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GR Class Notes
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Last class recap:

$A \cdot B = A^i B^j g_{ij}$

$(A \times B)_k = \epsilon_{ijk} A^i A^j$

$V^i = \frac{dx^i(t)}{dt}$

$F_i = \frac{\partial f(x)}{\partial x^i}$

$V^i F_i = \frac{df(x^i(t))}{dt}$

Some class generated definitions,

vector:
-components transform like coordinates (only applies to half)
-magnitude and direction
-element of a vector space
-associated with directional derivative, $\vec{n}\frac{\partial f}{\partial \vec{n}}$

tensor:
-linear machine, takes (vectors, covectors) to scalars
-multi-indexed object, each index transforms like appropriate vector

$$T^{transform-like-vector}_{transform-like-covector}$$

-Tensor product of a collection of vectors manifold
-space where every open set is homeomorphic to an open set on $\mathbb{R}^n$
-embedable open sets in $\mathbb{R}^{a+b}$ for some M^a tangent vector
-tangent curve on a vector 1-form
-linear functional takes a vector to a scalar

$$p(\vec{v}) = f$$

A few comments:
-position vectors not good vectors, don't transform properly under changes in origin
-velocity vectors ok
-coordinates (x^i) are not physical!
-distance between neighboring points is physical (difficult to give up notion of coordinates being physical, find things that are physical)

Regarding Excercise 3,
$\Delta l^2 = \Sigma_i (\Delta x^i)^2$
-the scalar produced has an implied metric

-Euclidean $g_{ij} = diagonal(1,1,1)$
-there are other metrics you could write down, such as polar:

$$\Delta l^2 = r^2(\Delta\theta)^2 + (\Delta r)^2 \implies g_{ij} = \{1, 0, r^2, 0\}$$

Different representation of the Gradient(see Griffiths inside cover), are determined by orthonormal basis

Last time,
- the statement $\vec{V} = V^i \hat{e}_i$, assumes an orthonormal basis
-this will not always be the case, and so $\vec{F} = V^i e_i$
-Generally there will be a coordinate basis such as:

$$\vec{F} = \vec{df} = \frac{\partial f}{\partial x^i} e^i = \frac{\partial f}{\partial x^i} dx^i$$

$$\vec{V} = v^i e_i = V^i \frac{\partial}{\partial x^i}$$

-sometimes there is no choice, some spacetime can't be written as orthogonal
-coordinate basis could be more convenient than normal
-generally think of vectors as operators
-$\frac{\partial v^i}{\partial x^j}$ doesn't transform as a tensor properly
-to get tensorial behavior, there is additional term

$$e^j \frac{\partial}{\partial x^j} \left(V^i e_i \right) = \left(\frac{\partial V^i}{\partial x^j} + \Gamma^i_{jk} V^k \right) e_i e^j$$

where this corresponds to

$$\Gamma^i_{jk} = \left(\frac{\partial e_k}{\partial x^j} \right)^i$$