PHZ 3113, Section 3924, Fall 2013, Homework 11

Due at the start of class on Friday, November 22. Half credit will be available for homework submitted after the deadline but no later than the start of class on Monday, November 25.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. a) Let $f(t) = e^{-\alpha|t|}$, and denote its Fourier transform by $g(\omega)$. Find $g(\omega)$.

   b) Use your result from part a), to find the inverse Fourier transform of $g(\omega)$, *i.e.*,
$$\int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega.$$

   Hint: you will need to use contour integration, and to carry out the integration separately for positive and negative t.

2. Let $V(t)$, the voltage across an RL series circuit, be given by:
$$V(t) = \frac{V_0}{2\pi} \int_{-\infty}^{\infty} \frac{-2i\omega}{\alpha^2 + \omega^2} e^{i\omega t} d\omega.$$

   a) Evaluate the integral expression for $V(t)$ given above, in the interval $-\infty < t < \infty$.

   b) Obtain a differential equation for the current, $I(t)$, in the circuit.

   c) Use Fourier techniques to obtain a non-differential equation for $\tilde{I}(\omega)$, the Fourier transform of $I(t)$, in terms of a complex impedance.

   d) Hence find $I(t)$ from its Fourier transform, again for $-\infty < t < \infty$. Assume $\alpha \neq R/L$.

   e) What is the sign of $I(0)$? Explain the different behaviors of $I(t)$ and $V(t)$ at $t = 0$.

3. Consider a particle governed by the following equation:
$$m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega t.$$

   a) Suppose that F_0 and b are zero. Write down the frequency, ω_0, of the undamped motion.

   b) Assuming that only F_0 is zero, and that $k > b^2/(4m)$, write down both the frequency, ω_1, and the characteristic decay time, τ, for the damped oscillatory motion.

   c) Suppose that, when $F_0 \neq 0$, the steady state response is $x = A\cos(\omega t - \phi)$. Obtain expressions for A and ϕ in terms of generic parameters for the problem.

   d) When $F_0 \neq 0$, suppose the peak amplitude of the response occurs at $\omega = \omega_2$, equal to one-third of the frequency of the undamped motion, ω_0. Express the decay time, τ, and damped oscillation frequency, ω_1, of part b) in terms of the undamped frequency, ω_0.

   e) For the same conditions as in part d), what would be the phase angle ϕ when ω is two-thirds the undamped frequency, ω_0. At what frequency does $\phi = \pi/2$?