Weak Isospin

★ The Weak Interaction arises from SU(2) local phase transformations

$$\psi \rightarrow \psi' = \psi e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e the three Pauli spin matrices

3 Gauge Bosons $W_1^\mu,\ W_2^\mu,\ W_3^\mu$

★ The wave-functions have two components which, in analogy with isospin, are represented by "weak isospin"

★ The fermions are placed in isospin doublets and the local phase transformation corresponds to

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}' = e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

★ Weak Interaction only couples to LH particles/RH anti-particles, hence only place LH particles/RH anti-particles in weak isospin doublets: $I_W = \frac{1}{2}$
RH particles/LH anti-particles placed in weak isospin singlets: $I_W = 0$

Weak Isospin

$$I_W = \tfrac{1}{2} \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \begin{matrix} \longleftarrow I_W^3 = +\frac{1}{2} \\ \longleftarrow I_W^3 = -\frac{1}{2} \end{matrix}$$

$$I_W = 0 \quad (\nu_e)_R,\ (e^-)_R, \ldots (u)_R,\ (d)_R, \ldots$$

RH/LH refer to chiral states

Electroweak Unification

★ Tempting to identify the W^3 as the Z

★ However this is not the case, have two physical neutral spin-1 gauge bosons, γ, Z and the W^3 is a mixture of the two,

★ Equivalently write the photon and Z in terms of the W^3 and a new neutral spin-1 boson the B

★ The <u>physical</u> bosons (the Z and photon field, A) are:

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

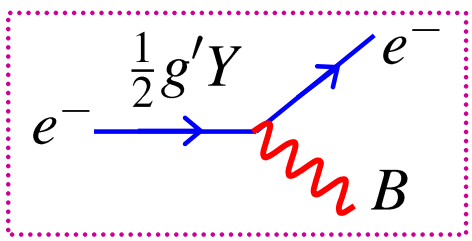
θ_W is the weak mixing angle

★ The new boson is associated with a new gauge symmetry similar to that of electromagnetism : $U(1)_Y$

★ The charge of this symmetry is called WEAK HYPERCHARGE Y

$$Y = 2Q - 2I^3_W$$

Q is the EM charge of a particle
I^3_W is the third comp. of weak isospin



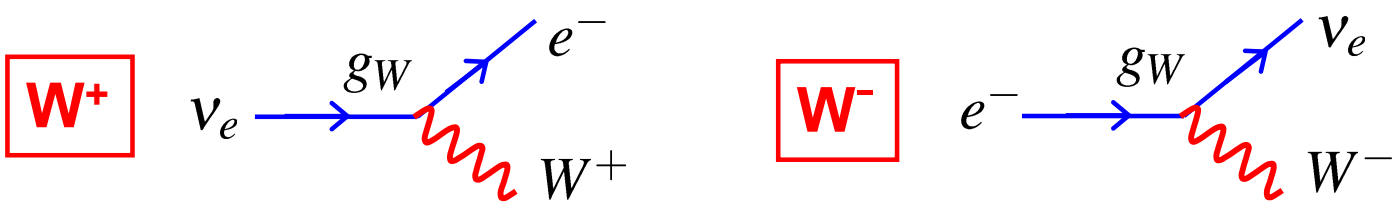
• By convention the coupling to the B_μ is $\frac{1}{2}g'Y$

$e_L : \; Y = 2(-1) - 2(-\frac{1}{2}) = -1$ $\qquad$ $\nu_L : \; Y = +1$

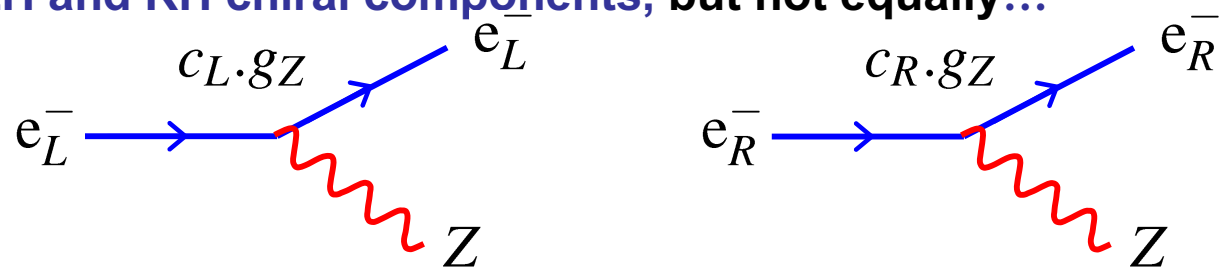
$e_R : \; Y = 2(-1) - 2(0) = -2$ $\qquad$ $\nu_R : \; Y = 0$

(this identification of hypercharge in terms of Q and I_3 makes all of the following work out)

★ the W Boson couples LH chiral component: LH coupling=g_W. RH coupling = 0



★ Unlike for the Charged Current Weak interaction (W) the Z Boson couples to both LH and RH chiral components, but not equally…



$$c_L = I_W^3 - Q\sin^2\theta_W \qquad \boxed{c_R = -Q\sin^2\theta_W}$$

W^3 part of Z couples only to LH components (like $W^\pm$)

B_μ part of Z couples equally to LH and RH components

$$\sin^2\theta_W \approx 0.23$$

Fermion	Q	I_W^3	c_L	c_R
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08

Summary

★ The Standard Model interactions are mediated by spin-1 gauge bosons

★ The form of the interactions are completely specified by the assuming an underlying local phase transformation ➡ GAUGE INVARIANCE

$U(1)_{em}$ ➡ QED

$SU(2)_L$ ➡ Charged Current Weak Interaction + W^3

$SU(3)_{col}$ ➡ QCD

★ In order to "unify" the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry : U(1) hypercharge

$U(1)_Y$ ➡ B_μ

★ The physical Z boson and the photon are mixtures of the neutral W boson and B determined by the Weak Mixing angle

$$\sin\theta_W \approx 0.23$$

★ Have we unified the EM and Weak interactions ? Well not really…

- Started with two independent theories with coupling constants g_W, e
- Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model θ_W