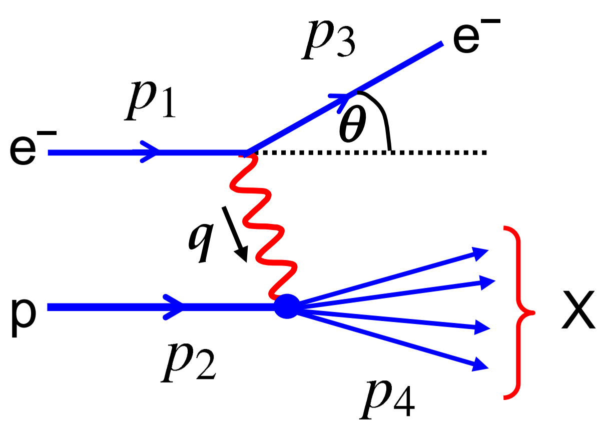
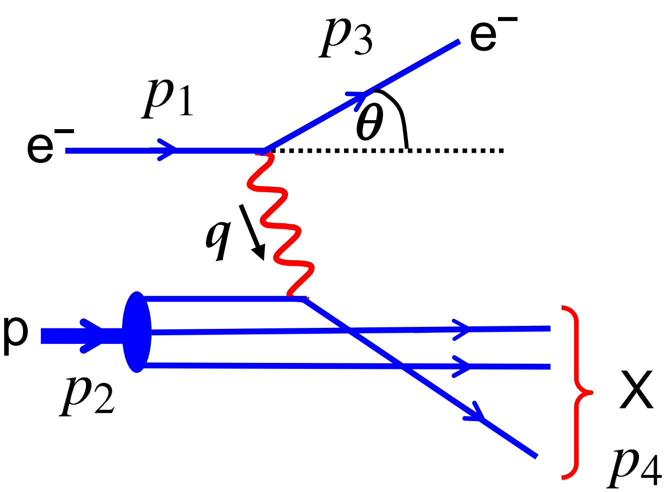
The Quark-Parton Model

- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents "**partons**"
- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.





Scattering from a proton with structure functions → **Scattering from a point-like quark within the proton**

e⁻ p Elastic Scattering at Very High q^2

★At high q^2 the Rosenbluth expression for elastic scattering becomes

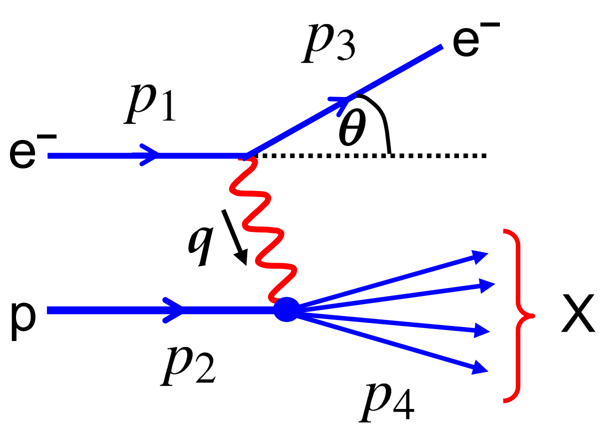
$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\frac{E_3}{E_1}\left(\frac{q^2}{2M^2}G_M^2\sin^2\frac{\theta}{2}\right) \qquad \tau = -\frac{q^2}{4M^2} \gg 1$$

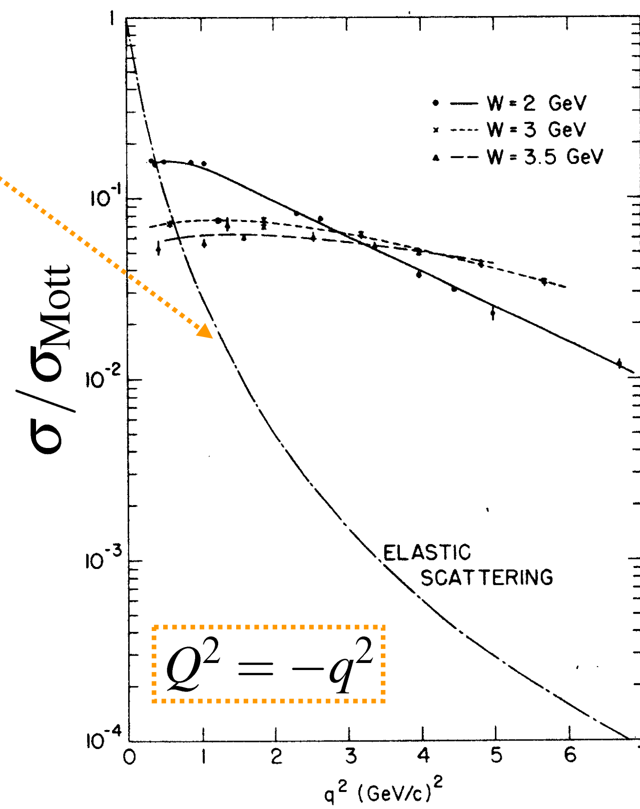
•From e⁻ p elastic scattering, the proton magnetic form factor is

$$G_M(q^2) \approx \frac{1}{(1+q^2/0.71\mathrm{GeV}^2)^2} \Rightarrow G_M(q^2) \propto q^{-4} \quad \text{at high } q^2$$

$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{elastic} \propto q^{-6}$$

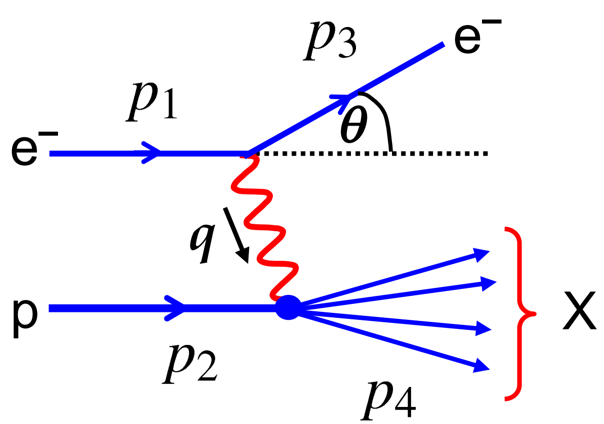
•Due to the finite proton size, elastic scattering at high q^2 is unlikely and inelastic reactions where the proton breaks up dominate.





M.Breidenbach et al., Phys. Rev. Lett. 23 (1969) 935

Kinematics of Inelastic Scattering



- For inelastic scattering the mass of the final state hadronic system is no longer the proton mass, M
- The final state hadronic system must contain at least one **baryon** which implies the final state invariant mass $M_X > M$

$$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$$

★ For inelastic scattering introduce four new kinematic variables: $\boxed{x, y, \nu, Q^2}$

★ **Define:**

$$\boxed{x \equiv \frac{Q^2}{2p_2.q}}$$

Bjorken x (Lorentz Invariant)

where $\boxed{Q^2 \equiv -q^2}$ $\quad Q^2 > 0$

- Here $M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2.q + M^2$

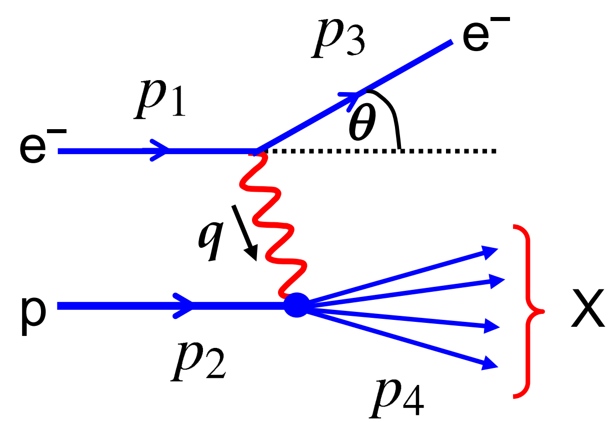
$\Rightarrow$ $Q^2 = 2p_2.q + M^2 - M_X^2$ $\Rightarrow$ $Q^2 \leq 2p_2.q$

Note: in many text books W is often used in place of M_X

hence $\boxed{0 < x < 1 \text{ inelastic}}$ $\quad \boxed{x = 1 \quad \text{elastic}}$

Proton intact
$M_X = M$

★ **Define:** $\boxed{y \equiv \frac{p_2.q}{p_2.p_1}}$ **(Lorentz Invariant)**



•**In the Lab. Frame:**

$$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$$

$$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$$

$$\rightarrow \quad y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

So y is the fractional energy loss of the incoming particle

$$\boxed{0 < y < 1}$$

•**In the C.o.M. Frame (neglecting the electron and proton masses):**

$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$

$$\rightarrow \quad y = \tfrac{1}{2}(1 - \cos\theta^*) \qquad \text{for} \quad E \gg M$$

★ **Finally Define:** $\boxed{\nu \equiv \frac{p_2.q}{M}}$ **(Lorentz Invariant)**

•**In the Lab. Frame:** $\nu = E_1 - E_3$

ν **is the energy lost by the incoming particle**

Relationships between Kinematic Variables

- Can rewrite the new kinematic variables in terms of the squared centre-of-mass energy, s, for the electron-proton collision

$$e^- \xrightarrow{p_1} \quad \xleftarrow{p_2} p$$

$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1.p_2 = 2p_1.p_2 + M^2 + \cancel{m_e^2}$$

Neglect mass of electron

$$2p_1.p_2 = s - M^2$$

- For a fixed centre-of-mass energy, it can then be shown that the four kinematic variables

$$Q^2 \equiv -q^2 \qquad x \equiv \frac{Q^2}{2p_2.q} \qquad y \equiv \frac{p_2.q}{p_2.p_1} \qquad \nu \equiv \frac{p_2.q}{M}$$

are not independent.

- i.e. the scaling variables x and y can be expressed as

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{2M}{s - M^2}\nu$$

Note the simple relationship between y and ν

and $$xy = \frac{Q^2}{s - M^2} \quad \Rightarrow \quad Q^2 = (s - M^2)xy$$

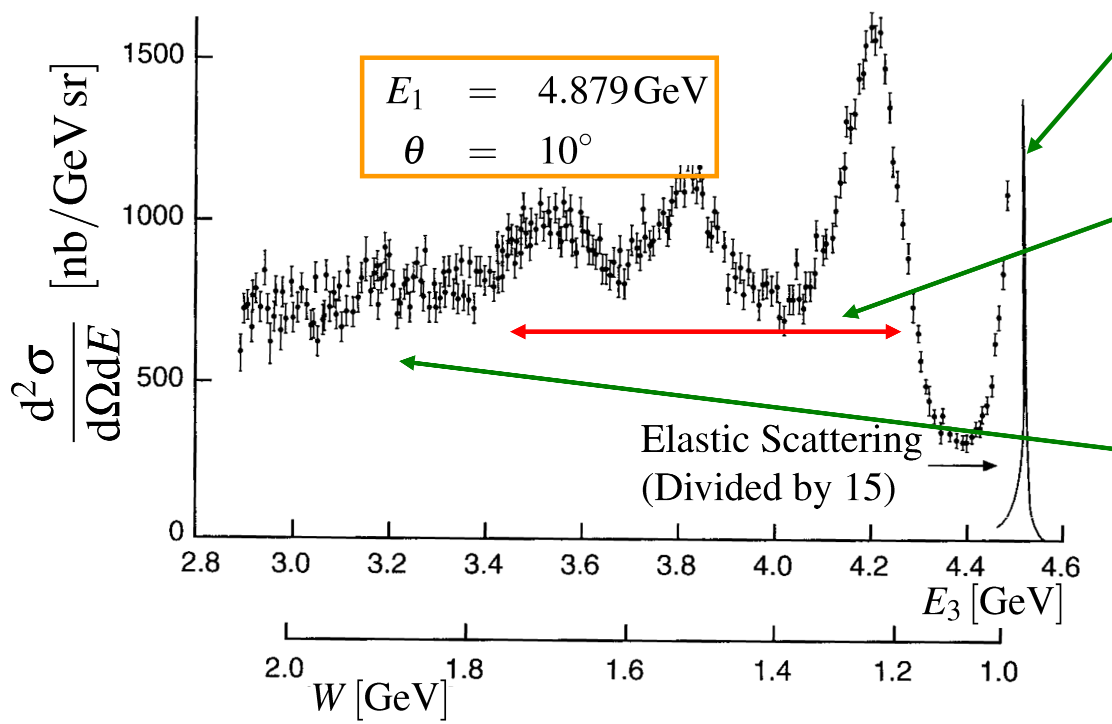
- For a fixed centre of mass energy, the interaction kinematics are completely defined by any **two** of the above kinematic variables (except y and ν)

- For elastic scattering $(x = 1)$ there is only one independent variable. As we saw previously if you measure electron scattering angle know everything else.

Inelastic Scattering

Example: Scattering of 4.879 GeV electrons from protons at rest

- Place detector at **10°** to beam and measure the **energies** of scattered **e⁻**
- Kinematics fully determined from the **electron energy** <u>and</u> **angle !**
- e.g. for <u>this energy and angle</u> : the invariant mass of the final state hadronic system $W^2 = M_X^2 = 10.06 - 2.03E_3$

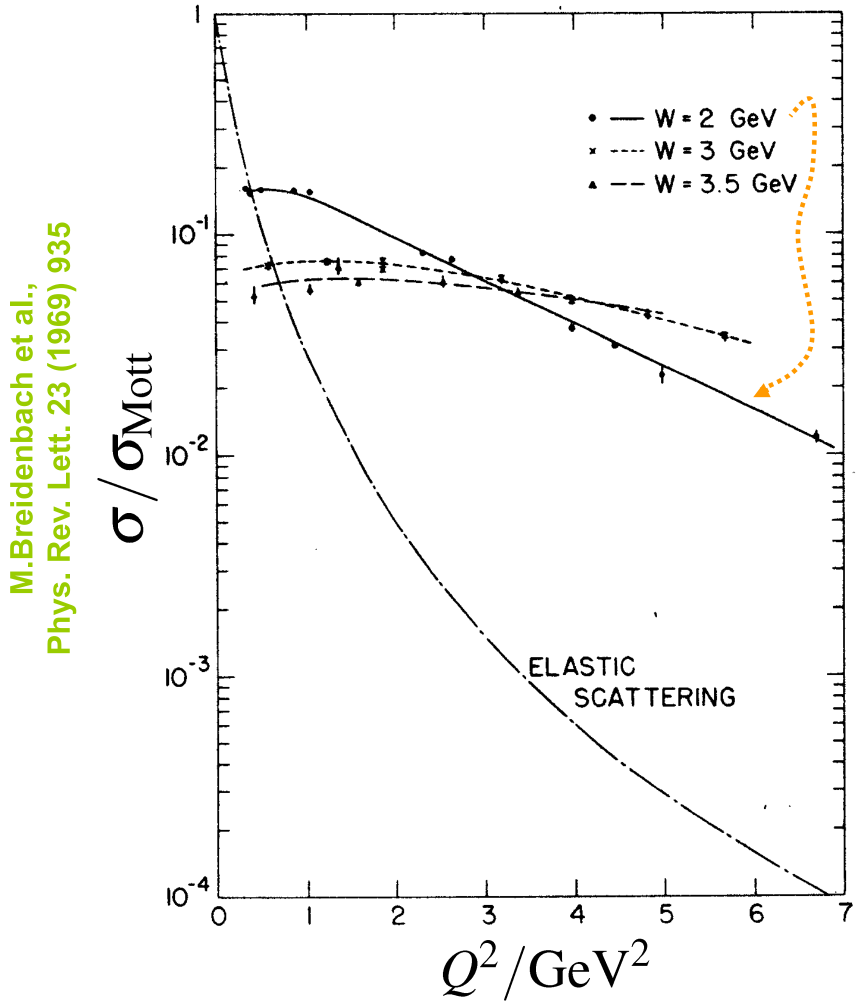


- **Elastic Scattering**
  proton remains intact
  $W = M$
- **Inelastic Scattering**
  produce "excited states" of proton e.g. $\Delta^+(1232)$
  $W = M_\Delta$
- **Deep Inelastic Scattering**
  proton breaks up resulting in a many particle final state

DIS = large W

Inelastic Cross Sections

- Repeat experiments at different angles/beam energies and determine q^2 dependence of elastic and inelastic cross-sections



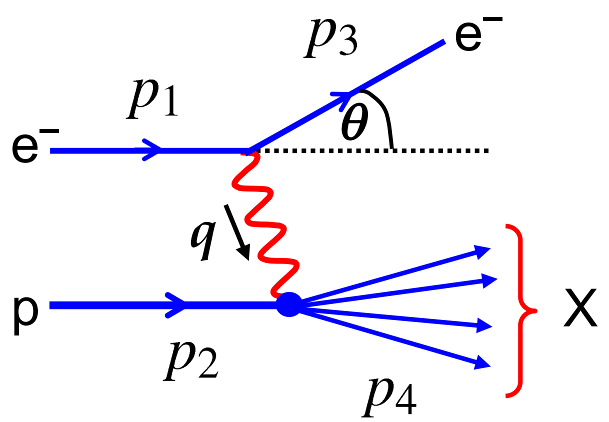
- Elastic scattering falls of rapidly with q^2 due to the proton not being point-like (i.e. form factors)
- Inelastic scattering cross sections only weakly dependent on q^2
- Deep Inelastic scattering cross sections almost independent of q^2 !

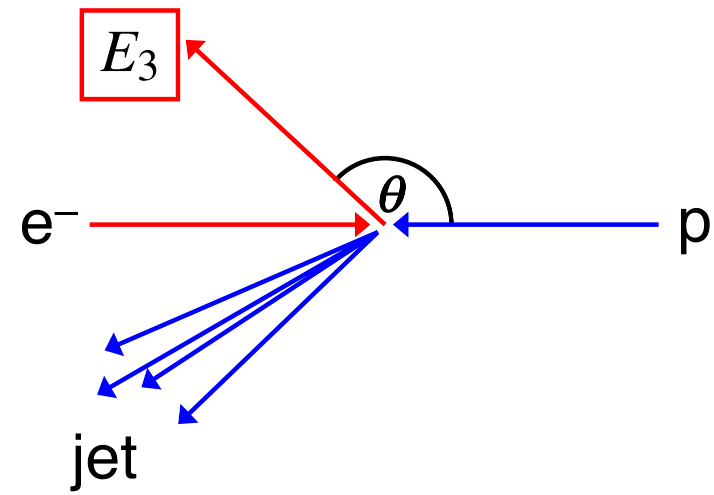
i.e. "Form factor" → 1

➡ **Scattering from point-like objects within the proton !**

Inelastic Cross Sections

- In the Lab. frame it is convenient to express the cross section in terms of the angle, θ , and energy, E_3, of the scattered electron – **experimentally well measured.**





$$Q^2 = 4E_1E_3\sin^2\theta/2; \quad x = \frac{Q^2}{2M(E_1 - E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad \nu = E_1 - E_3$$

- X-section in the Lab. frame

$$\boxed{\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\left[\frac{1}{\nu}F_2(x,Q^2)\cos^2\frac{\theta}{2} + \frac{2}{M}F_1(x,Q^2)\sin^2\frac{\theta}{2}\right]} \qquad \textbf{(3)}$$

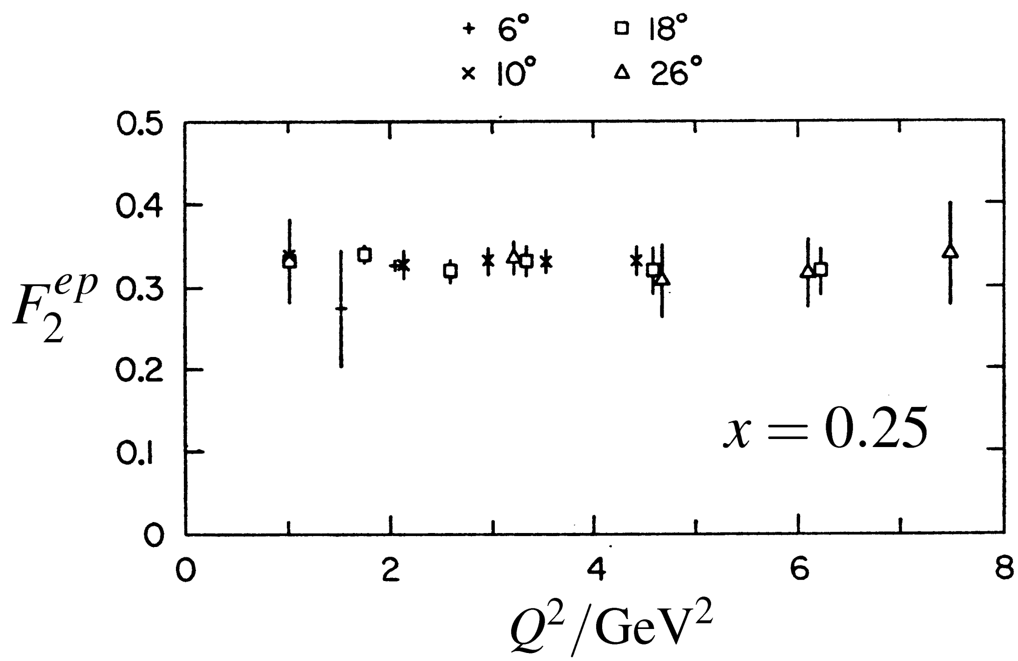
Electromagnetic Structure Function

Pure Magnetic Structure Function

Measuring the Structure Functions

★ To determine $F_1(x,Q^2)$ and $F_2(x,Q^2)$ for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies (see Q13 on examples sheet)

Example: electron-proton scattering F_2 vs. Q^2 at fixed x



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203

♦ Experimentally it is observed that both F_1 and F_2 are (almost) independent of Q^2

Bjorken Scaling and the Callan-Gross Relation

★ The near (see later) independence of the structure functions on Q^2 is known as **Bjorken Scaling**, i.e.

$$\boxed{F_1(x,Q^2) \rightarrow F_1(x) \qquad F_2(x,Q^2) \rightarrow F_2(x)}$$

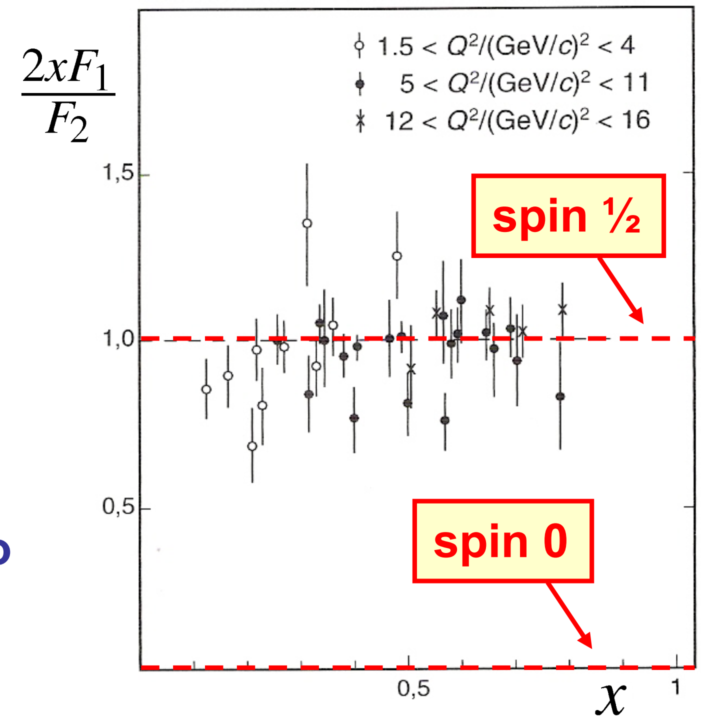
- It is strongly suggestive of scattering from **point-like** constituents within the proton

★ It is also observed that $F_1(x)$ and $F_2(x)$ are not independent but satisfy the **Callan-Gross relation**

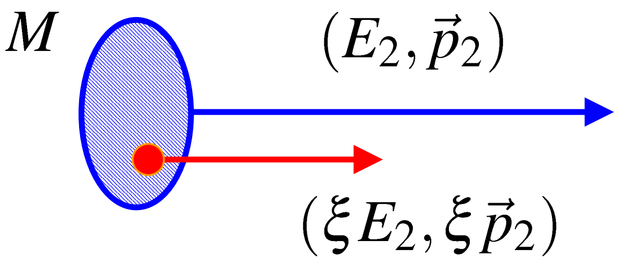
$$\boxed{F_2(x) = 2xF_1(x)}$$

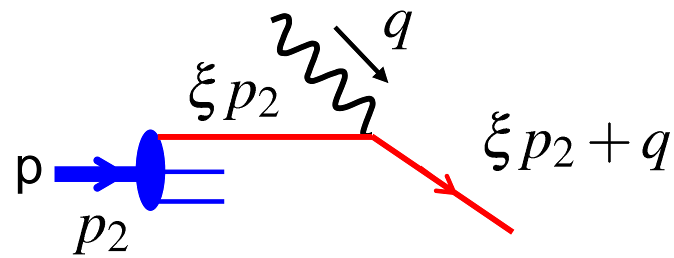
- As we shall soon see this is exactly what is expected for scattering from **spin-half** quarks.

- **<u>Note</u>** if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e. $F_1(x) = 0$



- In the parton model the basic interaction is **ELASTIC** scattering from a **"quasi-free"** spin-½ quark in the proton, i.e. treat the quark as a free particle!
- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the **"infinite momentum frame"**, where we can neglect the proton mass and $p_2 = (E_2, 0, 0, E_2)$
- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.
- Let the quark carry a fraction ξ of the proton's four-momentum.





- After the interaction the struck quark's four-momentum is $\xi p_2 + q$

$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad \Rightarrow \quad \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2.q = 0 \qquad (\xi^2 p_2^2 = m_q^2 \approx 0)$$

$$\Rightarrow \quad \xi = \frac{Q^2}{2p_2.q} = x$$

Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has very high energy)

The parton model predicts:

- **Bjorken Scaling** $F_1(x,Q^2) \rightarrow F_1(x) \qquad F_2(x,Q^2) \rightarrow F_2(x)$
  - ✶ **Due to scattering from point-like particles within the proton**
- **Callan-Gross Relation** $F_2(x) = 2xF_1(x)$
  - ✶ **Due to scattering from spin half Dirac particles where the magnetic moment is directly related to the charge; hence the "electro-magnetic" and "pure magnetic" terms are fixed with respect to each other.**

★ **At present parton distributions cannot be calculated from QCD**

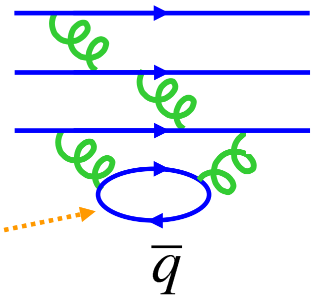
- **Can't use perturbation theory due to large coupling constant**

★ **Measurements of the structure functions enable us to determine the parton distribution functions !**

★ **For electron-proton scattering we have:**

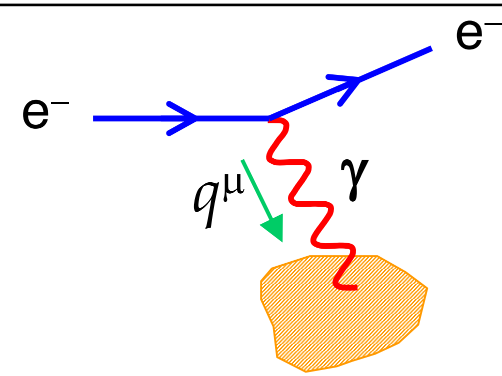
$$F_2^{\mathrm{p}}(x) = x\sum_q e_q^2 q^{\mathrm{p}}(x)$$

- **Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks**
  (will neglect the small contributions from heavier quarks)



Scaling Violations

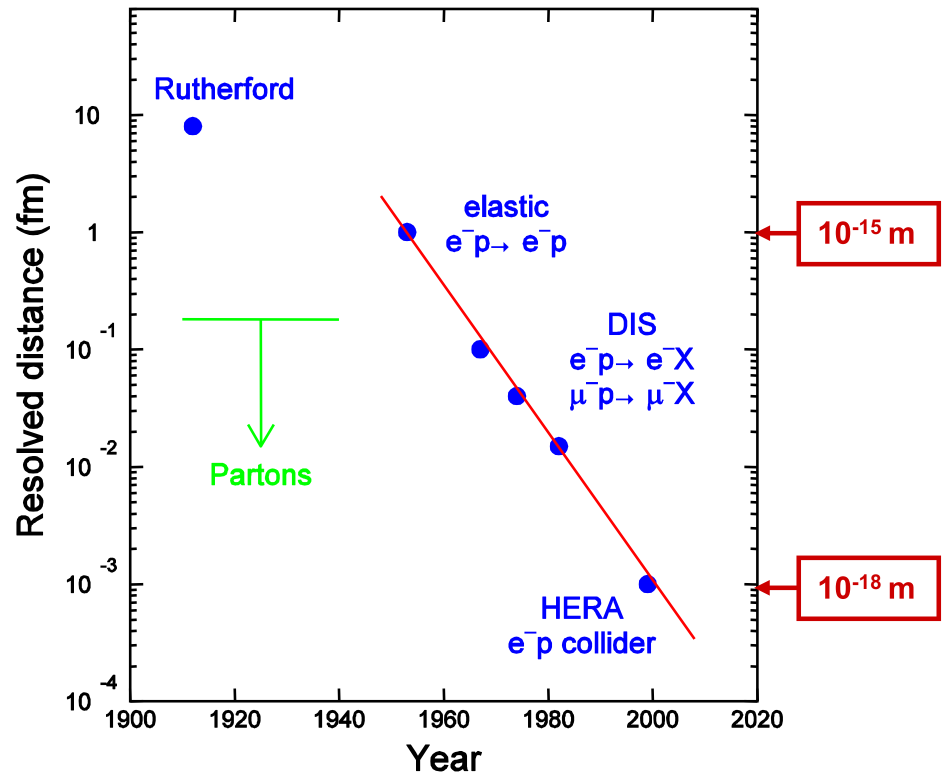
- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy
- Non-point like nature of the scattering becomes apparent when λ_γ ~ size of scattering centre



$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1\,\mathrm{GeV\,fm}}{|\vec{q}|(\mathrm{GeV})}$$

- Scattering from point-like quarks gives rise to **Bjorken scaling**: no q^2 cross section dependence
- IF quarks were not point-like, at high q^2 (when the wavelength of the virtual photon ~ size of quark) would observe rapid decrease in cross section with increasing q^2.
- To search for quark sub-structure want to go to highest q^2

HERA

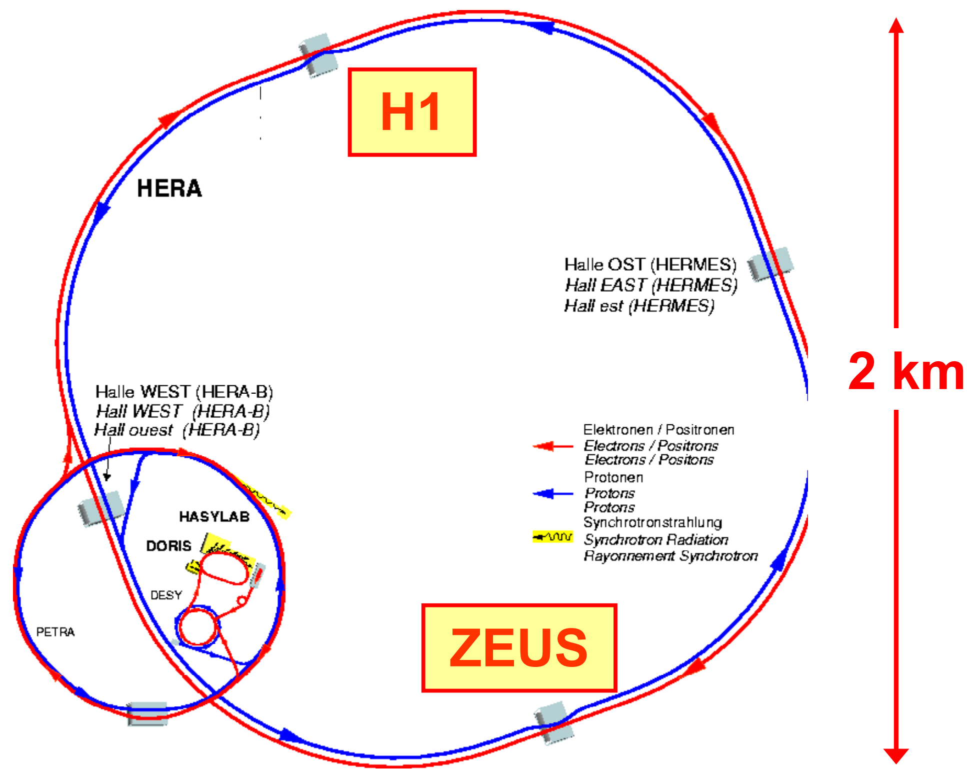


HERA $e^{\pm}p$ Collider : 1991-2007

★ DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany

$e^{\pm}$ 27.5 GeV → ← 820 GeV p

$\sqrt{s} = 300$ GeV

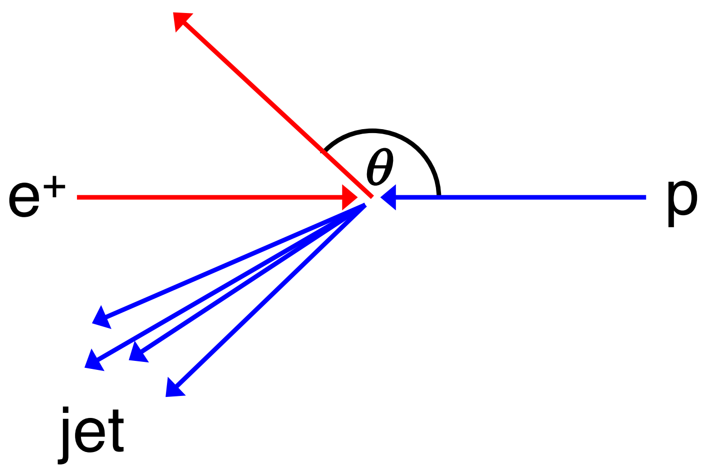


★ Two large experiments : H1 and ZEUS

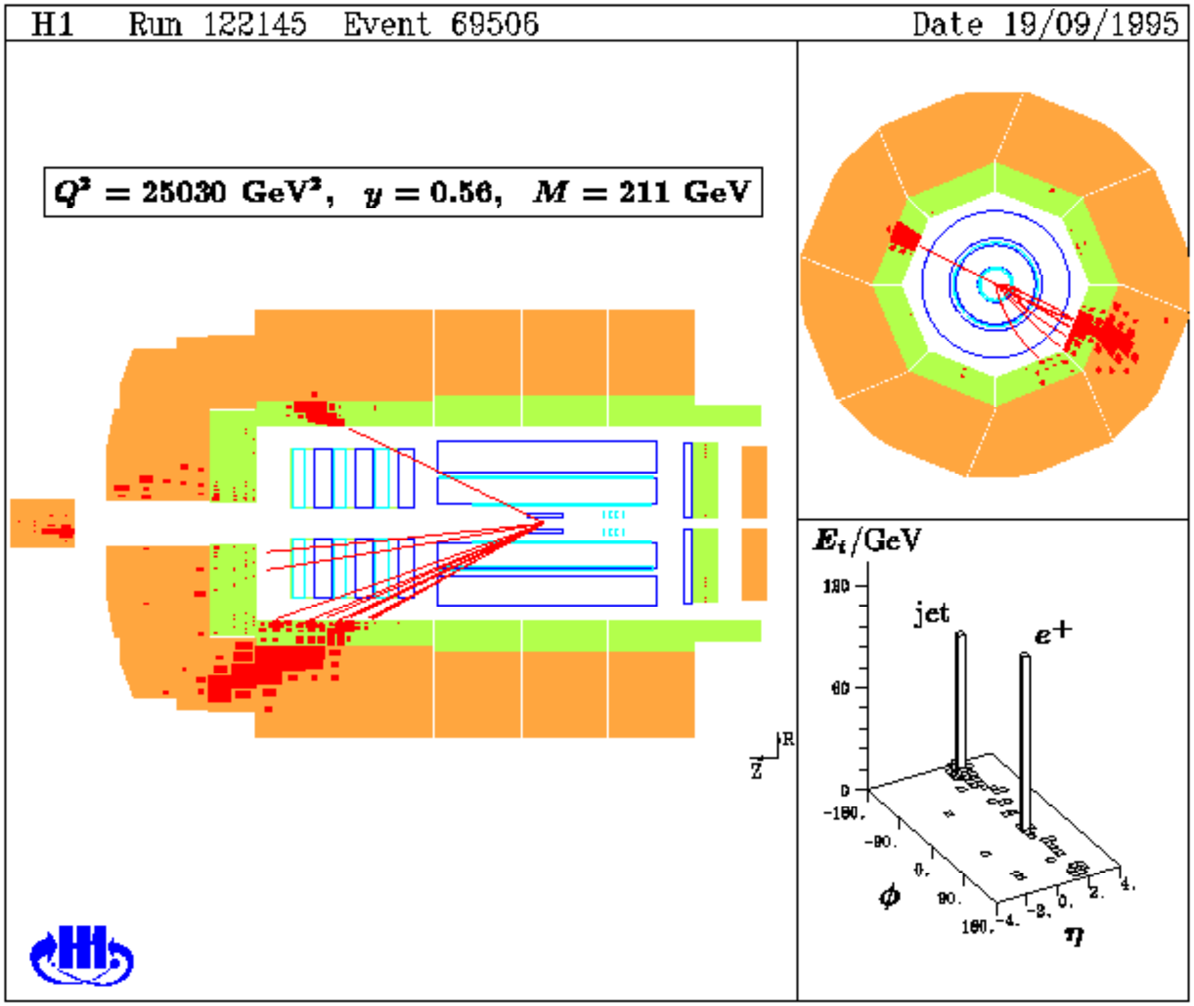
★ Probe proton at very high Q^2 and very low x

Example of a High Q^2 Event in H1

- ✶ **Event kinematics determined from electron angle and energy**



- ✶ **Also measure hadronic system (although not as precisely) – gives some redundancy**

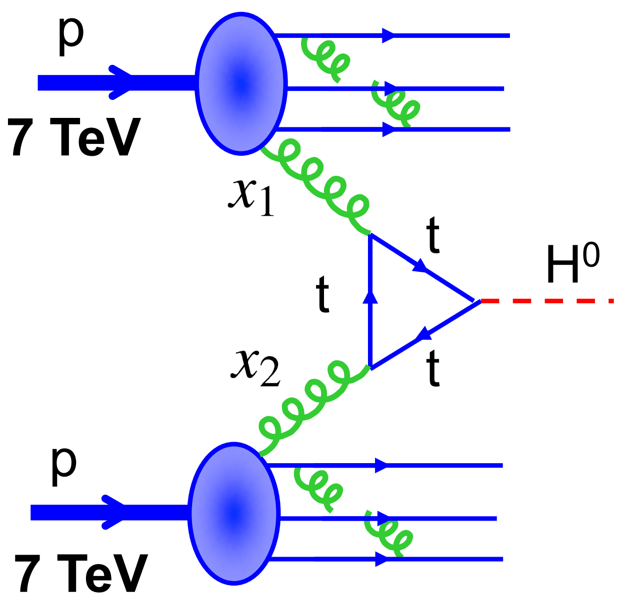


Proton-Proton Collisions at the LHC

★ Measurements of structure functions not only provide a powerful test of QCD, the parton distribution functions are essential for the calculation of cross sections at pp and p$\bar{\text{p}}$ colliders.

•**Example:** Higgs production at the Large Hadron Collider LHC (2009-)

- The LHC will collide 7 TeV protons on 7 TeV protons
- However underlying collisions are between partons
- Higgs production the LHC dominated by "gluon-gluon fusion"



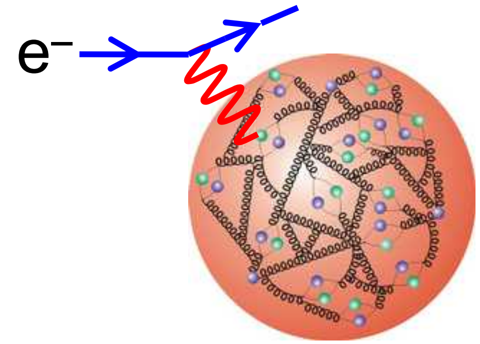
- Cross section depends on gluon PDFs

$$\sigma(pp \to HX) \sim \int_0^1 \int_0^1 g(x_1) g(x_2) \sigma(gg \to H) \mathrm{d}x_1 \mathrm{d}x_2$$

- Uncertainty in gluon PDFs lead to a ±5 % uncertainty in Higgs production cross section

- Prior to HERA data uncertainty was ±25 %

Summary



- At very high electron energies $\lambda \ll r_p$:
  the proton appears to be a sea of
  quarks and gluons.
- Deep Inelastic Scattering = Elastic scattering
  from the quasi-free constituent quarks
  - ➡ Bjorken Scaling $F_1(x, Q^2) \rightarrow F_1(x)$ point-like scattering
  - ➡ Callan-Gross $F_2(x) = 2xF_1(x)$ Scattering from spin-1/2
- Describe scattering in terms of parton distribution functions $u(x), d(x), \ldots$
  which describe momentum distribution inside a nucleon
- The proton is much more complex than just uud - sea of anti-quarks/gluons
- Quarks carry only 50 % of the protons momentum – the rest is due to
  low energy gluons