

PHZ 3113, Section 3924, Fall 2013, Homework 4

Due at the start of class on Friday, September 20. Half credit will be available for homework submitted after the deadline but no later than the start of class on Monday, September 23.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working. Throughout the questions below,  $\hat{\mathbf{x}}$ ,  $\hat{\mathbf{y}}$  and  $\hat{\mathbf{z}}$  are unit vectors along the Cartesian  $x$ ,  $y$  and  $z$  axes, respectively.

1. Integrate  $2x^2 - y$  over the interior of a circle of radius  $R$  centered on  $(R, 0)$ .

2. Evaluate:

$$\int_0^2 \int_0^{(4-x)/2} (x+2y)^{1/2}(2x-3y)dydx$$

by transforming to the variables  $u = x + 2y$ ,  $v = 2x - 3y$ .

3. Evaluate:

$$\int_0^\infty \int_0^\infty \frac{2x^2 + \pi y^2}{1 + (2x^2 - \pi y^2)^2} \exp(-xy/\pi) dy dx$$

by transforming to the variables  $u = 2x^2 - \pi y^2$ ,  $v = xy/\pi$ .

4. Suppose that  $\mathbf{A} = \hat{\mathbf{x}} - 2\hat{\mathbf{y}} + 2\hat{\mathbf{z}}$  and  $\mathbf{B} = 6\hat{\mathbf{x}} - 2\hat{\mathbf{y}} - 3\hat{\mathbf{z}}$ .

- Write  $\mathbf{A}$  and  $\mathbf{B}$  in Cartesian component notation, e.g.,  $(A_x, A_y, A_z)$ .
- Calculate  $|\mathbf{A}|$  and  $\hat{\mathbf{A}} = \mathbf{A}/|\mathbf{A}|$ .
- Calculate  $\mathbf{A} \cdot \mathbf{B}$  and  $\mathbf{A} \times \mathbf{B}$ .
- Find the angle between vectors  $\mathbf{A}$  and  $\mathbf{B}$ .
- Find the area of the parallelogram whose vertices are at the positions  $\mathbf{O}$  (the origin),  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{A} + \mathbf{B}$ .
- Find a unit vector perpendicular to both  $\mathbf{A}$  and  $\mathbf{B}$ .

Any vector provided as an answer to parts b)–f) may be written either as a linear combination of the unit vectors  $\mathbf{x}$ ,  $\mathbf{y}$  and  $\mathbf{z}$ , or in terms of its cartesian components.

5. Any plane in three-dimensional space can be defined as the set of all points  $\mathbf{r}$  satisfying  $\mathbf{r} \cdot \mathbf{p} = c$ , where  $\mathbf{p}$  is a fixed vector perpendicular to the plain and  $c$  is a real constant.

- Find a *unit* vector  $\mathbf{p}$  and a corresponding value  $c$  that define the plane  $2x - 3y + 6z = 6$ .
- Find the line of intersection between the plane defined in a) and the plane given by  $\mathbf{r} \cdot (2\hat{\mathbf{x}} - \hat{\mathbf{y}} + 2\hat{\mathbf{z}}) = -3$ . Your answer should express  $y$  and  $z$  as functions of  $x$ .
- Find the smallest (in magnitude) angle in degrees between the normals to the two planes considered in b).