

GR

So last time we talked about the event horizons of the Schwartchild black holes. For a black hole of mass M we broke up space time into 4 region. We defined the horizons as the boundary of the set of the time-like curves leading to the scri+ boundary (calling scri+ and scri- γ^+ and γ^- respectively), or $EH = \partial J^-(\gamma^+)$. If we have a contained surface formed from the light rays originating from some region of space, the surface at which they start curving back towards each other is called the apparent horizon. We call space-like infinite i_0 to be the edge of Region I furthest from the origin, where γ^+ and γ^- meet, and the spot where γ^+ hits the event horizon (at positive infinity time) between regions II and I is called i_+ and the one at negative infinite time where γ^- his the event horizon between regions I and IV are called i_- .

As we add mass we have a change in where the γ^\pm surfaces are then moved outwards, then because these are moved outwards the event horizon shifts out so the null curve that used to make up the event horizon no longer does, however it will form an apparent horizon.

So how does the presence of a black hole change our perception? Suppose we have a particle traveling into a black hole, if we look into the past we a particle before it goes in. At $r = 2M$ we have our event horizon, but looking at r larger than this segments we can observe through the past light cone regions where the particles were significantly close to this. However with large enough r , in getting to us this light would be extremely redshifted. So things that formed the black hole, their past light cones would be extremely redshifted, but we can see things that are in somewhat close orbit around the black hole, and similarly we can affect the black hole since it is always within our future light cone.

Now let's talk about the surface gravity. Suppose we are sitting outside of the compact body that formed the black hole, with fixed position in time or our position $X^\mu = (t, r_0, \theta_0, \phi_0)$ so that our velocity $U^\mu = \frac{dt}{d\tau} (1, 0, 0, 0)$. Using the $ds^2 = -(1 - \frac{2M}{r}) dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$ gives us $U^\mu U^\nu g_{\mu\nu} = -1 = -(1 - \frac{2M}{r}) (\frac{dt}{d\tau})^2$ or $\frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{2M}{r}}}$ which we can use to rewrite $U^\mu = \frac{1}{\sqrt{1 - \frac{2M}{r}}} (1, 0, 0, 0)$. Then the acceleration is given by $A^\mu = U^\nu \nabla_\nu U^\mu = \frac{1}{\sqrt{1 - \frac{2M}{r}}} \Gamma_{t\sigma}^\mu U^\sigma = \frac{1}{\sqrt{1 - \frac{2M}{r}}} \Gamma_{tt}^\mu$. Using that $\Gamma_{tt}^\mu = \frac{-1}{2} \frac{1}{g_{\mu\mu}} \partial_\mu g_{tt} = \delta_r^\mu \frac{1}{2g_{rr}} \partial_r g_{tt}$ so that $A^\mu = \frac{M}{r^2} \delta_r^\mu$. This gives us a magnitude of acceleration $a = |A| = \frac{GM}{r^2 \sqrt{1 - \frac{2M}{r}}}$

Now for the redshift factor. We have several killing vectors, $T = \partial_t$ and $\Phi = \partial_\phi$ are both, from

which we get killing horizons. Then $T^\mu = (1, 0, 0, 0) = VU^\mu$ where $V = \sqrt{1 - \frac{2M}{r}}$ is the redshift factor. Then we have the conserved quantities $p_t = -E = p_\mu T^\mu$ for a particle falling towards the black hole. We use this because if we have a photon of a certain energy $\omega_\mu = (-\omega, \vec{k})$ then we see from the above relation $-\omega = \omega_\mu U^\mu = \frac{-E}{V}$ by definition or in other words we see the frequency is dependent on V which depends on r , or $\frac{\omega_2}{\omega_1} = \frac{V_1}{V_2}$, the ratio of the redshift, although factors. However, then we see that that $\kappa = Va = \frac{GM}{r^2}$ is well behaved for any r .

Now suppose we have X^μ a killing vector, and suppose X is null on some null hyper surface Σ . Then Σ is called the killing horizon of X^μ . Suppose we have $ds^2 = -dt^2 + dx^2 = -dt'^2 + dx'^2$. Then $B^\mu = \frac{\partial}{\partial t'} = x\partial_t + t\partial_x$ is a killing vector and $-x^2 + t^2 \rightarrow 0$ on the line $t = x$ so this is the killing horizon for B^μ since it becomes null there. Then we see that the event horizon of the sch. black hole is the killing horizon for $T^\mu = \partial_t$ and ∂_t is also a geodesic on the killing horizon. $T^\mu \nabla_\mu T^\nu = -\kappa T^\nu$ and we see $\kappa^2 = (\nabla_\mu T_\nu)(\nabla^\mu T^\nu) = (\nabla_\mu V)(\nabla^\mu V)$.