

Lecture note 9/16

B. Bond, L. Yan

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To have a better understanding of the equations in General Relativity we should look at similar equations, mainly Maxwell's Equations, to get a better idea of what each term represents and how we can compare them.

ϕ_E		ϕ_G
$E = \nabla\phi_E$		$a = -\nabla\phi_G$
$F_E = qE = m_i a_E$		$F_G = m_g a$
$F = q(E + v \times B)$		$F = ?$
$\Rightarrow \nabla_\mu F^{\mu\nu} = J^\nu$		$R_{\mu\nu}(g_{\mu\nu}) = \dots$
$F^{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$	$\nabla_\mu J^\mu = 0$	$G^{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R^{\sigma\rho}g_{\sigma\rho}) = 8\pi G_\nu T_{\mu\nu}$
$\nabla_\nu \nabla_\mu F^{\mu\nu} = 0$		$\nabla_\mu G^{\mu\nu} \equiv 0 \Rightarrow \nabla_\mu T^{\mu\nu} = 0$

When comparing the two sets of equations with each other we can see that the charge (q) from Maxwell's Equations is similar to the mass (m_g) in the force of gravity and similar we can say that the electric field (E) plays a similar role to a from the force of gravity.

The last equation listed for Maxwell's equations is by definition of $F^{\mu\nu}$.

When requiring that $G^{\mu\nu}$ is covariantly constant in the gravity equations this implies that $T^{\mu\nu}$ must be covariantly constant as well from the equation above.

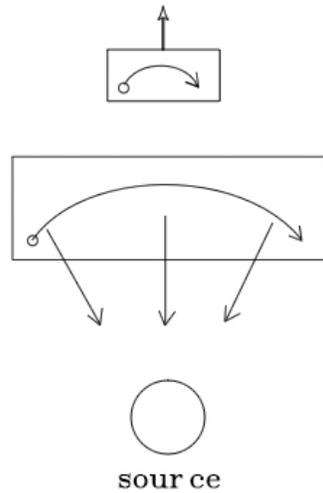
$$F = m_i a \quad F = m_g a_g$$

$$\frac{m_a}{m_i} = \text{constant} = 1 \text{ for all physics laws except gravity}$$

If $m_i a = m_g a_g$ paths are the same for all m_g

WEP

“motions(paths) of freely-falling particles are the same in a gravitational field and a uniformly accelerated frame, in small enough regions of space time.”



The above figure shows that in small enough spacetime, the acceleration due to gravity or the accelerated frame cannot be distinguished, while in big enough spacetime, it can.

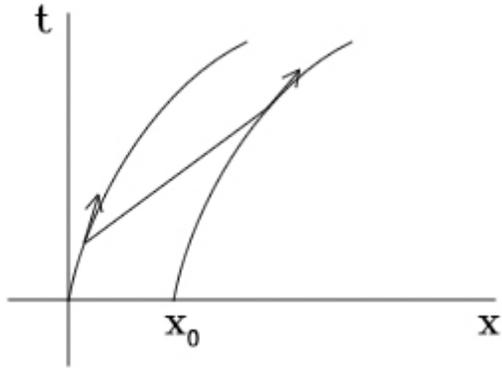
②

EEP

“For all laws of physics, they reduce to those of special relativity in small enough regions of spacetime.” Therefore,

⇒ Gravity is not a force

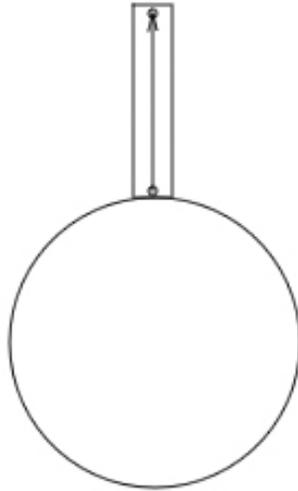
Freely-falling ⇒ not accelerating



Two accelerating objects apart by x_0 . The behind one sends a light to the ahead one. when the light is detected, there will be a change in wavelength(Doppler effect).

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta v}{c}$$

$$\Delta v = x_0 t$$



Send a light from the bottom of a high tower to the top. Doppler effect occurs due to the gravitational field.

$$\frac{\Delta\lambda}{\lambda_0} \propto a$$