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Last time we looked at the minkowski metric perturbed slightly

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Then we found the zero component was proportional to the field potential, $h_{00} = -2\phi$. What we went through to deduce this was the minimal requirements necessary such that the gravity from the geodesics and the stress tensor being conserved were satisfied. We then found that the stress tensor was proportional to $G_{\mu\nu}$, the einstein tensor. We looked at the divergence of the geodesic equation, however we didn't actually try to solve for them. Because of this, If we were to substitute this requirement into the geodesic equations not all of the einstein equations would be solved. then we have

$$\mathrm{d}s^2 = -c^2(1+2\phi)\mathrm{d}t^2 + \overbrace{(1-2\phi)\mathrm{d}r^2 + \underbrace{r^2\,\mathrm{d}\theta^2 + r^2\sin^2\mathrm{d}\theta}_{2}}^{1}$$

were we see we had to add an extra ϕ dependence on g_{rr} to "approximately" solve the einstein equations.
Now we can look at parts of the metric and find which parts characterize spherical symmetry, the part labeled 2, and the part that is the spatial length and clearly not flat, 1.
WE can then rewrite this metric in a way that is equivalent to low order terms,

$$\mathrm{d}s^2 = -c^2\left(1+2\phi(R)\right)\mathrm{d}t^2 + \overbrace{\left(1-2\phi(R)\right)\left(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right)}^{flat}$$

where we have $R^2 = x^2+y^2+z^2$, and $r = R(1+\frac{\phi}{2R})$. Now the two are both approximations of the exact solutions to the same order.
One of the problems from when we were looking at the problem yesterday was that we had assumed it was all stationary, however since this would be representing two bodies the source would be moving due to the potential of the second as well. So we would like to look at perturbations to the metric that are not stationary due to the source moving about the center of mass. We want to the perturbation to be proportional to the gravitation potential that will

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be linear, or representative of gravitational waves,

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \frac{2}{c^2} \begin{pmatrix} \phi(t,x,y,z) & \cdots & \cdots & \phi \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \phi & \cdots & \cdots & \phi \end{pmatrix}$$

Where via coordinate change we can make this second matrix traceless. We will call it $h_{\mu\nu}$, which we call the transverse traceless gauge, or

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} \ddots & \cdots & & \\ & & & \\ \vdots & h_+ & h_x & \\ & h_x & -h_+ & \\ & & & \end{pmatrix}$$

We could write everything out in spherical coordinates and it is important to make use of spherical harmonics, call the spin weighted representation of the waves.
Now in reality usually bodies are spinning, so we can examine the relativistic effects of this. In fact if you have rotating space time there is necessarily a cross coupling between the terms.
Next we want to look at conformally flat geometries, or space with a flat spatial part multiplied by a function of time.

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)$$

, however relabeling $\mathrm{d}\eta^2 = \mathrm{d}t^2/a(t)^2$ we get the 4-d conformally flat space

$$\mathrm{d}s^2 = a^2(t)\left(-\mathrm{d}\eta^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)$$