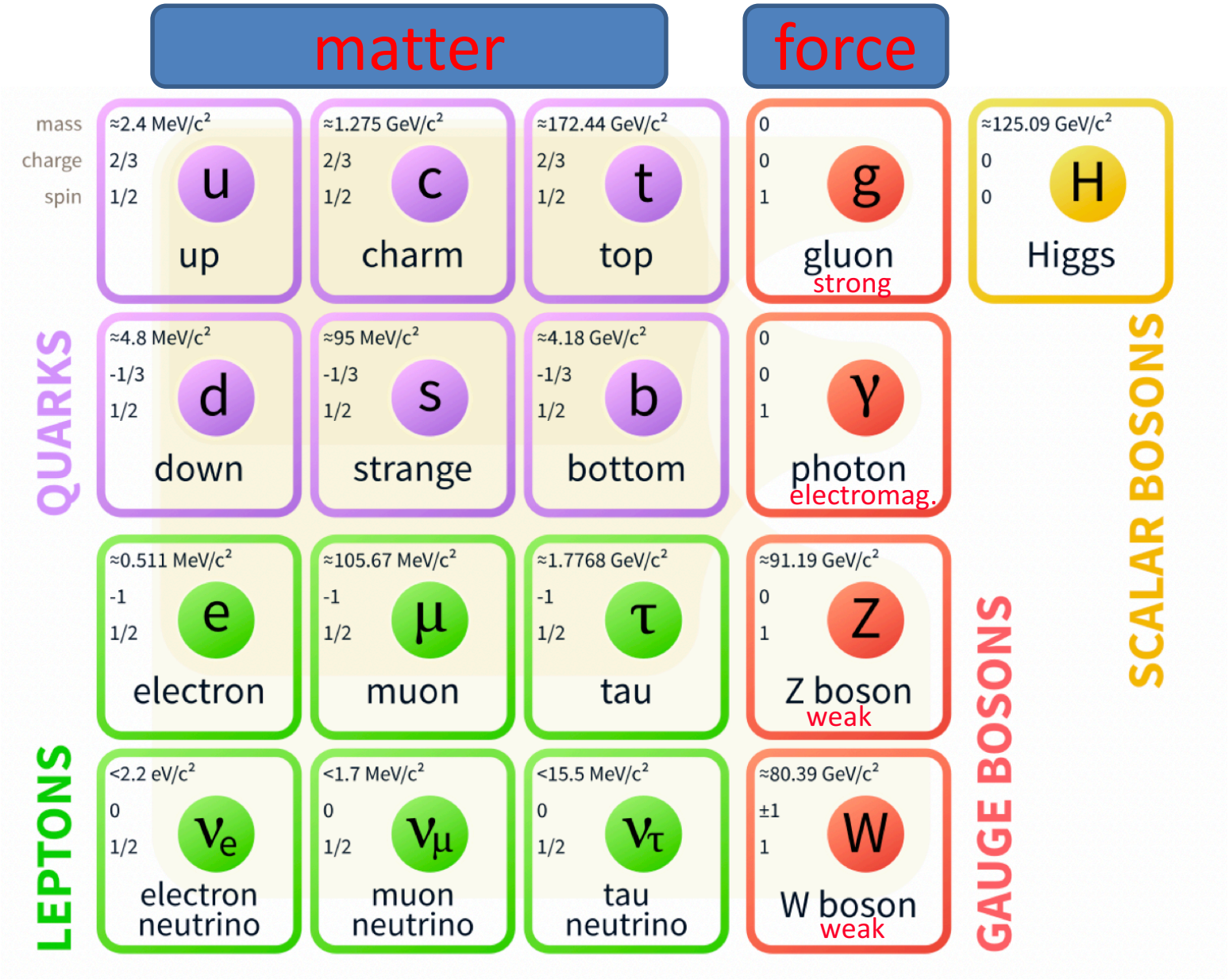
Standard Model building blocks



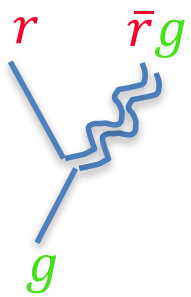
Gauge Fields

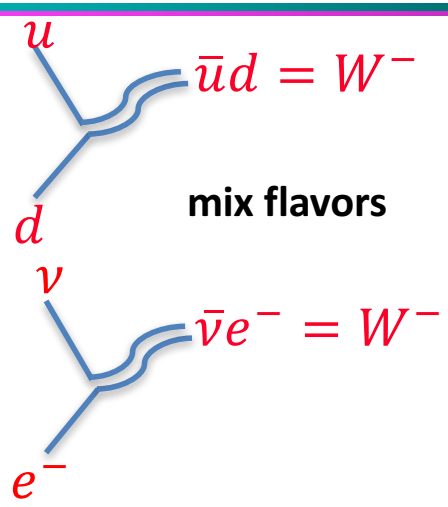
- QED is a gauge theory – 6 components (**E**,**B**) arising from the vector potential **A**=(Ao,Ai), $\boldsymbol{E} = \boldsymbol{\nabla} A$, **B**= $\boldsymbol{\nabla} \times A$
- Conservation of charge: - Charge is a source of the Gauge field - Gauss law - Maxwell like fields
- Gauge fields assume conservation of charge --> symmetry: $A'_\mu = A_\mu e^{i\theta}$
- Not any conservation law comes with the gauge field: For example, baryon number is conserved, but it does not come with any gauge field.
- Color charge has a gauge field – gluons
- G_{ij} - gluon field
- What is the symmetry operation? It is multiply by the unitary operation U[I,j]*q[j]=q'[i],
- antiquarks (antiparticles) are presented by complex conjugate fields q
- Gluons define transitions q[i] → q[j] = q[i]-A[I,j]-q[j]
- G_{ij} - gluon field transforms as quark-antiquark $q_i^* q_j$
- q-----Q exchange qluons (field lines) which interact and bundle together → confinement
- What about weak force (W and Z bosons)

Gauge Symmetry

quarks	u	d	c	s	t	b
red	u_r	d_r	c_r	s_r	t_r	b_r
blue	u_b	d_b	c_b	s_b	t_b	b_b
green	u_g	d_g	c_g	s_g	t_g	b_g
leptons	ν_e	e^-	ν_μ	μ^-	ν_τ	τ^-

mix colors





- Color symmetry unitary matrix mix 3 colors, not quarks
  - SU(3) – 8 generators – 8 gauge bosons we call “gluons”
- Weak interactions mix u-d, c-s and t-b. → SU(2) group
  - one more gauge “flavor” symmetry – should be gauge bosons
  - leptons like the 4th color
  - e- + anti-neutrino mix = W- → d+antiU
  - e+ + neutrino mix = W+ → antiD+u
- SU(2) - 3 generators, 3 gauge bosons, W-, W+, Z
- SU(1) – 1 gauge boson - photon

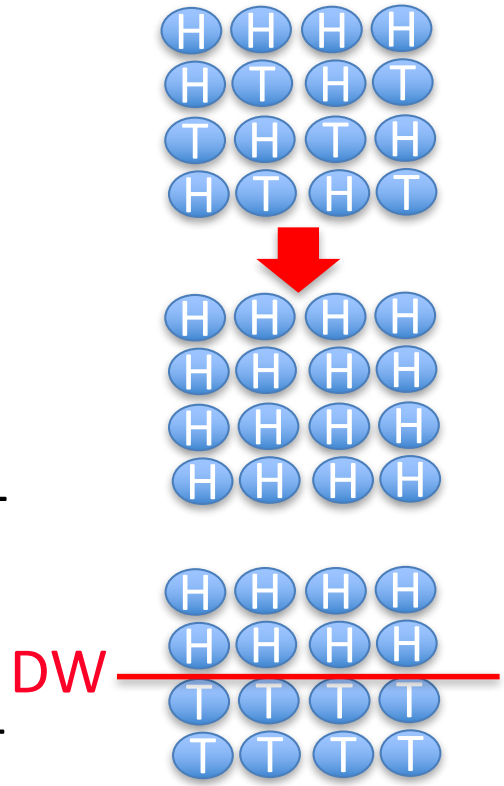
SU(3) x SU(2) x SU(1)

α_s α_w α $\frac{e}{2\sqrt{2\varepsilon}} = g_w \sin(\theta_w) = g_z \cos(\theta_w)$

Symmetry Breaking

- SU(3) x SU(2) x U(1)
  - Why leptons do not participate in color interactions???
  - Why 3 flavors???

- Spontaneous symmetry breaking:
  - Round table -> forks -> what fork to use -> small perturbation causes to use either left or right fork
  - Heads/tails universe: all heads = all tails = ground state (e.g. two nearby head and tail costs energy). But only one ground state can be realized because of boundary conditions far away.
  - Explicit symmetry braking – what is the difference – difference in the ground states of heads and tails →Spontaneous SB has domain walls dividing heads from tails. DW can not exist if there is explicit SB. Presence of DW is indication of SB. Magnetic field – example of explicit symmetry breaking



Discrete vs Continuous Symmetry Breaking

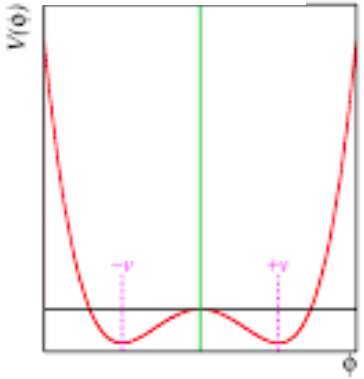
- How a particle like a photon can gain a mass? Two key concepts
  - Spontaneous symmetry breaking: if one spin frozen all neighbors will align
  - Explicit SB: an external magnetic field will define orientation of spins
  - Example of a domain wall in a lattice with two different boundary conditions. Spontaneous SB will divide a lattice in two domains
- Field ϕ and its dynamic is defined by a Lagrangian

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

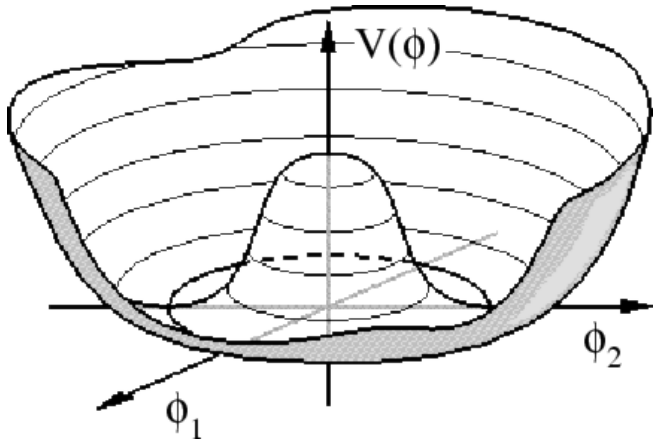
- Discrete vs continuous symmetry: e.g. spin flip vs spin rotation

no symmetry breaking

symmetry breaking



continuous symmetry breaking



Goldstone Boson

- Continuous case – complex field ϕ
  - $L = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - V(\phi\phi^*)$
  - Small variation of the gradient – only first term in L – massless particle
  - Let say $\phi = \rho e^{i\alpha}$

$$\partial_\mu\phi = (\partial_\mu\rho + i\rho\partial_\mu\alpha)e^{i\alpha}$$
$$\partial_\mu\phi^* = (\partial_\mu\rho - i\rho\partial_\mu\alpha)e^{-i\alpha}$$
$$\mathrm{L} = (\partial_\mu\rho)^2 + \rho^2(\partial_\mu\alpha)2 - \mathrm{V}(\rho)$$

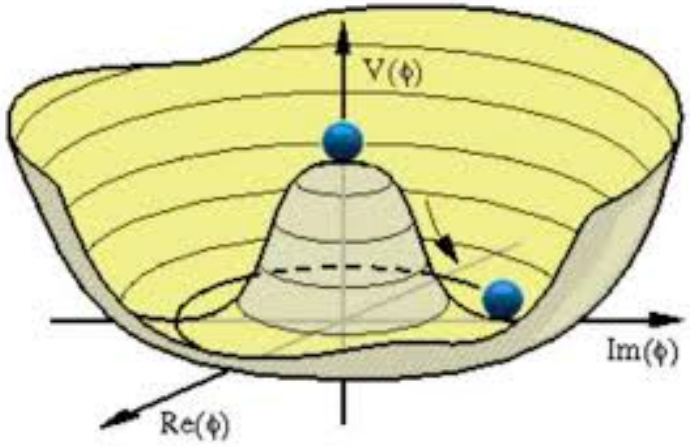
  - Two fields <--> Two oscillations

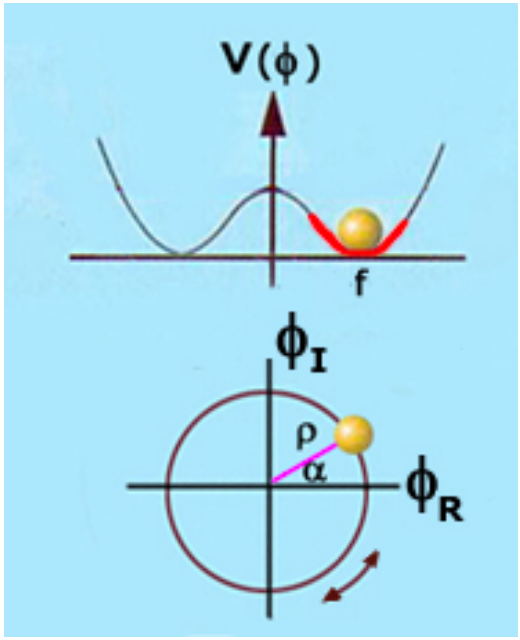
$(\partial_\mu\rho)^2 - \mathrm{V}(\rho)$ - massive field – restoring force

$(\partial_\mu\rho\alpha)^2 = (\partial_\mu\beta)^2$ -massless field – no restoring force

  - Goldstone Bosons – only gradient, like motion along the rim of the Mexican hat – boson of U(1) symmetry
    - ✓ like a) spin waves in ferrimagnets when spin orientation gradually changes - massless particles (rotational symmetry), b) air pressure wave, slinky (horizontal translation), c) long string (vertical translation)

$$\mathrm{V} = -a\phi^*\phi + b(\phi\phi^*)^2$$





Gauge Invariance

- Charges carried by fields are associated with complex fields – phase transformation - can multiply ϕ by any phase exp(iθ). θ(x) – just a coordinate dependent parameter allowed to vary from place to place
- Gauge invariance –symmetry wrt θ(x)
  - Physics should not depend on θ(x) – L is invariant

$$L = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - V(\phi\phi^*)$$

  - Can multiply ϕ by any phase exp(iθ)
  - V($\phi^*\phi$) invariant wrt θ but

$$\partial_\mu \phi' = (\partial_\mu \phi + i\phi\partial_\mu\theta)e^{i\theta}$$
$$\partial_\mu \phi'^* = (\partial_\mu \phi^* - i\phi^*\partial_\mu\theta)e^{-i\theta}$$
$$(\partial_\mu \phi + i\phi\partial_\mu\theta)\,(\partial_\mu \phi - i\phi\partial_\mu\theta)$$ – L is not invariant!

- How to restore Gauge invariance? – introduce more fields
  - Transformation of the vector potential **A**
  - $A'_\mu = A_\mu - \partial_\mu\theta$
  - Gauge covariant derivative

$$D_\mu\phi = \partial_\mu\phi + iA_\mu\phi, \quad D_\mu\phi^* = \partial_\mu\phi^* - iA_\mu\phi^*$$
$$D_\mu\phi' = (\partial_\mu\phi + i\phi\partial_\mu\theta + iA_\mu\phi - i\phi\partial_\mu\theta))e^{i\theta} = D_\mu\phi e^{i\theta}$$
$$D_\mu\phi^{*\prime} = (\partial_\mu\phi^* - i\phi^*\partial_\mu\theta + iA_\mu\phi^* + i\phi^*\partial_\mu\,\theta))e^{-i\theta} = D_\mu\phi^* e^{-i\theta}$$

Add vector potential **A**

- EM gauge invariance assumes interaction $e^- \rightarrow e^-\gamma$
- Add a "photon"-like particle with vector potential A

$$e^- \rightarrow e^-W^0, \nu_e \rightarrow \nu_e W^0, e^- \rightarrow \nu_e W^-, \nu_e \rightarrow e^-W^+, e^- \rightarrow e^-B^0, \nu_e \rightarrow \nu_e B^0$$

$$\gamma = B^0 cos\theta_W + W^0 sin\theta_W, Z^0 = W^0 cos\theta_W - B^0 sin\theta_W$$

- tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Lagrangian: $F_{\mu\nu}F^{\mu\nu}$ - gauge invariant
- Total gauge invariant Lagrangian of vector field + field which quanta are "charged" particles

$$L = \frac{1}{2} D_\mu \phi^* D^\mu \phi - V(\phi\phi^*) + F_{\mu\nu}F^{\mu\nu}$$

- Get local symmetry at the cost of an extra field

  $D_\mu \phi = \partial_\mu \phi + iA_\mu \phi$, $A_\mu \phi$ - interaction of A and scalar field
- Question: How can we get rid of massless Goldstone boson and make "photon" massive?
- Can we add mass term for a photon $m^2 A_\mu A^\mu$? Technically NO - not gauge invariant.
- How gauge bosons (like W and Z) get a mass?

Higgs Phenomena

- How gauge bosons get a mass?
  - Photon can get a mass – when propagate in a superconductor
  - E-p -> hw - hk plane w=ck → no mass – linear dispersion (k=1/λ)
  - Particle with mass: E=p^2+m – dispersion relation
  - Mass – oscillation of the whole field lattice (k=0=p)
- Spontaneous symmetry breaking creates Goldstone boson (oscillations along min of potential)

$$L = (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \alpha) 2 - \mathrm{V}(\rho)$$

$\rho = \mathrm{f} + H$ - H is Higgs field

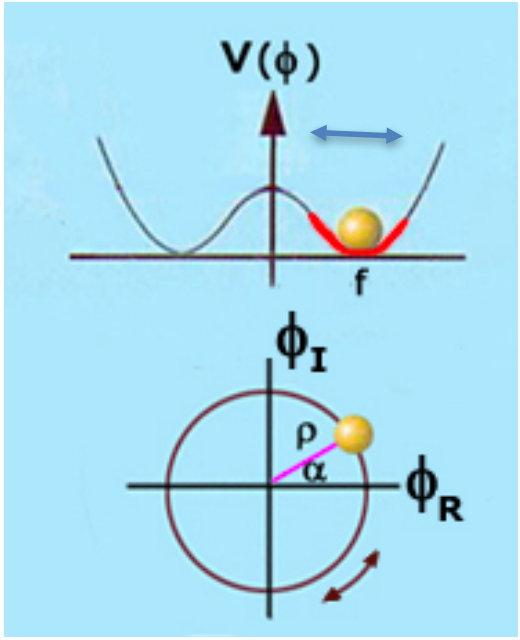
$$L = \mathrm{f}^2 (\partial_\mu \alpha) 2 = (\partial_\mu \beta)^2$$

- Gauge invariance kills Goldstone boson (oscillations along min of potential) and give "photon" a mass

$\left(\partial_\mu \phi + i A_\mu \phi\right)\left(\partial_\mu \phi^* - i A_\mu \phi^*\right) \rightarrow -i\mathrm{f}(\partial_\mu \alpha + A_\mu)\, i\mathrm{f}(\partial_\mu \alpha + A_\mu)$ →gauge transform → $\mathrm{f}^2 A_\mu A_\mu$ - angular degree of freedom "disappeared" and "photon" gets a mass f ! (3 polarizations instead of 2)

$\phi = \rho e^{i\alpha}$

Oscillation in ρ is massive Higgs boson



Oscillation in α is massless Goldstone boson

Higgs and masses of fermions

- Left and right (parity) is a symmetry in QCD and QED
- W boson is emitted by only left-handed particles – electron in beta-decay is always left-handed. Looks like left-handed particles carry weak charge and right-handed not.
- Dirac equation

$$i\frac{\partial\Psi}{\partial t}+i\alpha_i\frac{\partial\Psi}{\partial x_i}=m\beta\Psi$$

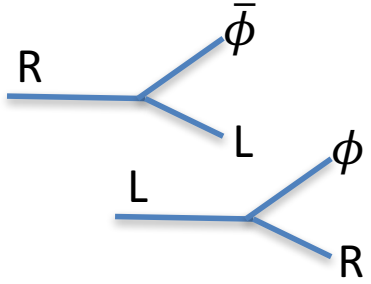
$$i\frac{\partial\Psi_R}{\partial t}+i\alpha_i\frac{\partial\Psi_R}{\partial x_i}=m\Psi_L,\quad i\frac{\partial\Psi_L}{\partial t}-i\alpha_i\frac{\partial\Psi_L}{\partial x_i}=m\Psi_R$$

  - mass mixes up Ψ_R and Ψ_L
  - what if only Ψ_R has a weak charge q – mass term would violate q conservation

    $\Psi_R \to e^{i\theta}\Psi_R$, $\Psi_L \to \Psi_L$: U(1) destroys symmetry of Dirac Eq
  - Lets take a boson field $\phi \to e^{i\theta}\phi$

$$i\frac{\partial\Psi_R}{\partial t}+i\alpha_i\frac{\partial\Psi_R}{\partial x_i}=g\phi^*\Psi_L$$

$$i\frac{\partial\Psi_L}{\partial t}-i\alpha_i\frac{\partial\Psi_L}{\partial x_i}=g\phi\Psi_R$$



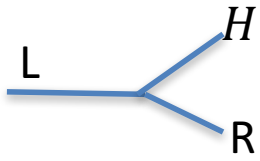
  - L and R can turn to each other by emitting a boson ($\phi \to e^{i\theta}\mathrm{f} \to \mathrm{f}$)
  - $m_f = g\mathrm{f} \sim gM_z$. g is Yukawa coupling – different for different fermions

Higgs and masses of fermions

- Lets explicitly generate Higgs field

$$i\frac{\partial \Psi_R}{\partial t} + i\alpha_i \frac{\partial \Psi_R}{\partial x_i} = g(\phi + h)^* \Psi_L$$

$$i\frac{\partial \Psi_L}{\partial t} - i\alpha_i \frac{\partial \Psi_L}{\partial x_i} = g(\phi + h) \Psi_R$$



- Probability of decay $H \to f\bar{f}$ is proportional to g_f^2
  - Higgs prefers to decay to massive fermions
- All particles in the standard model have a mass due to a shift f in the Higgs field – all masses are proportional to f
  - Bosons get mass due to spontaneous symmetry breaking
  - Fermions gets mass due to a peculiar property of the weak interaction (only "left" weak charge) + shift in the Higgs field
  -

$$\Gamma(H \to f\bar{f}) = N\alpha_W \left(\frac{m_f}{M_W}\right)^2 M_H$$

$$\Gamma(H \to b\bar{b}) : \Gamma(H \to c\bar{c}) : \Gamma(H \to \tau\bar{\tau}) = 3m_b^2 = 3m_c^2 = m_\tau^2$$

- Decays to light fermions are suppressed. Main decay modes are
  - $b\bar{b}, b\bar{b}, \tau\bar{\tau}, \mathrm{gg}, W^{+-} f\bar{f}', Z^0 f\bar{f}$
- Main production mechanism is gluon fusion + $t\bar{t}$ loop