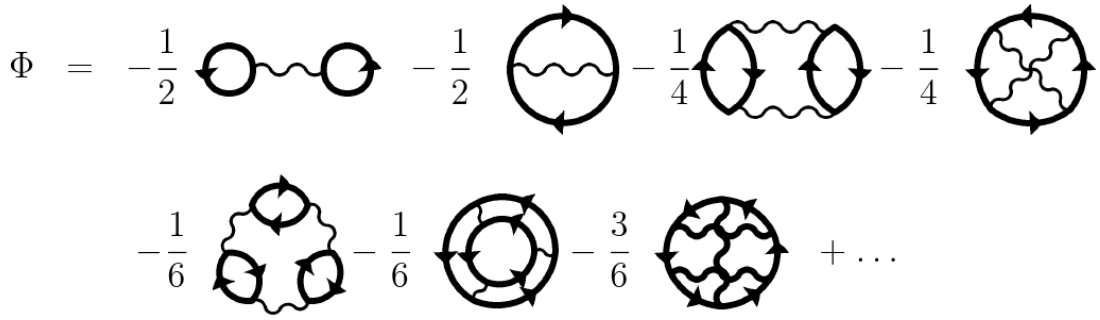

Figure: Generating Φ-diagrams. Prefactor is given by $n_\Sigma/2n$.

The functional Φ can be constructed by summation over irreducible self-energy diagrams closed with an additional Green's function line and multiplied by appropriate numerical factors:

$$\Phi[G] \quad = \quad \sum_{n,k} \frac{1}{2n} \int \mathrm{d}1\mathrm{d}2 \; \Sigma_k^{(n)}(1,\bar{1})\, G(\bar{1},1^+)$$

- n denotes the number of interaction lines
- k labels Σ-diagrams
- n_Σ is the number of topologically different Σ-diagrams that can be generated from the Φ-diagram

$G^{\gtrless}$ in relative and center of mass coordinates

back

We recall that

$$G^{<}(\mathbf{p}\omega, \mathbf{R}T) = \int \mathrm{d}^3 r \, \mathrm{d}t \; e^{-i\mathbf{p}\mathbf{r}+i\omega t} \left[\pm i G^{<}(\mathbf{r}t, \mathbf{R}T)\right]$$

$$G^{>}(\mathbf{p}\omega, \mathbf{R}T) = \int \mathrm{d}^3 r \, \mathrm{d}t \; e^{-i\mathbf{p}\mathbf{r}+i\omega t} \; i \, G^{>}(\mathbf{r}t, \mathbf{R}T)$$

$$\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_{\bar{1}}}{2} , \qquad \mathbf{T} = \frac{t_1 + t_{\bar{1}}}{2} , \qquad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_{\bar{1}} , \qquad \mathbf{t} = t_1 - t_{\bar{1}}$$

- $G^{<}(\mathbf{p}\omega, \mathbf{R}T)$ can be interpreted as the density of particles with momentum $\mathbf{p}$ and energy ω at the space time point $(\mathbf{R}, t)$
- Correspondingly, $G^{>}(\mathbf{p}\omega, \mathbf{R}T)$ denotes the density of states available to a particle that is added to the system at $(\mathbf{R}, t)$ with momentum $\mathbf{p}$ and energy ω

GW approximation

◂ back

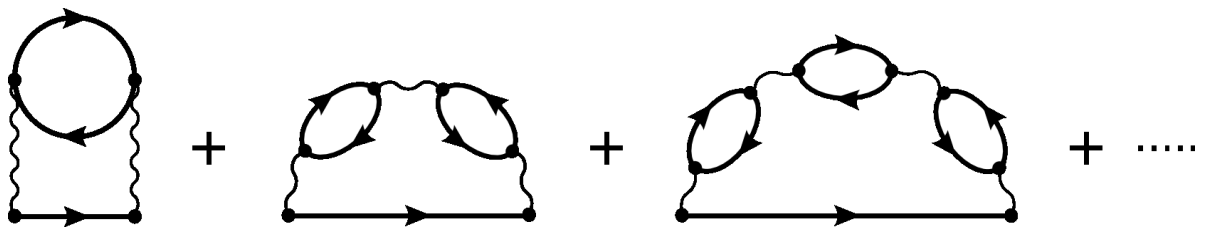

Figure: GW self-energy ($\sim$ dynamical screened potential)—summation of all bubble-like diagrams [A. Stan et al, Europhys. Lett. **76**, 298 (2006)]