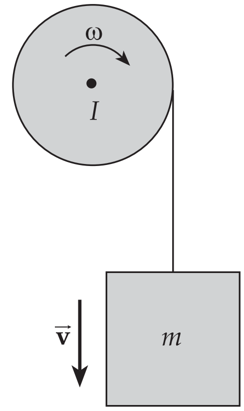
PHY 2053 Homework #9 Solutions

8.40 As the bucket drops, it loses gravitational potential energy. The spool gains rotational kinetic energy and the bucket gains translational kinetic energy. Since the string does not slip on the spool, $v = r\omega$ where r is the radius of the spool. The moment of inertia of the spool is $I = \frac{1}{2}Mr^2$, where M is the mass of the spool. Conservation of energy gives



$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy_f = 0 + 0 + mgy_i$$

or $$\frac{1}{2}m\left(r\omega\right)^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 = mg\left(y_i - y_f\right)$$

This gives $$\omega = \sqrt{\frac{2mg\left(y_i - y_f\right)}{\left(m + \frac{1}{2}M\right)r^2}} = \sqrt{\frac{2\left(3.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(4.00\text{ m}\right)}{\left[3.00\text{ kg} + \frac{1}{2}\left(5.00\text{ kg}\right)\right]\left(0.600\text{ m}\right)^2}} = \boxed{10.9\text{ rad/s}}$$

8.42 (a) The moment of inertial of the flywheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(500\text{ kg}\right)\left(2.00\text{ m}\right)^2 = 1.00\times10^3\text{ kg}\cdot\text{m}^2$$

and the angular velocity is

$$\omega = \left(5000\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 524\text{ rad/s}$$

Therefore, the stored kinetic energy is

$$KE_{stored} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(1.00\times10^3\text{ kg}\cdot\text{m}^2\right)\left(524\text{ rad/s}\right)^2 = \boxed{1.37\times10^8\text{ J}}$$

(b) A 10.0-hp motor supplies energy at the rate of

$$\tilde{A} = \left(10.0\text{ hp}\right)\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 7.46\times10^3\text{ J/s}$$

The time the flywheel could supply energy at this rate is

$$t = \frac{KE_{stored}}{\tilde{A}} = \frac{1.37\times10^8\text{ J}}{7.46\times10^3\text{ J/s}} = 1.84\times10^4\text{ s} = \boxed{5.10\text{ h}}$$

8.44 Using conservation of mechanical energy,

$$\left(KE_{trans} + KE_{rot} + PE_g\right)_f = \left(KE_{trans} + KE_{rot} + PE_g\right)_i$$

or $$\frac{1}{2}M v_t^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + M g\left(L\sin\theta\right)$$

Since $I = \frac{2}{5}M R^2$ for a solid sphere and $v_t = R\omega$ when rolling without slipping, this becomes $\frac{1}{2}M R^2\omega^2 + \frac{1}{5}M R^2\omega^2 = M g\left(L\sin\theta\right)$ and reduces to

8.50 The total angular momentum of the system is

$$I_{total} = I_{masses} + I_{student} = 2\left(m r^2\right) + 3.0 \text{ kg}\cdot\text{m}^2$$

Initially, $r = 1.0 \text{ m}$, and $I_i = 2\left[\left(3.0 \text{ kg}\right)\left(1.0 \text{ m}\right)^2\right] + 3.0 \text{ kg}\cdot\text{m}^2 = 9.0 \text{ kg}\cdot\text{m}^2$

Afterward, $r = 0.30 \text{ m}$, so

$$I_f = 2\left[\left(3.0 \text{ kg}\right)\left(0.30 \text{ m}\right)^2\right] + 3.0 \text{ kg}\cdot\text{m}^2 = 3.5 \text{ kg}\cdot\text{m}^2$$

(a) From conservation of angular momentum, $I_f\omega_f = I_i\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.0 \text{ kg}\cdot\text{m}^2}{3.5 \text{ kg}\cdot\text{m}^2}\right)\left(0.75 \text{ rad/s}\right) = \boxed{1.9 \text{ rad/s}}$$

(b) $$KE_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(9.0 \text{ kg}\cdot\text{m}^2\right)\left(0.75 \text{ rad/s}\right)^2 = \boxed{2.5 \text{ J}}$$

8.54 For one of the crew, $\Sigma F_c = m\,a_c$ becomes $n = m\left(\frac{v_t^2}{r}\right) = m\,r\omega_i^2$

We require $n = m\,g$, so the initial angular velocity must be $\omega_i = \sqrt{\frac{g}{r}}$

From conservation of angular momentum, $I_f\omega_f = I_i\omega_i$, or $\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$

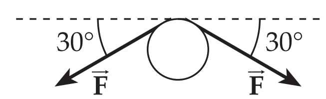
Thus, the angular velocity of the station during the union meeting is

$$\omega_f = \left(\frac{I_i}{I_f}\right)\sqrt{\frac{g}{r}} = \left[\frac{5.00\times 10^8\ \text{kg}\cdot\text{m}^2 + 150(65.0\ \text{kg})(100\ \text{m})^2}{5.00\times 10^8\ \text{kg}\cdot\text{m}^2 + 50(65.0\ \text{kg})(100\ \text{m})^2}\right]\sqrt{\frac{g}{r}} = 1.12\sqrt{\frac{g}{r}}$$

The centripetal acceleration experienced by the managers still on the rim is

$$a_c = r\omega_f^2 = r12.3\ \text{m/s}^2(1.12)^2\frac{g}{r} = (1.12)^2\left(9.80\ \text{m/s}^2\right) = \boxed{12.3\ \text{m/s}^2}$$

9.6 From $Y = \frac{F L_0}{A(\Delta L)}$, the tension needed to stretch the wire by 0.10 mm is



$$F = \frac{Y A(\Delta L)}{L_0} = \frac{Y\left(\pi d^2\right)(\Delta L)}{4L_0}$$

$$= \frac{\left(18\times 10^{10}\ \text{Pa}\right)\pi\left(0.22\times 10^{-3}\ \text{m}\right)^2\left(0.10\times 10^{-3}\ \text{m}\right)}{4\left(3.1\times 10^{-2}\ \text{m}\right)} = 22\ \text{N}$$

The tension in the wire exerts a force of magnitude *F* on the tooth in each direction along the length of the wire as shown in the above sketch. The resultant force exerted on the tooth has an *x*-component of $R_x = \Sigma F_x = -F\cos 30° + F\cos 30° = 0$, and a *y*-component of $R_y = \Sigma F_y = -F\sin 30° - F\sin 30° = -F = -22\ \text{N}$.

Thus, the resultant force is

$\vec{\mathbf{R}} = \boxed{22\ \text{N directed down the page in the diagram}}$.

9.10 (a) When at rest, the tension in the cable equals the weight of the 800-kg object, 7.84×10^3 N . Thus, from $Y=\dfrac{FL_0}{A(\Delta L)}$, the initial elongation of the cable is

$$\Delta L=\frac{F\cdot L_0}{A\cdot Y}=\frac{\left(7.48\times 10^3 \text{ N}\right)(25.0 \text{ m})}{\left(4.00\times 10^{-4} \text{ m}^2\right)\left(20\times 10^{10} \text{ Pa}\right)}=2.45\times 10^{-3} \text{ m}=\boxed{2.5 \text{ mm}}$$

(b) When the load is accelerating upward, Newton's second law gives

$$F-mg=ma_y, \text{ or } F=m\left(g+a_y\right) \qquad (1)$$

If $m=800$ kg and $a_y=+3.0 \text{ m/s}^2$, the elongation of the cable will be

$$\Delta L=\frac{F\cdot L_0}{A\cdot Y}=\frac{\left[(800 \text{ kg})(9.80+3.0) \text{ m/s}^2\right](25.0 \text{ m})}{\left(4.00\times 10^{-4} \text{ m}^2\right)\left(20\times 10^{10} \text{ Pa}\right)}=3.2\times 10^{-3} \text{ m}=3.2 \text{ mm}$$

Thus, the increase in the elongation has been

$$increase=(\Delta L)-(\Delta L)_{initial}=3.20 \text{ mm}-2.45 \text{ mm}=\boxed{0.75 \text{ mm}}$$

9.12 The acceleration of the forearm has magnitude

$$a=\frac{|\Delta v|}{\Delta t}=\frac{80\dfrac{\text{km}}{\text{h}}\left(\dfrac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ h}}{3\,600 \text{ s}}\right)}{5.0\times 10^{-3} \text{ s}}=4.4\times 10^3 \text{ m/s}^2$$

The compression force exerted on the arm is $F=ma$ and the compressional stress on the bone material is

$$Stress=\frac{F}{A}=\frac{(3.0 \text{ kg})\left(4.4\times 10^3 \text{ m/s}^2\right)}{2.4 \text{ cm}^2\left(10^{-4} \text{ m}^2/1 \text{ cm}^2\right)}=\boxed{5.6\times 10^7 \text{ Pa}}$$

Since the stress is less than the allowed maximum, $\boxed{\text{the arm should survive}}$.