

PHZ 3113, Section 3924, Fall 2013, Homework 6

Due at the start of class on Friday, October 11. Half credit will be available for homework submitted after the deadline but no later than the start of class on Monday, October 14.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. In spherical polar coordinates, find $\nabla \cdot \mathbf{e}_r$, $\nabla \cdot \mathbf{e}_\theta$, $\nabla \times \mathbf{e}_\theta$ and $\nabla \times \mathbf{e}_\phi$,
 - i) by doing each whole calculation entirely in cartesian coordinates, and
 - ii) by repeating each calculation in spherical polar coordinates using results given in class.
2. Write out the metric, g_{ij} for spherical polar coordinates and hence find its inverse g^{ij} . Thus, write out explicitly the expression for $\nabla^2 u$ using the results given in class (in terms of g).
3. Show by explicit calculation (in cartesian coordinates) that $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \epsilon_{ijk} A^i B^j C^k$.
4. Consider elliptical coordinates $x = a \cosh(u) \cos(v)$, $y = a \sinh(u) \sin(v)$ and prove that the coordinate lines $u = \text{constant}$, and $v = \text{constant}$ are orthogonal.
5. In special relativity, for motion restricted to the x -direction, we consider the metric implied by $ds^2 = -c^2 dt^2 + dx^2$ (where c is the speed of light). Under the coordinate transformation:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

(the Lorentz transformation), show that $g_{i'j'}$ is identical to g_{ij} , *i.e.*, $ds^2 = -c^2 dt'^2 + dx'^2$.

6. Evaluate

$$\int_0^\pi \sin(x) \delta(\cos(3x)) dx.$$

7. Using $\mathbf{B}(\mathbf{x}, t) = \nabla \times \mathbf{A}(\mathbf{x}, t)$, $\mathbf{E}(\mathbf{x}, t) = -\nabla \phi(\mathbf{x}, t) - \partial \mathbf{A}(\mathbf{x}, t) / \partial t$ in two of Maxwell's Equations:

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

show they are satisfied identically. Also show that the other two of Maxwell's Equations:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

(where $\mu_0 \epsilon_0 = 1/c^2$) simplify significantly if one uses the (Lorentz gauge) condition:

$$\nabla \cdot \mathbf{A} = -\frac{1}{c^2} \frac{\partial \phi}{\partial t}$$

to eliminate \mathbf{A} from the $\nabla \cdot \mathbf{E}$ equation and ϕ for the $\nabla \times \mathbf{B}$ equation.

8. Show that $\sinh(ix) = i \sin(x)$, $\cosh(ix) = \cos(x)$, $\cos(ix) = \cosh(x)$ and $\sin(ix) = i \sinh(x)$.
9. $e^{nz} = (\cosh(z) + \sinh(z))^n = \cosh(nz) + \sinh(nz)$. Find $\sinh(3z)$ in terms of $\sinh(z)$ and $\cosh(z)$.
10. If $z = a/b$ and $1/(a+b) = 1/a + 1/b$, find z . Write your results in $re^{i\theta}$ form.