Finite-Size Scaling

Characteristic feature of a second-order phase transition is the divergence of the correlation length at a critical temperature $T_c = T_c(\infty)$

$$\xi(T) = \xi_0 + |1 - T/T_c|^{\nu} + \dots$$

This leads to the singularities of the specific heat, magnetization ($T < T_c$), susceptibility parameterized by the critical exponents (2D Ising model: $\nu = 1, \alpha = 0(\log), \gamma = 7/4,\ \beta = 1/8$)

$$C = C' + C_0|1 - T/T_c|^{-\alpha} + \dots, \quad m = m_0(1 - T/T_c)^{\beta} + \dots,$$
$$\chi = \chi_0|1 - T/T_c|^{-\gamma} + \dots$$

In any numerical simulation the system size is finite, and hence near T_c the role of ξ is taken over by the linear system size L

$$|1 - T/T_c(\infty)| \propto \xi(T)^{-1/\nu} \to |1 - T_c(L)/T_c(\infty)| \propto L^{-1/\nu} \qquad (2)$$

As a critical temperature of the finite lattice $T_c(L)$ we take the location of the specific-heat peak (or susceptibility). This leads to

$$T_c(\infty) - T_c(L) = aL^{-1/\nu}$$

Finite-Size Scaling: references

Finite value of the correlation length ξ implies that also all divergences of thermodynamic quantities are rounded and shifted. How this happens is described by the finite-size scaling theory. See, e.g.

- A. E. Ferdinand and M. E. Fisher, Phys. Rev. **185**, 832 (1969); D.P. Landau, Phys. Rev. B **13**, 2997 (1976)
- K. Binder, Phys. Rev. Lett. **47**, 693-696 (1981)
- D.P. Landau and K. Binder, Phys. Rev. B **31**, 5946-5953 (1985)
- V. Privman (ed.), *Finite-Size Scaling and Numerical Simulations of Statistical Systems* (World Scientific, Singapore, 1990)