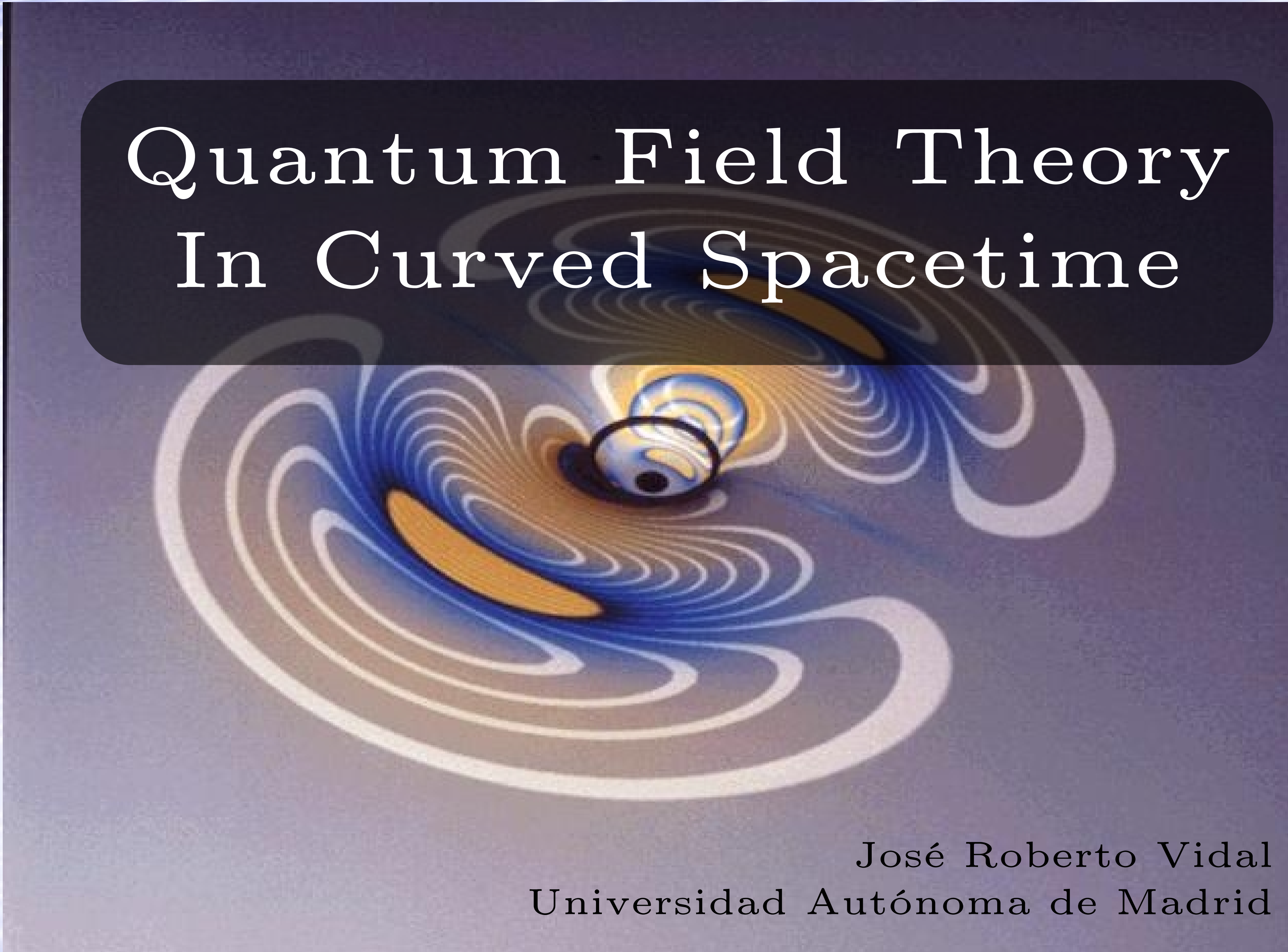
Quantum Field Theory In Curved Spacetime



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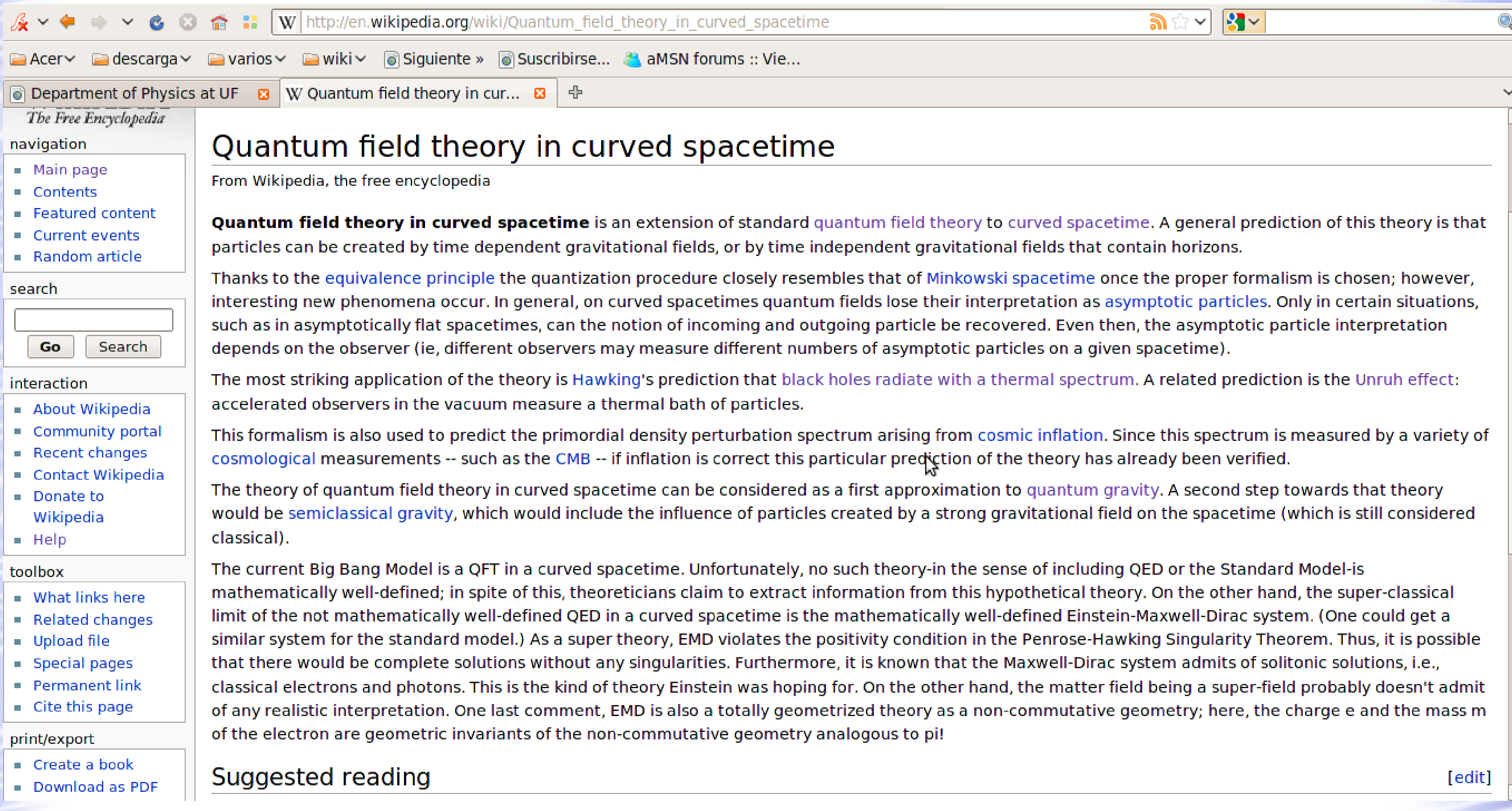
Why QFT in curved ST?



Quantum field theory in curved spacetime

From Wikipedia, the free encyclopedia

Quantum field theory in curved spacetime is an extension of standard quantum field theory to curved spacetime. A general prediction of this theory is that particles can be created by time dependent gravitational fields, or by time independent gravitational fields that contain horizons.

Thanks to the equivalence principle the quantization procedure closely resembles that of Minkowski spacetime once the proper formalism is chosen; however, interesting new phenomena occur. In general, on curved spacetimes quantum fields lose their interpretation as asymptotic particles. Only in certain situations, such as in asymptotically flat spacetimes, can the notion of incoming and outgoing particle be recovered. Even then, the asymptotic particle interpretation depends on the observer (ie, different observers may measure different numbers of asymptotic particles on a given spacetime).

The most striking application of the theory is Hawking's prediction that black holes radiate with a thermal spectrum. A related prediction is the Unruh effect: accelerated observers in the vacuum measure a thermal bath of particles.

This formalism is also used to predict the primordial density perturbation spectrum arising from cosmic inflation. Since this spectrum is measured by a variety of cosmological measurements -- such as the CMB -- if inflation is correct this particular prediction of the theory has already been verified.

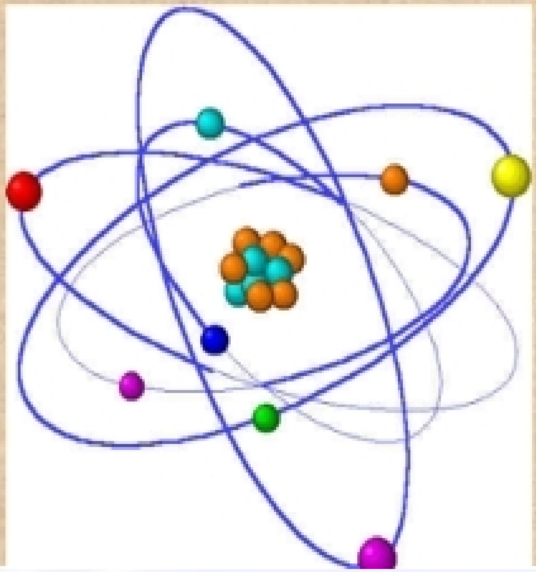
The theory of quantum field theory in curved spacetime can be considered as a first approximation to quantum gravity. A second step towards that theory would be semiclassical gravity, which would include the influence of particles created by a strong gravitational field on the spacetime (which is still considered classical).

The current Big Bang Model is a QFT in a curved spacetime. Unfortunately, no such theory-in the sense of including QED or the Standard Model-is mathematically well-defined; in spite of this, theoreticians claim to extract information from this hypothetical theory. On the other hand, the super-classical limit of the not mathematically well-defined QED in a curved spacetime is the mathematically well-defined Einstein-Maxwell-Dirac system. (One could get a similar system for the standard model.) As a super theory, EMD violates the positivity condition in the Penrose-Hawking Singularity Theorem. Thus, it is possible that there would be complete solutions without any singularities. Furthermore, it is known that the Maxwell-Dirac system admits of solitonic solutions, i.e., classical electrons and photons. This is the kind of theory Einstein was hoping for. On the other hand, the matter field being a super-field probably doesn't admit of any realistic interpretation. One last comment, EMD is also a totally geometrized theory as a non-commutative geometry; here, the charge e and the mass m of the electron are geometric invariants of the non-commutative geometry analogous to pi!

Suggested reading

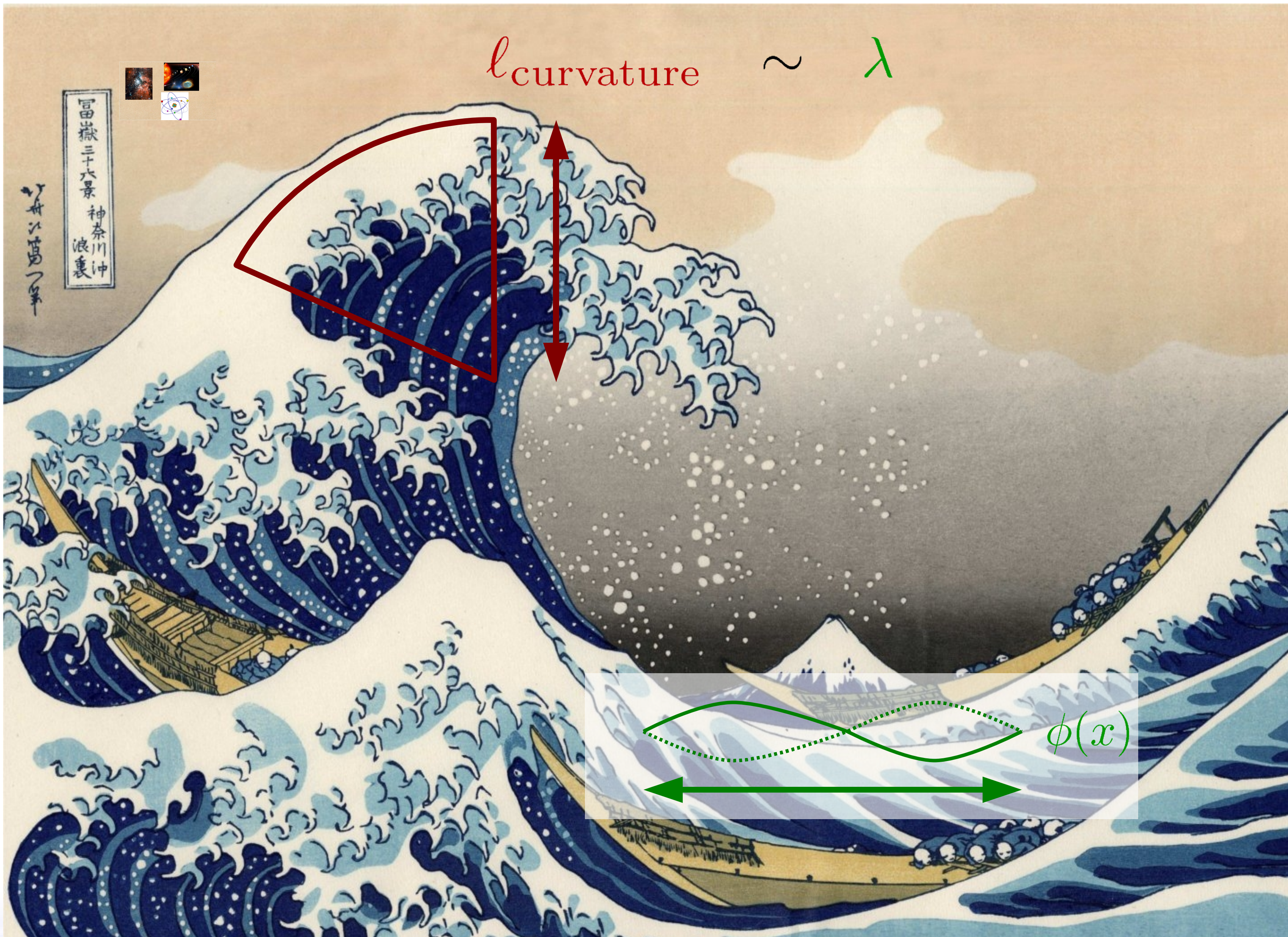
[edit]

Why QFT in curved ST?



Why QFT in curved ST?

Why QFT in curved ST?



Particles vs. fields

A la Weinberg

A la Peskin-Schroeder

Particles vs. fields

A la Weinberg

A la Peskin-Schroeder

- Based on symmetries

The Poincaré group tells the whole story:

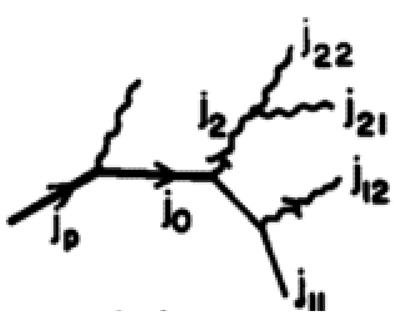
$$|m^2, J^2; p^\mu, \sigma\rangle$$

Particles vs. fields

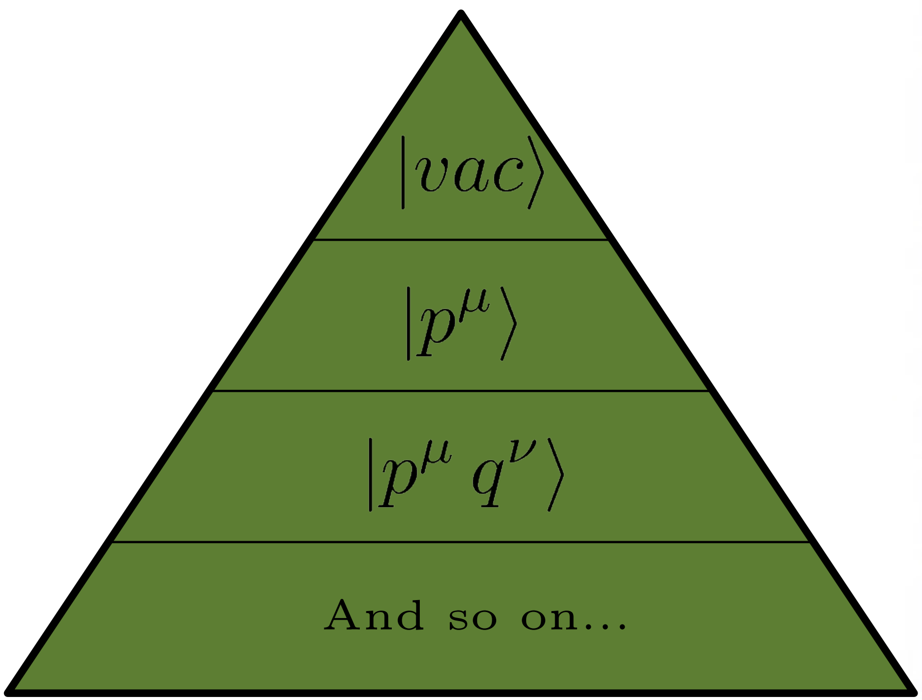
A la Weinberg

A la Peskin-Schroeder

- Based on symmetries
- Multiparticle states



We *need* the Fock space.



Particles vs. fields

A la Weinberg

A la Peskin-Schroeder

- Based on symmetries
- Multiparticle states
- Quantum fields are just a tool.

$$\hat{\phi}(x) \sim \hat{a}_k,\, \hat{a}_k^\dagger$$

$$\hat{\mathcal{H}}_{\text{int}} = \boxed{\text{Products of fields}}$$

Particles vs. fields

A la Weinberg

A la Peskin-Schroeder

- We start with a classical system.

Classical people:
Poisson, Lagrange, Hamilton...

$$\{\phi, \pi\} = \mathbb{I}$$

$$\mathcal{L} = \partial\phi^2 - V(\phi)$$

$$\mathcal{H} = \pi^2 + \nabla\phi^2 + V(\phi)$$

Particles vs. fields

A la Weinberg

A la Peskin-Schroeder

- We start with a classical system.
- Canonical quantization.

$$[\hat{\phi}, \hat{\pi}] = i\mathbb{I}$$

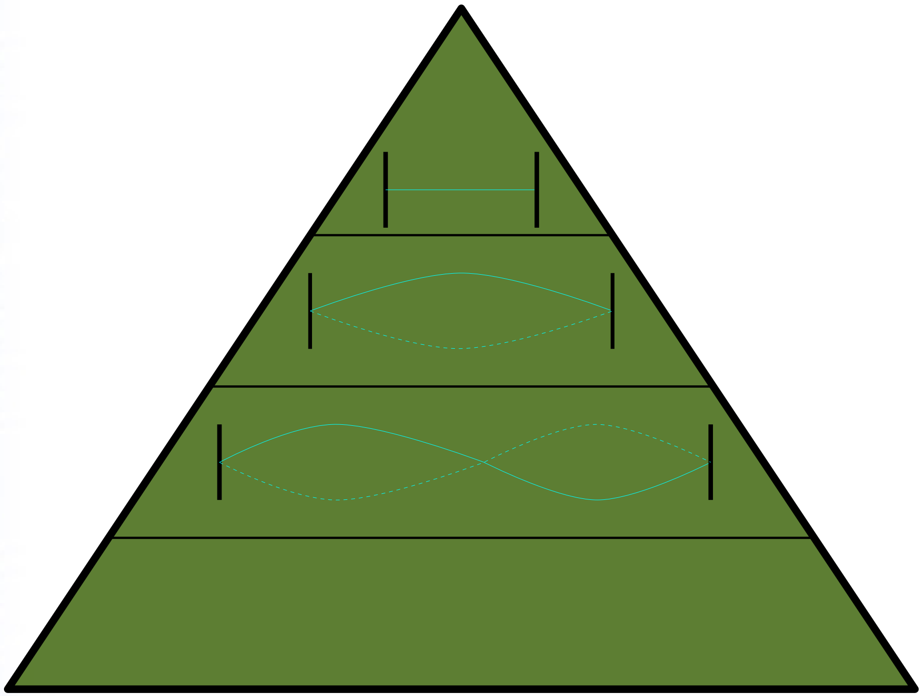
Diagonalization through Fourier transform:

$$\hat{H} = \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k$$

Particles vs. fields

A la Weinberg

A la Peskin-Schroeder



- We start with a classical system.
- Canonical quantization.
- A beatiful metaphore.

Particles are quantized excitations of the field

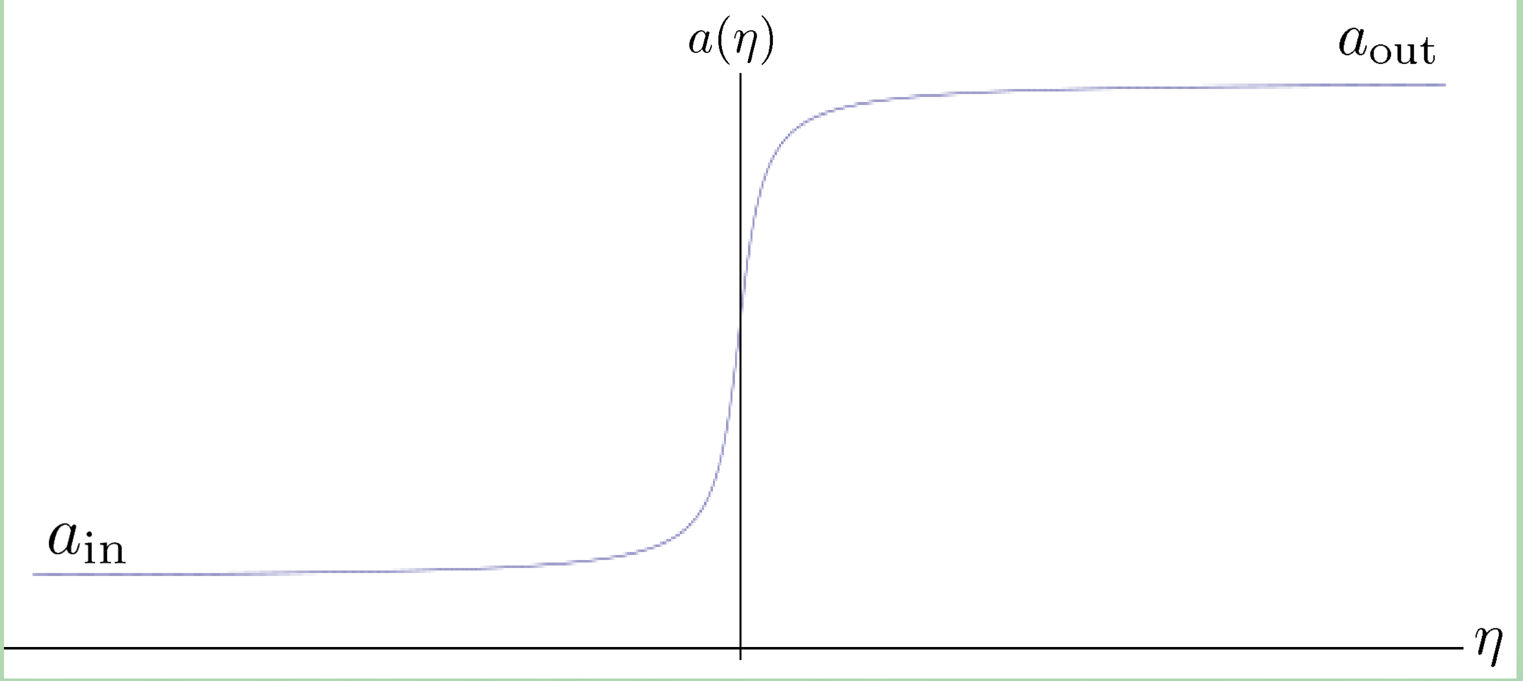
Part I

QFT in curved ST is
like Schwinger effect!

1+1 FRW universe

$$ds^2 = a(\eta)^2(d\eta^2 - dx^2)$$

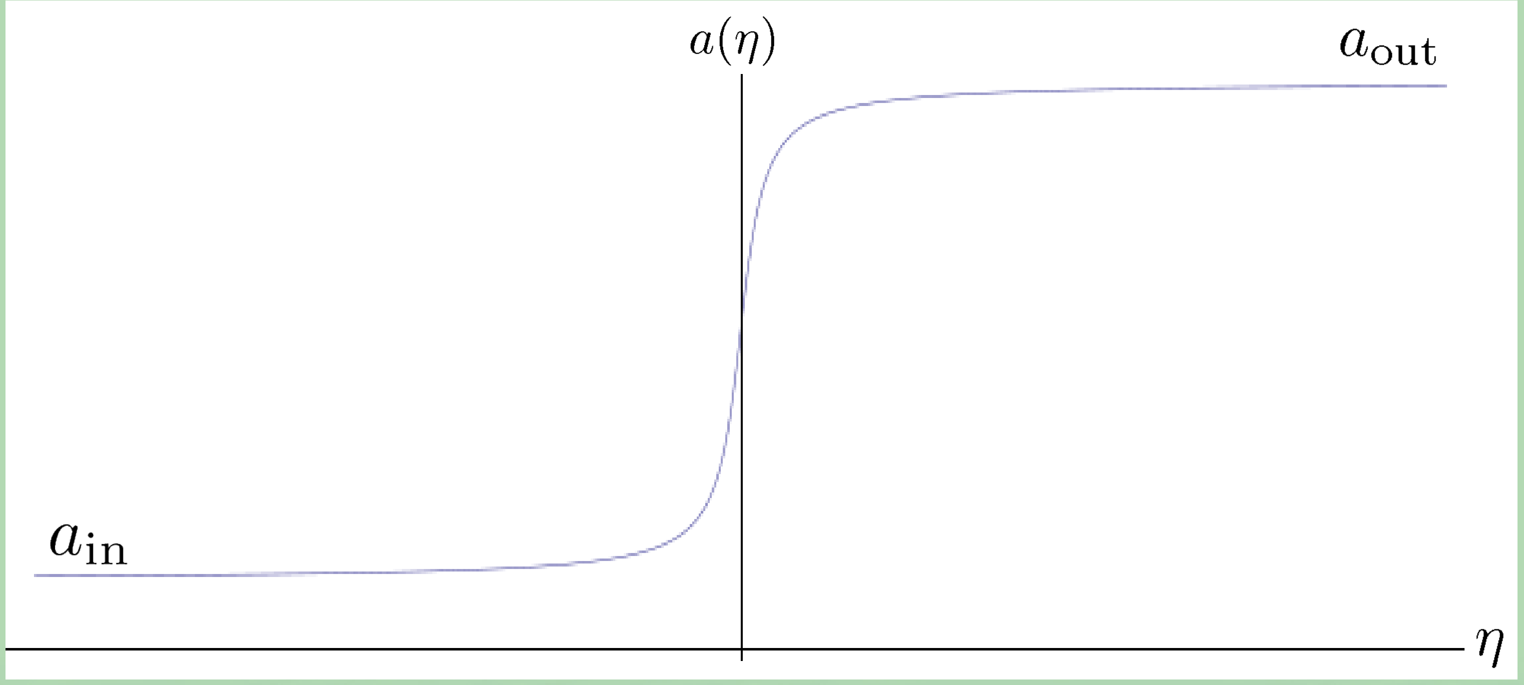
$$\sqrt{g}\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2a^2}{2}\phi^2$$



1+1 FRW universe

$$ds^2 = a(\eta)^2(d\eta^2 - dx^2)$$

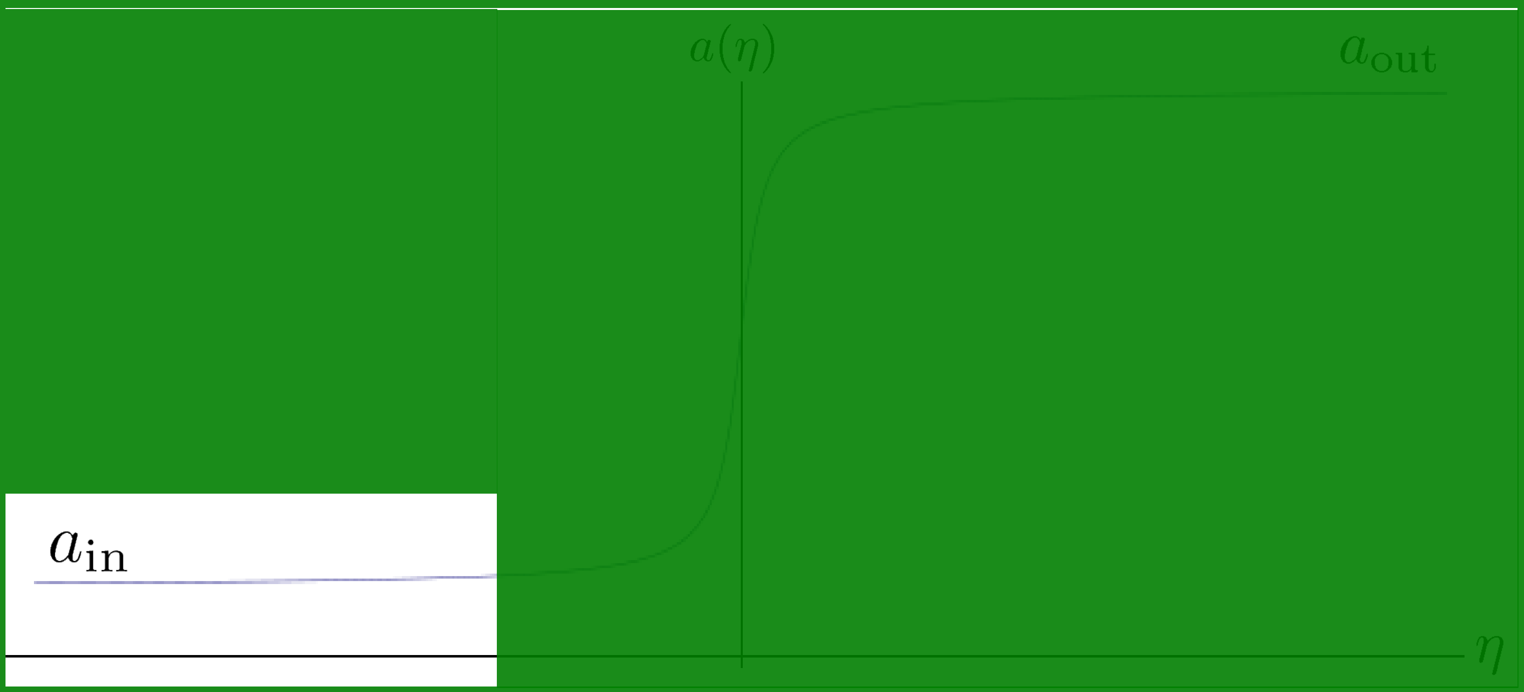
$$\Box\phi + m^2 a(\eta)^2 \phi = 0$$



In modes

$$\Box\phi + m^2 a_{\text{in}}^2 \phi = 0$$

$$\phi_k^{\text{in}} \overset{\eta\to-\infty}{\longrightarrow} e^{ikx} e^{-i\omega_{\text{in}}(k)\eta}$$



In modes

$$\hat{\phi} = \sum_k \hat{a}_k^{\text{in}}\, \phi_k^{\text{in}} + \hat{a}_k^{\text{in}\dagger}\, \phi_k^{\text{in}*}$$

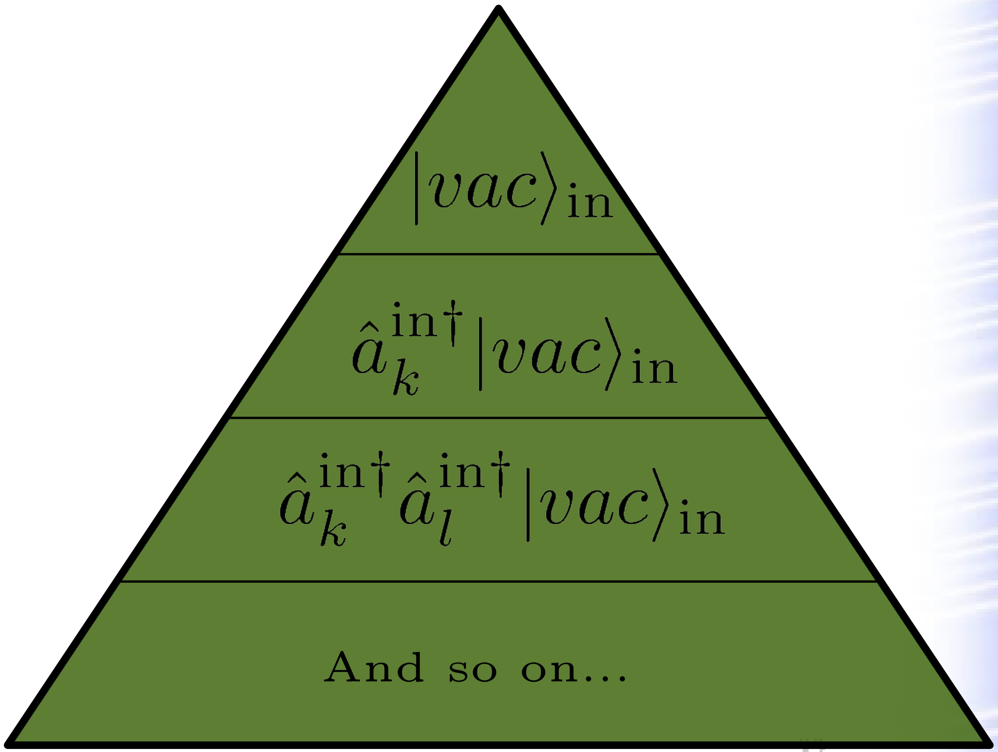
Complete and normalized basis

$$[\hat{a}_k^{\text{in}}, \hat{a}_l^{\text{in}\dagger}] = \delta_{kl}$$

$$\hat{N}^{\text{in}} = \sum_k \hat{a}_k^{\text{in}\dagger} \hat{a}_k^{\text{in}}$$

$|vac\rangle_{\text{in}}$

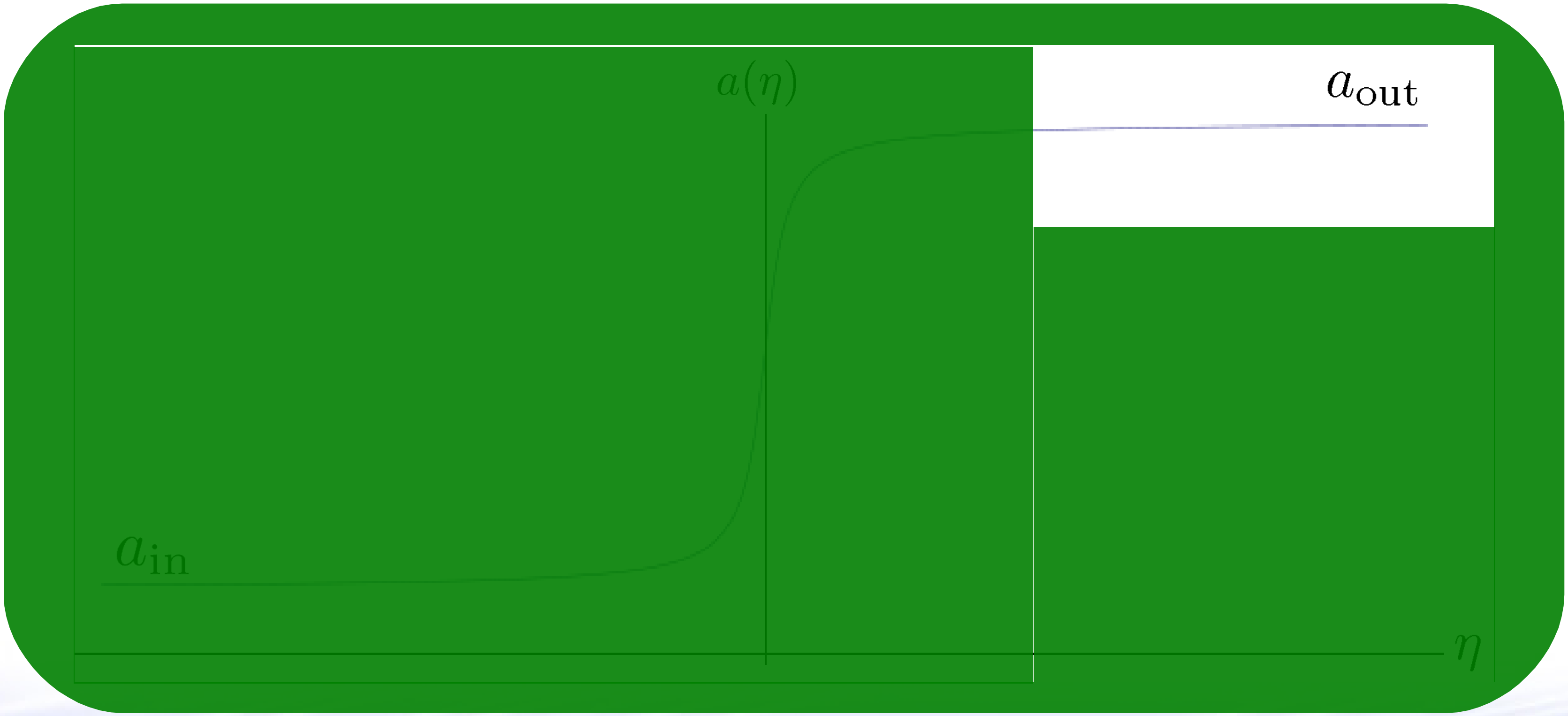
No particles in the dinosaurs era



Out modes

$$\Box\phi + m^2 a_{\rm out}^2 \phi = 0$$

$$\phi_k^{\rm out} \xrightarrow{\eta\rightarrow+\infty} e^{ikx} e^{-i\omega_{\rm out}(k)\eta}$$



Out modes

$$\hat{\phi} = \sum_k \hat{a}_k^{\text{out}} \, \phi_k^{\text{out}} + \hat{a}_k^{\text{out}\dagger} \, \phi_k^{\text{out}*}$$

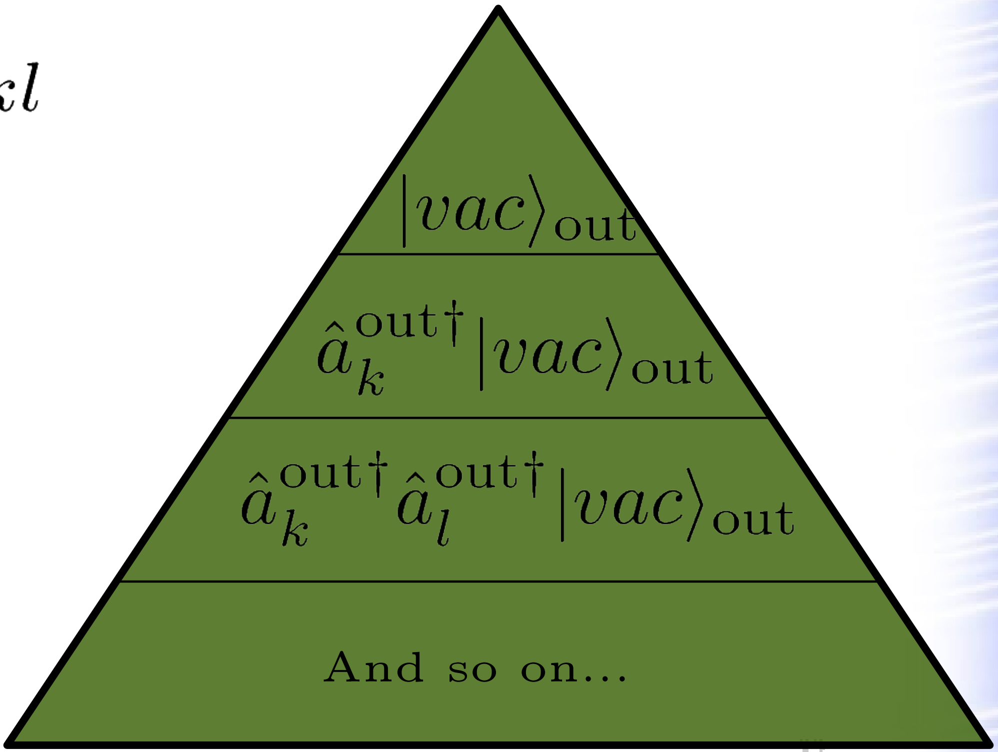
Complete too!!

$$[\hat{a}_k^{\text{out}}, \hat{a}_l^{\text{out}\dagger}] = \delta_{kl}$$

$$\hat{N}^{\text{out}} = \sum_k \hat{a}_k^{\text{out}\dagger} \hat{a}_k^{\text{out}}$$

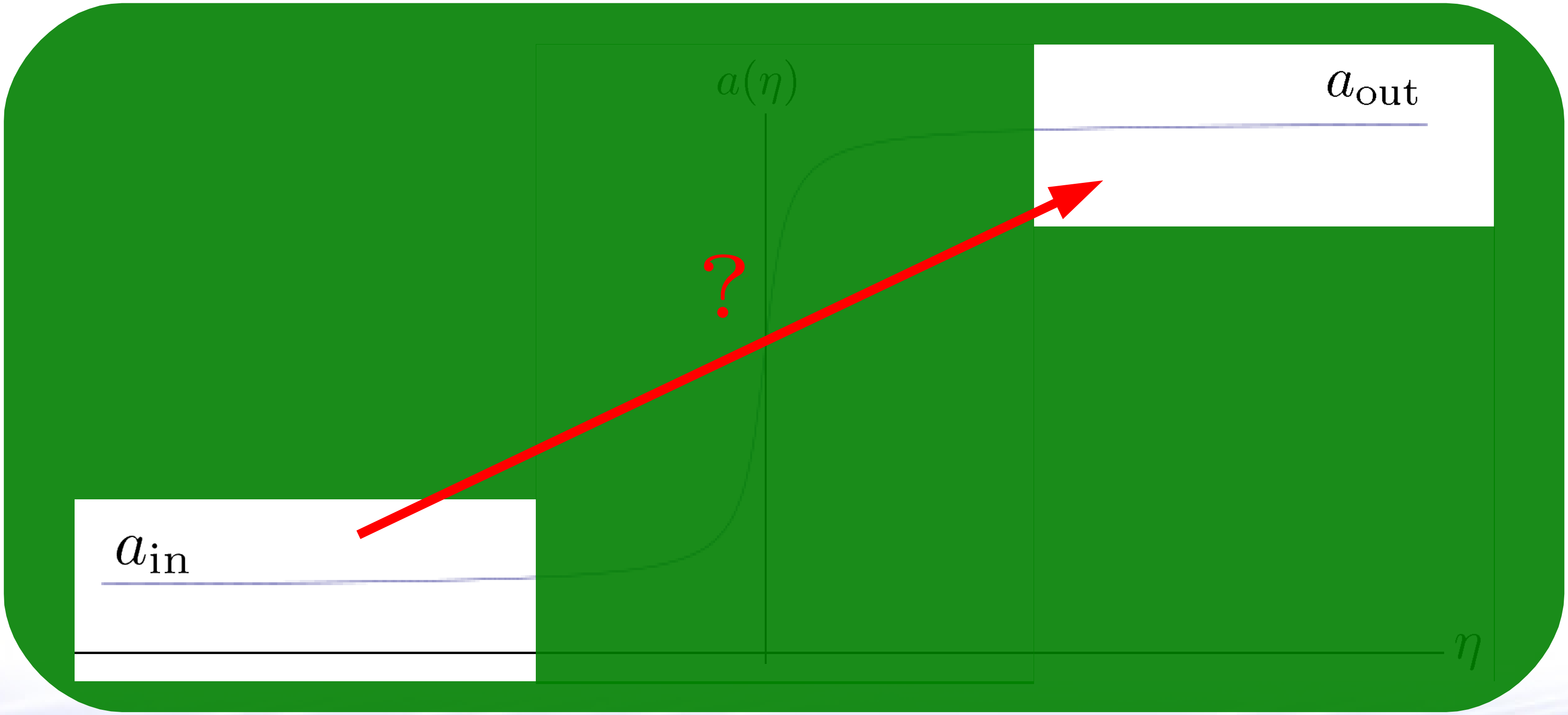
$|vac\rangle_{\text{out}}$

No particles in the spacecrafts era

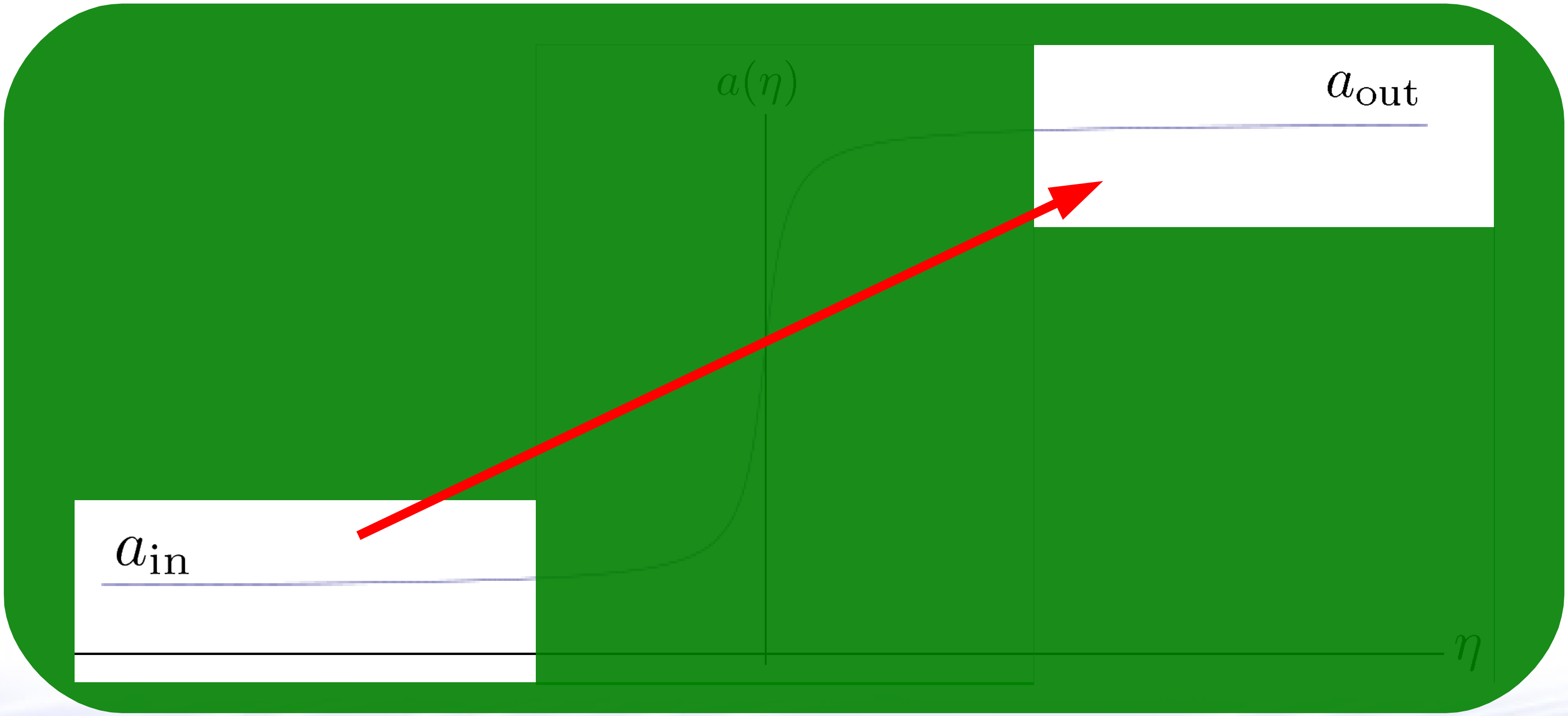


Particles from nowhere

$$|\text{stuff}\rangle_{\text{in}} \xrightarrow{\;?\;} |\text{stuff}\rangle_{\text{out}}$$



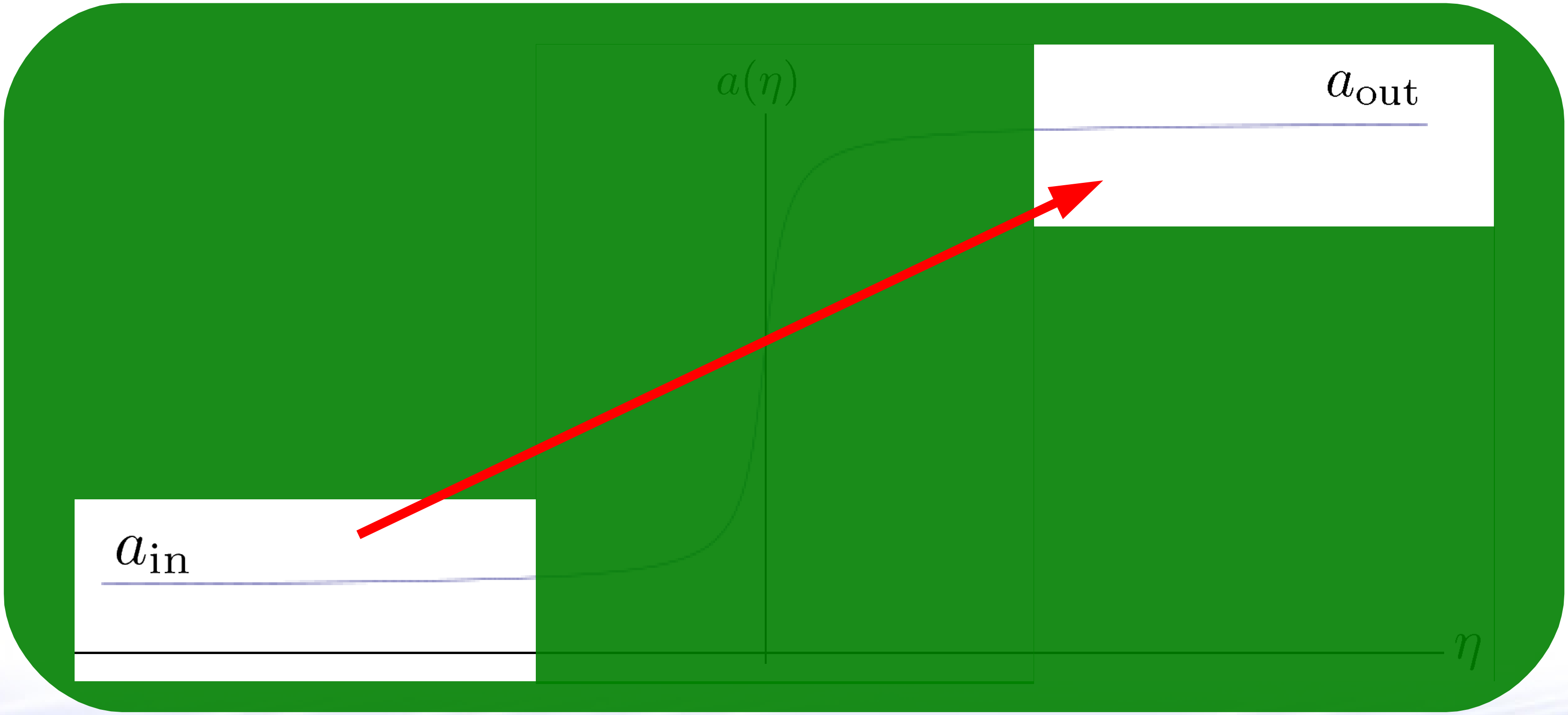
Particles from nowhere



Particles from nowhere

$$\phi_k^{\mathrm{in}} \overset{\eta\to-\infty}{\longrightarrow} e^{ikx} e^{-i\omega_{\mathrm{in}}(k)\eta}$$

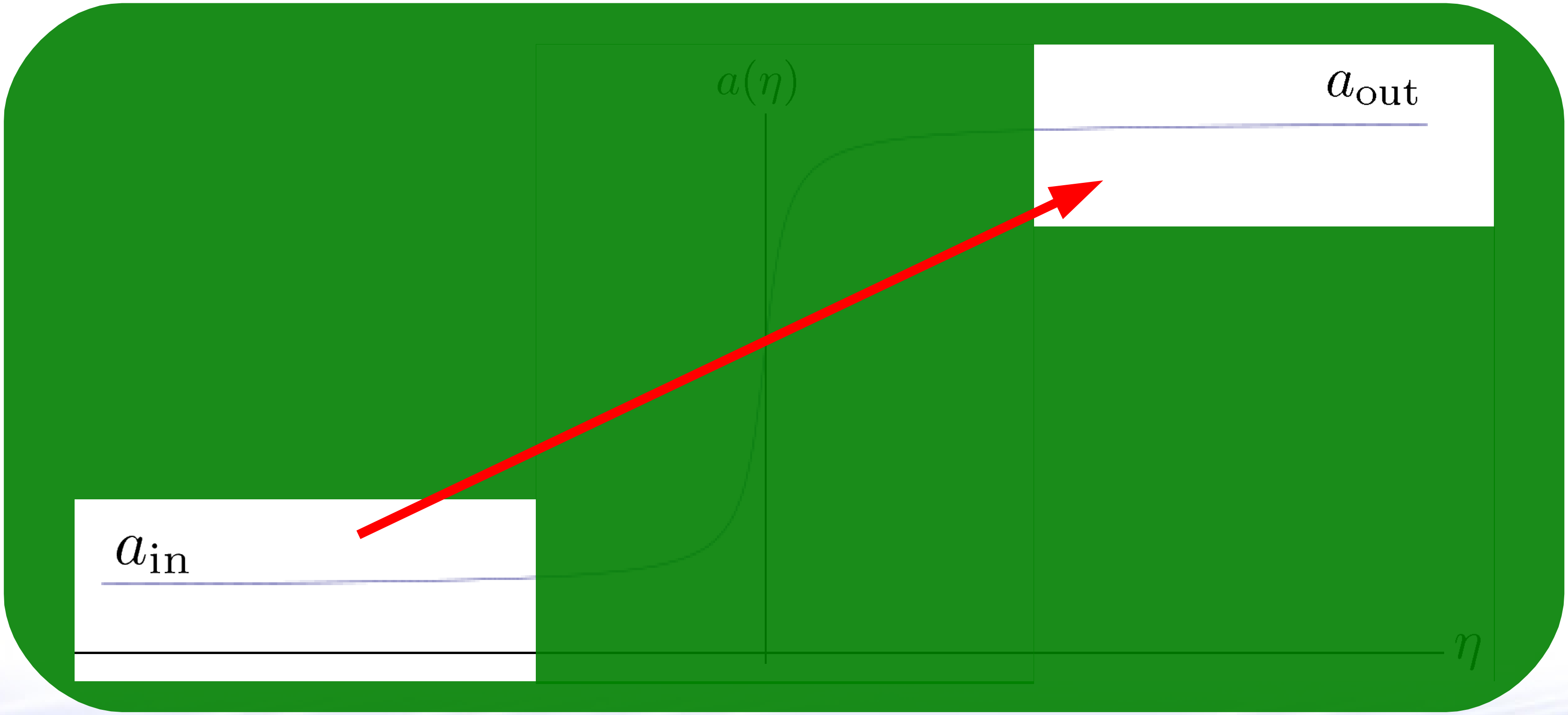
$$\overset{\eta\to+\infty}{\longrightarrow} A_k e^{ikx} e^{-i\omega_{\mathrm{out}}(k)\eta} + B_k e^{ikx} e^{i\omega_{\mathrm{out}}(k)\eta}$$



Particles from nowhere

$$\phi_k^{\text{in}} \xrightarrow{\eta \to -\infty} e^{ikx} e^{-i\omega_{\text{in}}(k)\eta}$$

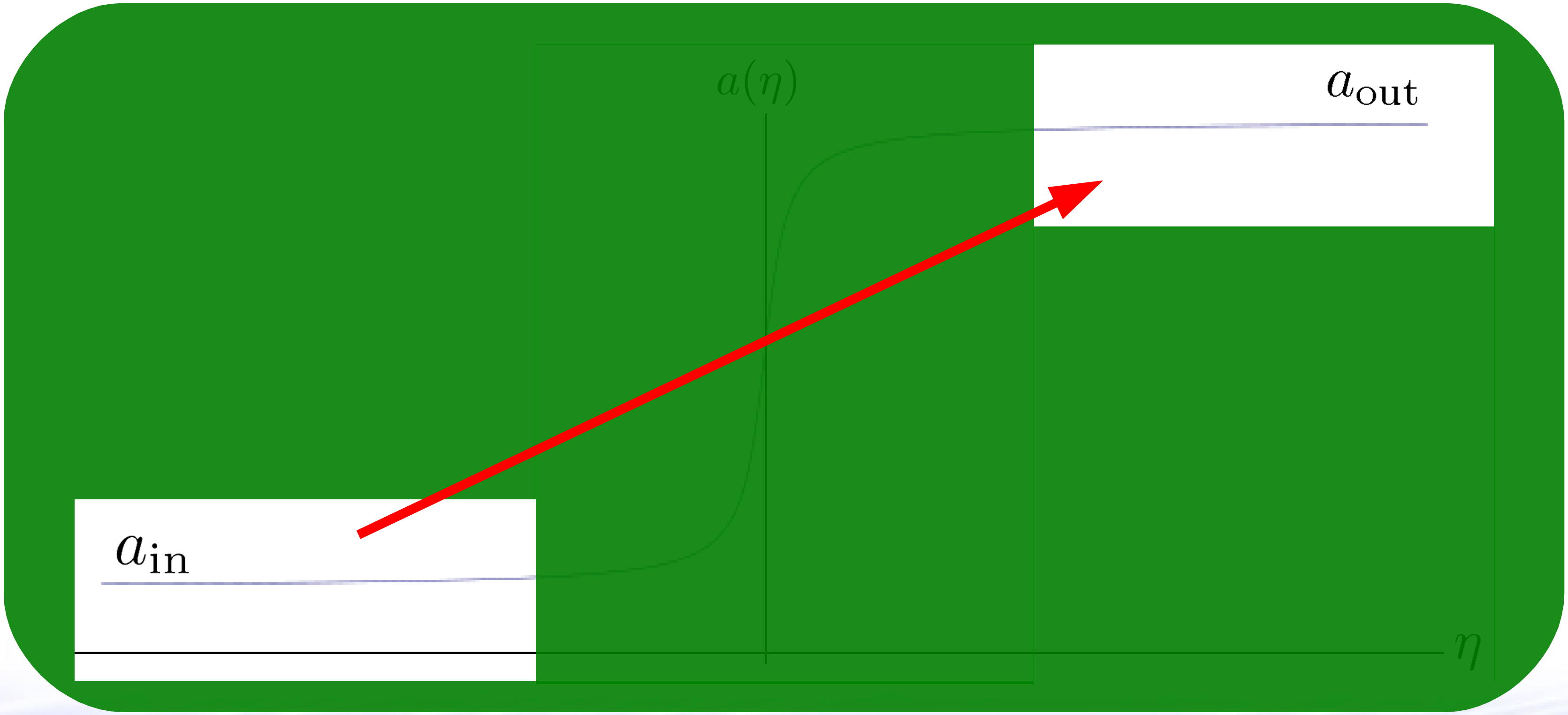
$$= A_k \phi_k^{\text{out}} + B_k \phi_k^{\text{out}*}$$



Particles from nowhere

$$\hat{a}_k^{\text{out}} = A_k \hat{a}_k^{\text{in}} + B_k^* \hat{a}_k^{\text{in}\dagger}$$

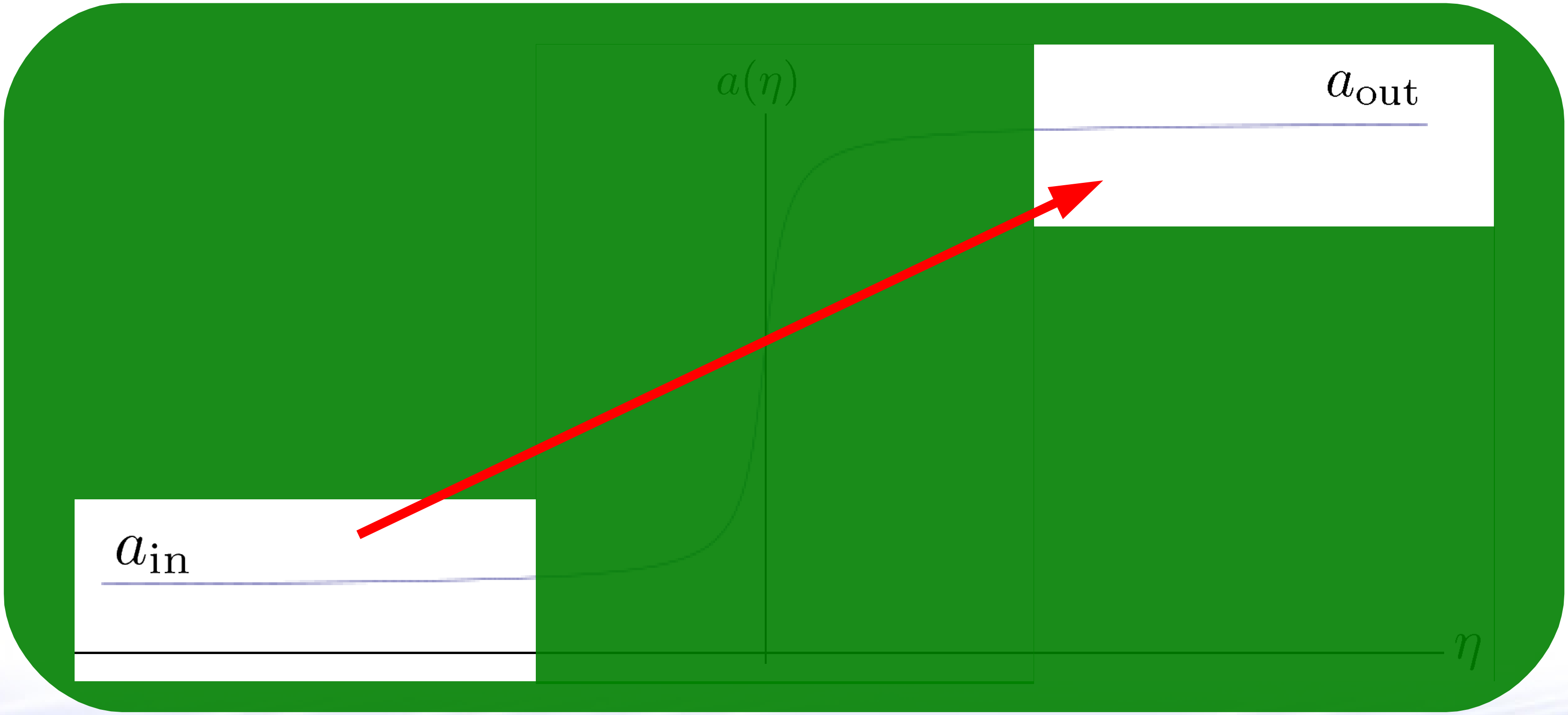
$$|vac\rangle_{\text{out}} = c^{(0)} |vac\rangle_{\text{in}} + c_{kl}^{(2)} |kl\rangle_{\text{in}} + \dots$$



Particles from nowhere

$$\hat{a}_k^{\rm out} = A_k \hat{a}_k^{\rm in} + B_k^* \hat{a}_k^{\rm in\dagger}$$

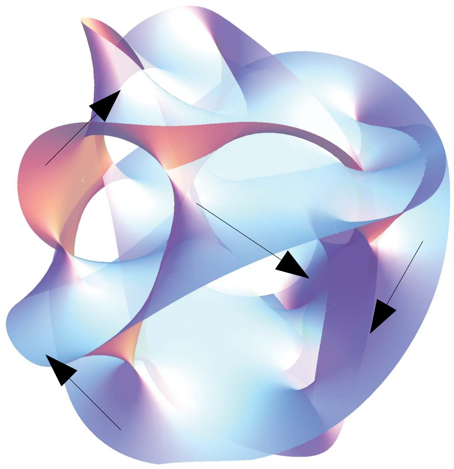
$$_{\rm in}\langle vac|\hat{N}_k^{\rm out}|vac\rangle_{\rm in} = |B_k|^2$$



General formalism

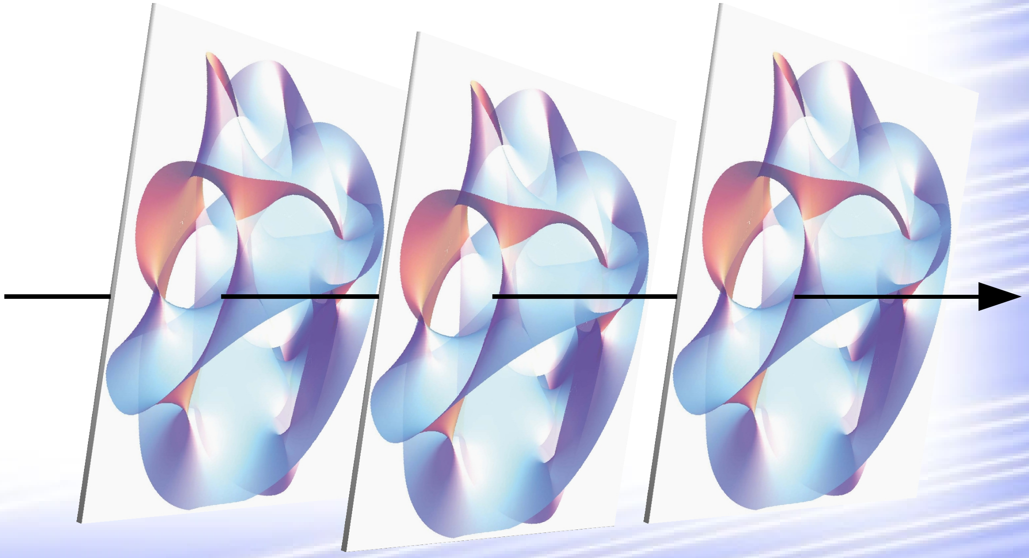
- Space *plus* time

Clear separation between space an time



??

Suitable spacetimes



General formalism

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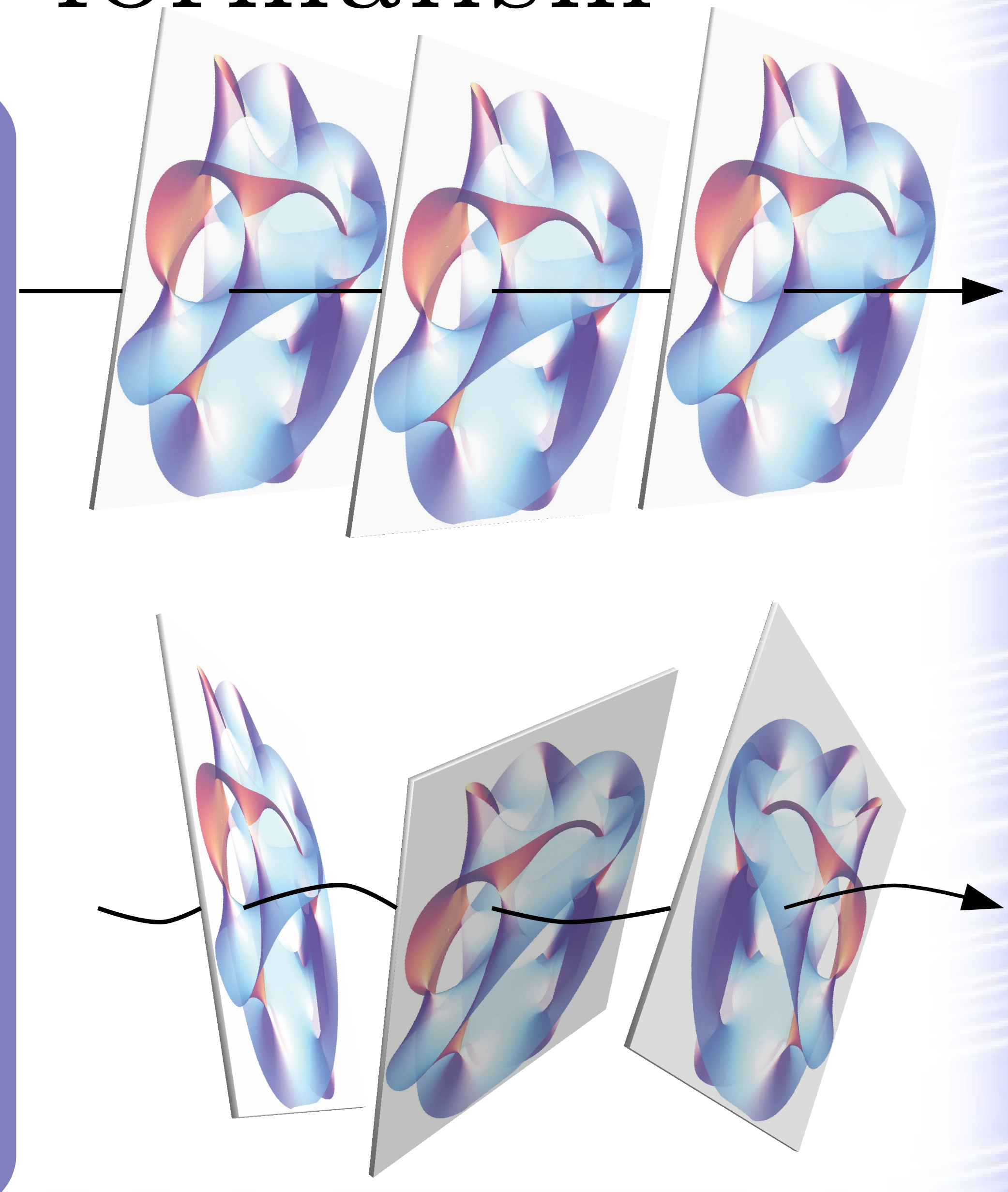

General formalism

- Space *plus* time
- Solve field equation

Find a complete and *normalized* collection of modes.

$$g^{\mu\nu}\nabla_\mu\nabla_\mu u_i + m^2 u_i = 0$$

$$\boxed{\begin{aligned}&(u_i, u_j) = \delta_{ij} = -(u_i^*, u_j^*)\\ &(u_i, u_j^*) = 0\end{aligned}}$$

$$(u, v) = i\int dx\,(u^*\dot{v} - \dot{u}^* v)|_{t=t_0}$$

General formalism

- Space *plus* time
- Solve field equation

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$$g^{\mu\nu}\nabla_\mu\nabla_\mu u_i + m^2 u_i = 0$$

$$\boxed{\begin{array}{l}(u_i, u_j) = \delta_{ij} = -(u_i^*, u_j^*) \\ (u_i, u_j^*) = 0\end{array}}$$

$$(u, v) = i\int dx\, W[u, v](x)$$

General formalism

- Space *plus* time
- Solve field equation

$$g^{\mu\nu}\nabla_\mu\nabla_\mu u_i + m^2 u_i = 0$$

Find a complete and *normalized* collection of modes.

$$\boxed{\begin{array}{l}(u_i, u_j) = \delta_{ij} = -(u_i^*, u_j^*) \\ (u_i, u_j^*) = 0\end{array}}$$

$$(u, v) = i\int_\Sigma d\sigma\,(u^*\dot{v} - \dot{u}^* v)$$

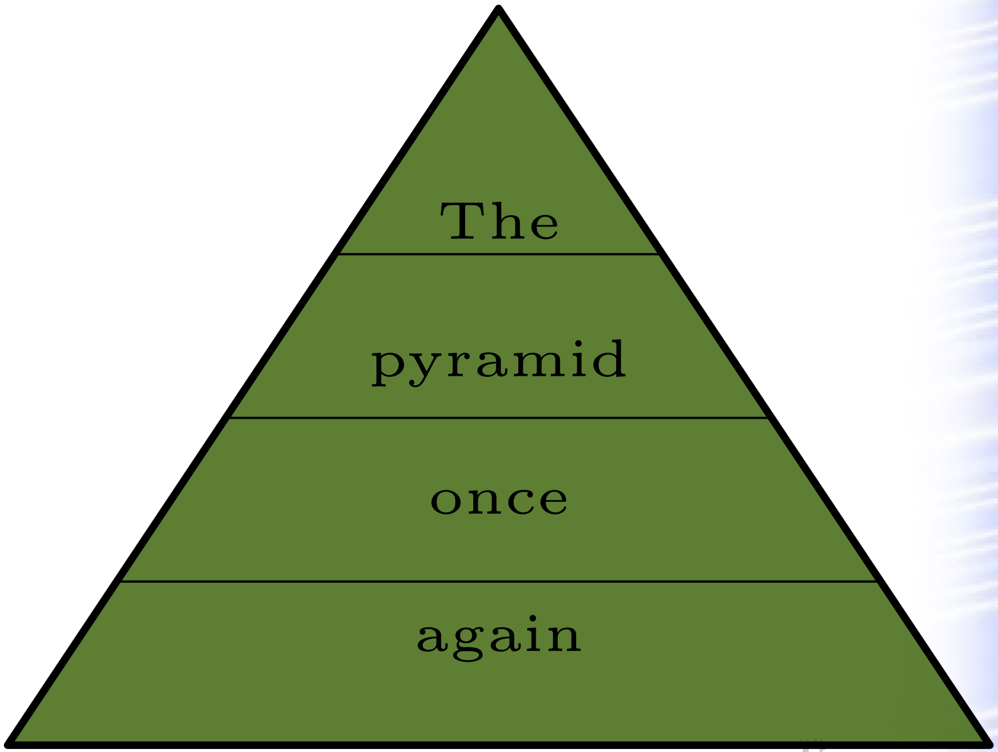
General formalism

- Space *plus* time
- Solve field equation
- Quantize!

We get automatically a well defined field operator plus a Hilbert space

$$\hat{\phi} = \sum_i u_i \hat{a}_i + u_i^* \hat{a}_i^\dagger$$

$$[\hat{\phi}(t,x), \dot{\hat{\phi}}(t,x')] = i\delta_\Sigma(x-x')$$



General formalism

- Space *plus* time
- Solve field equation
- Quantize!
- Ambiguity

Differents sets of modes give rise to differents "particles".

$$g^{\mu\nu}\nabla_\mu\nabla_\mu v_i + m^2 v_i = 0$$

$$(v_i, v_j) = \delta_{ij} \ldots$$

$$\hat{\phi} = \sum_i v_i \hat{b}_i + v_i^* \hat{b}_i^\dagger$$

General formalism

- Space *plus* time
- Solve field equation
- Quantize!
- Ambiguity

Differents sets of modes give rise to differents "particles".

Bogoliubov transformations

$$v_i = \alpha_{ij} u_j + \beta_{ij} u_j^*$$

$$\hat{b}_i = \alpha_{ij}^* \hat{a}_j - \beta_{ij}^* \hat{a}_j^\dagger$$

$${}_a\langle vac | N_i^{(b)} | vac \rangle_a = \sum_j |\beta_{ji}|^2$$

General formalism

- Space *plus* time
- Solve field equation
- Quantize!
- Ambiguity
- And much more!

Many other observables

- Expectation values

$$\langle \hat{\phi}(x)\hat{\phi}(y) \rangle$$

→ EM Tensor

- Transition rates

$$\frac{\langle \psi | \hat{\phi}(x)\hat{\phi}(y) | \chi \rangle}{\langle \psi | \chi \rangle}$$

Part II

QFT in curved ST is not Schwinger effect!

This is not Schwinger effect!

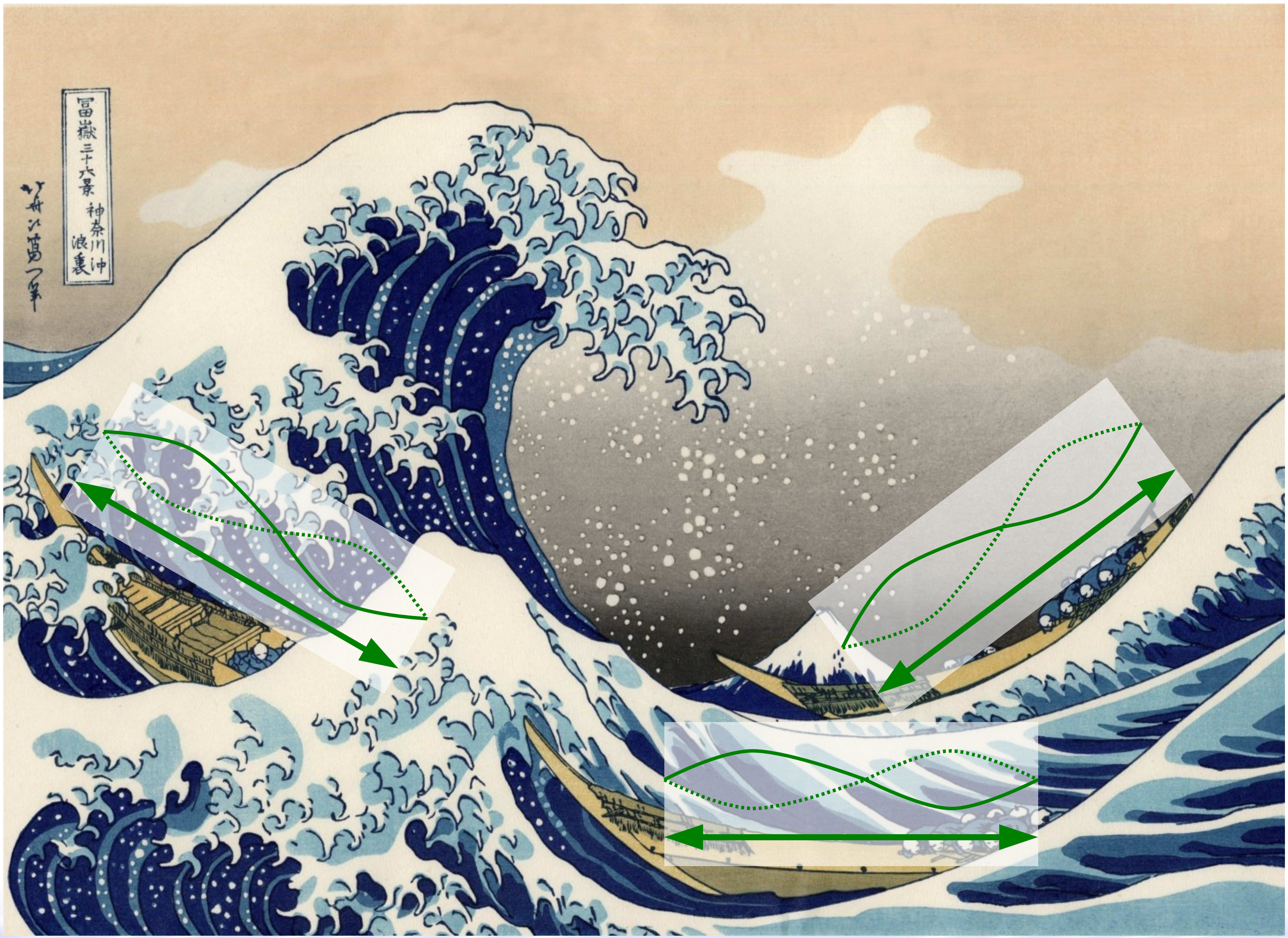
$$\sqrt{g}\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2a^2}{2}\phi^2$$

$$\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m(t)^2}{2}\phi^2$$

$$\langle vac_{\text{out}}|vac_{\text{in}}\rangle \neq 1$$

$$[\hat{\phi}(\xi), \hat{\phi}(\xi')] = \ldots$$

What particles?!?

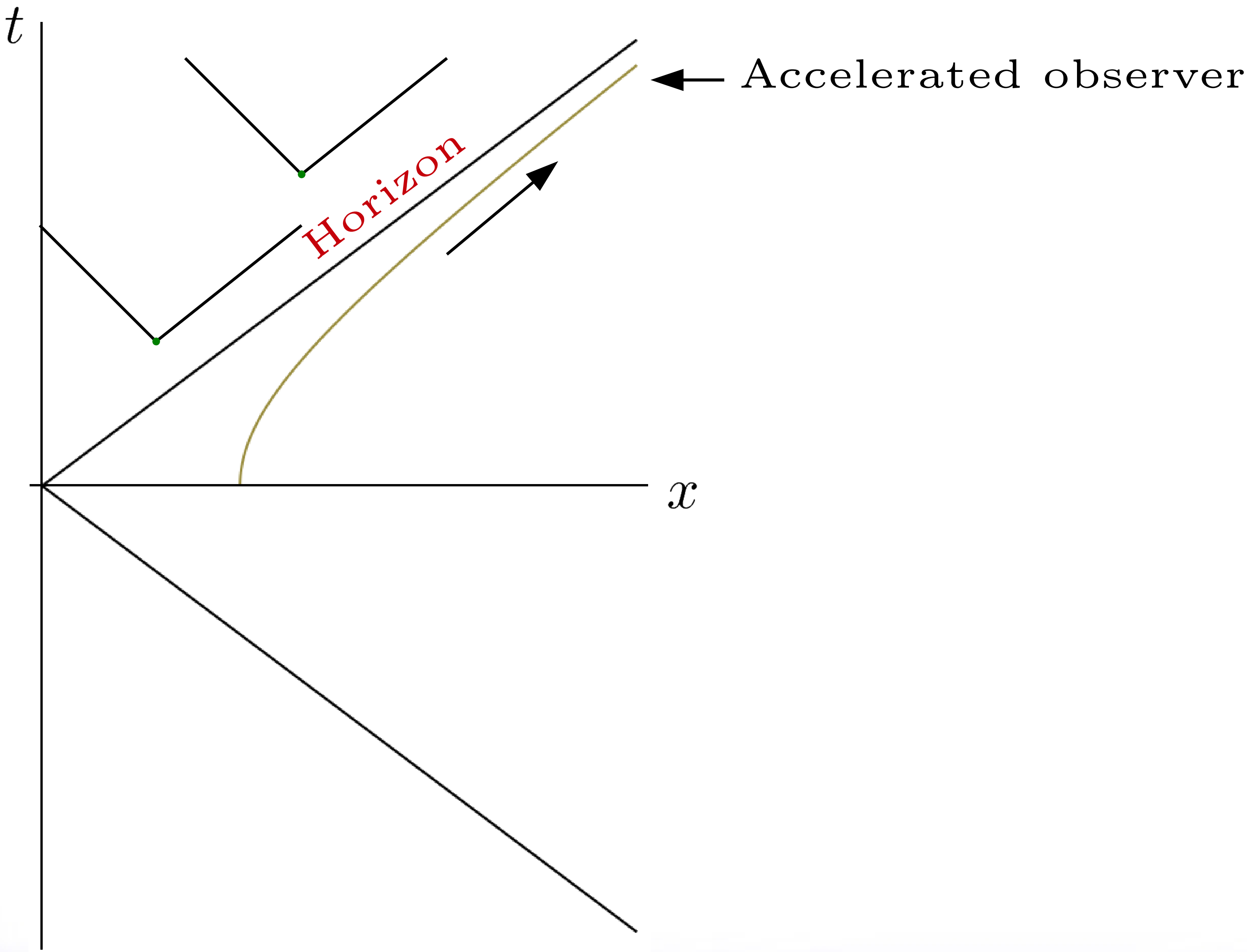


What particles?!?

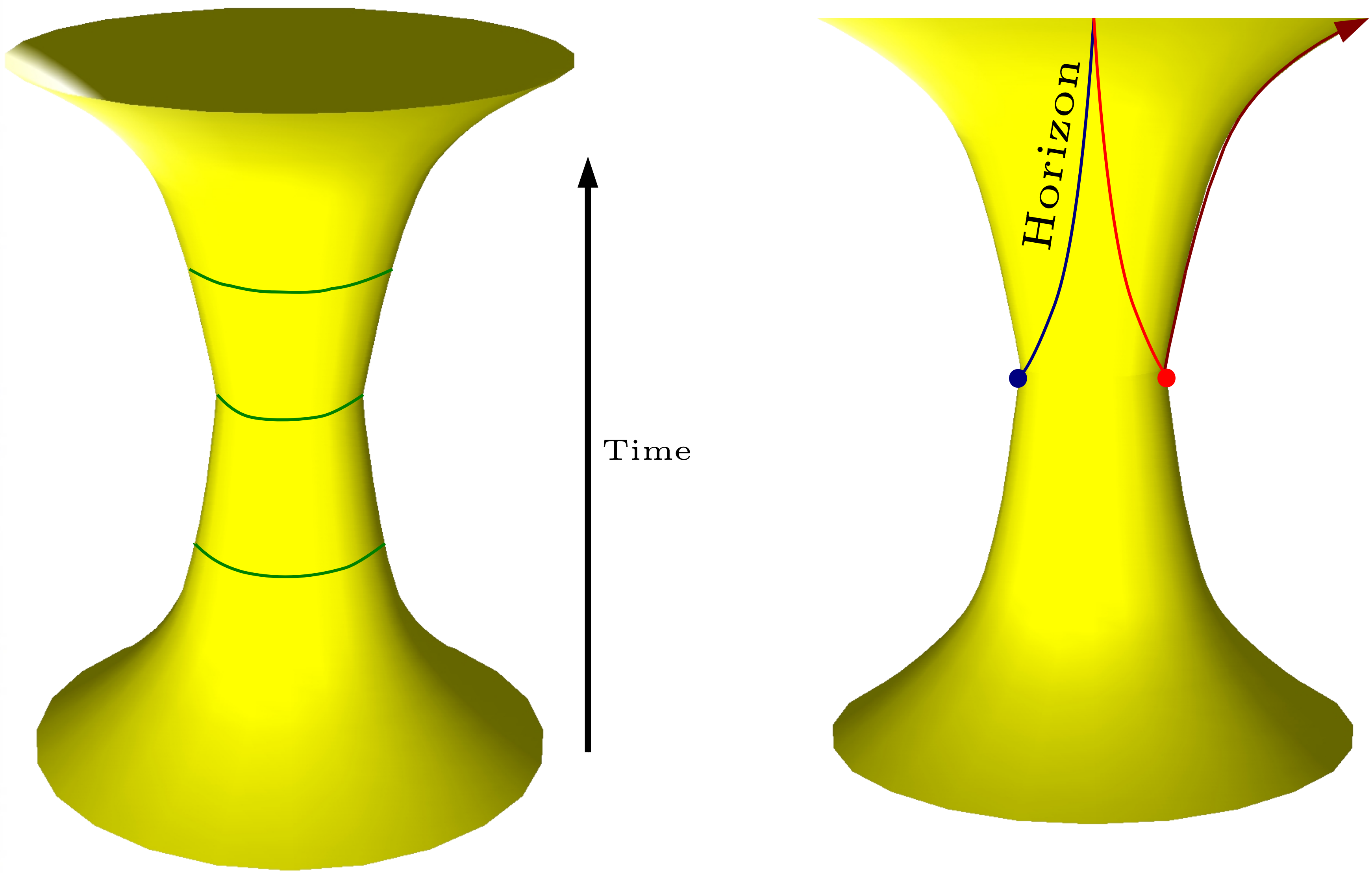
Bonus

Horizons and temperature

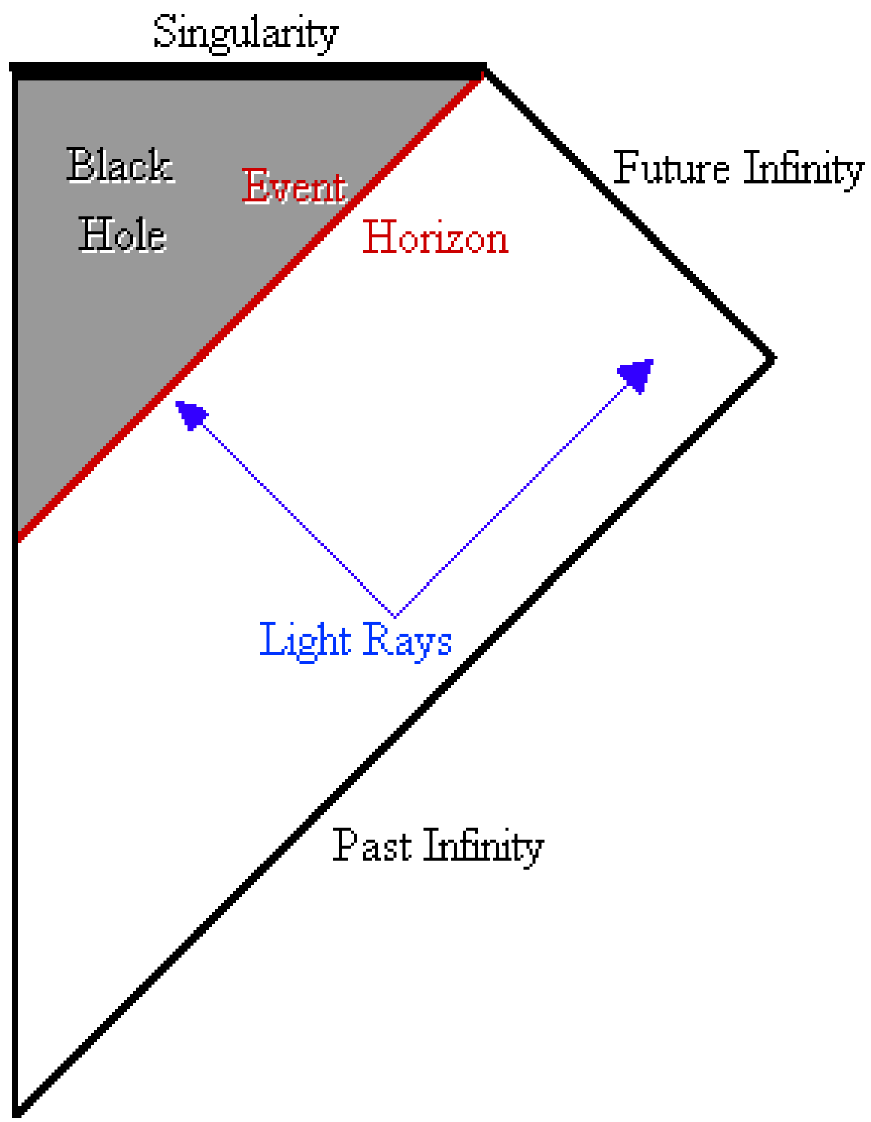
Bonus 1: Minkowski ST



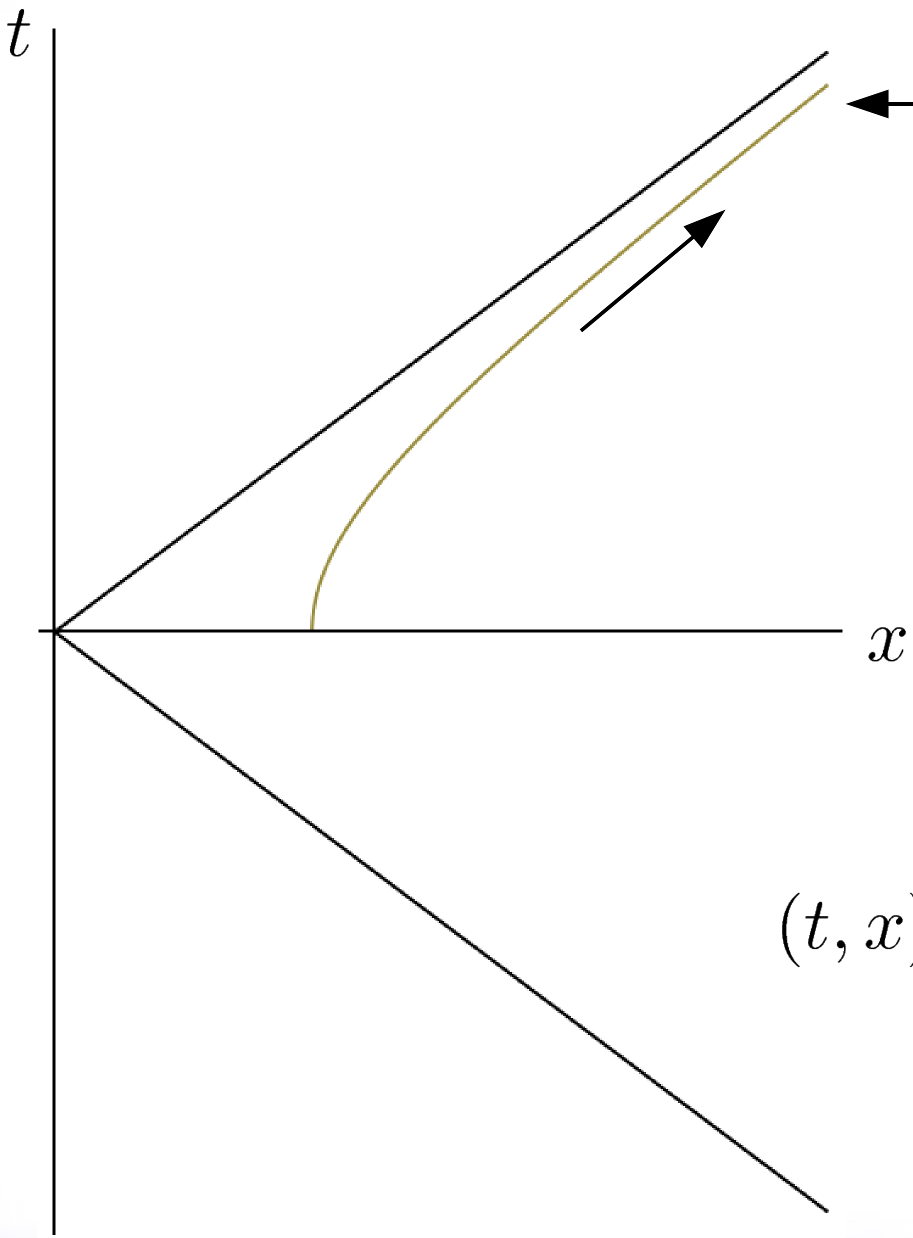
Bonus 2: de Sitter ST



Bonus 3: Black Holes



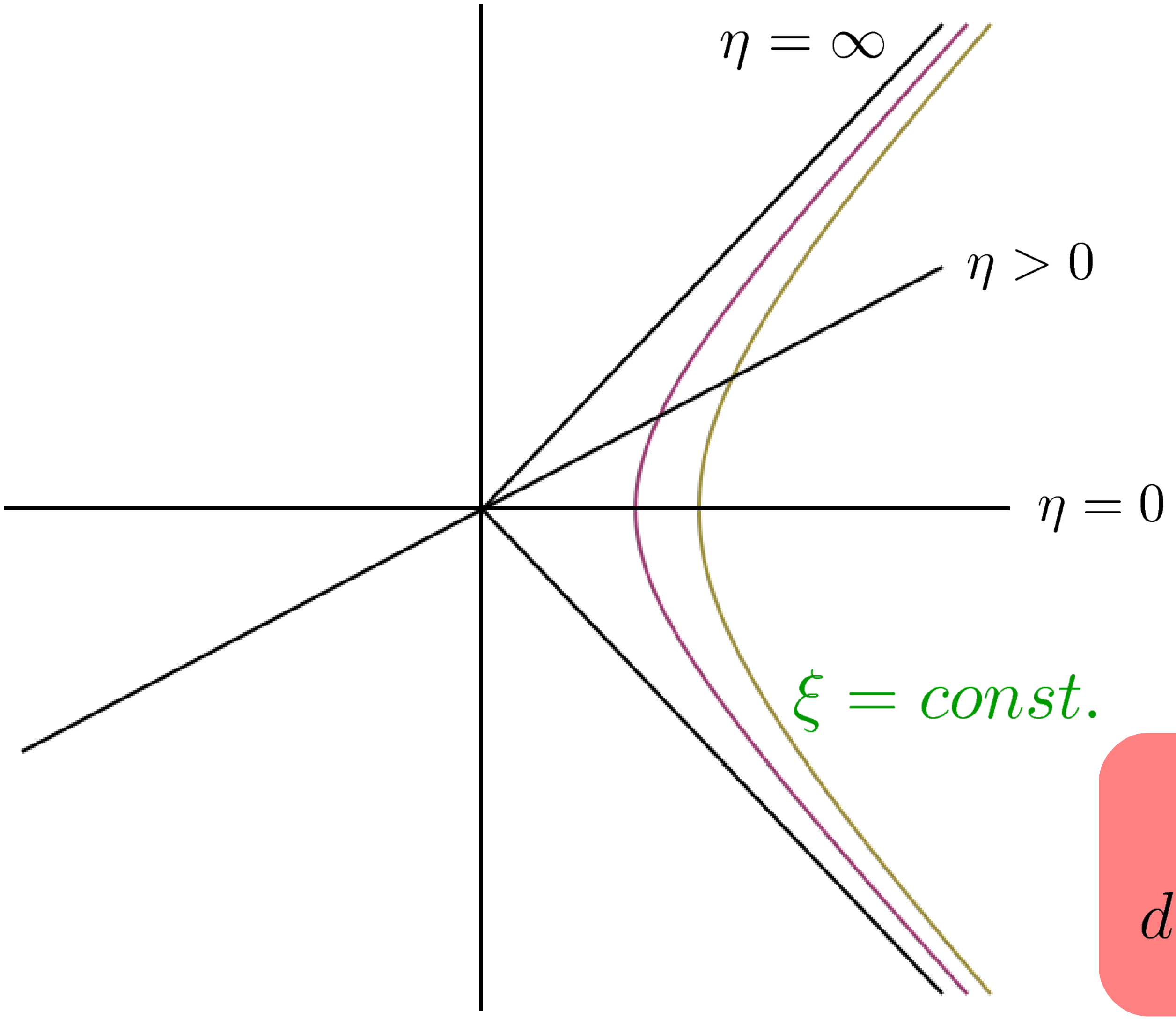
Unruh effect



$$x = const. + \frac{1}{a}\sqrt{1 + a^2 t^2}$$

$$(t, x) \sim (e^{a\xi} \sinh a\eta, e^{a\xi} \cosh a\eta)$$

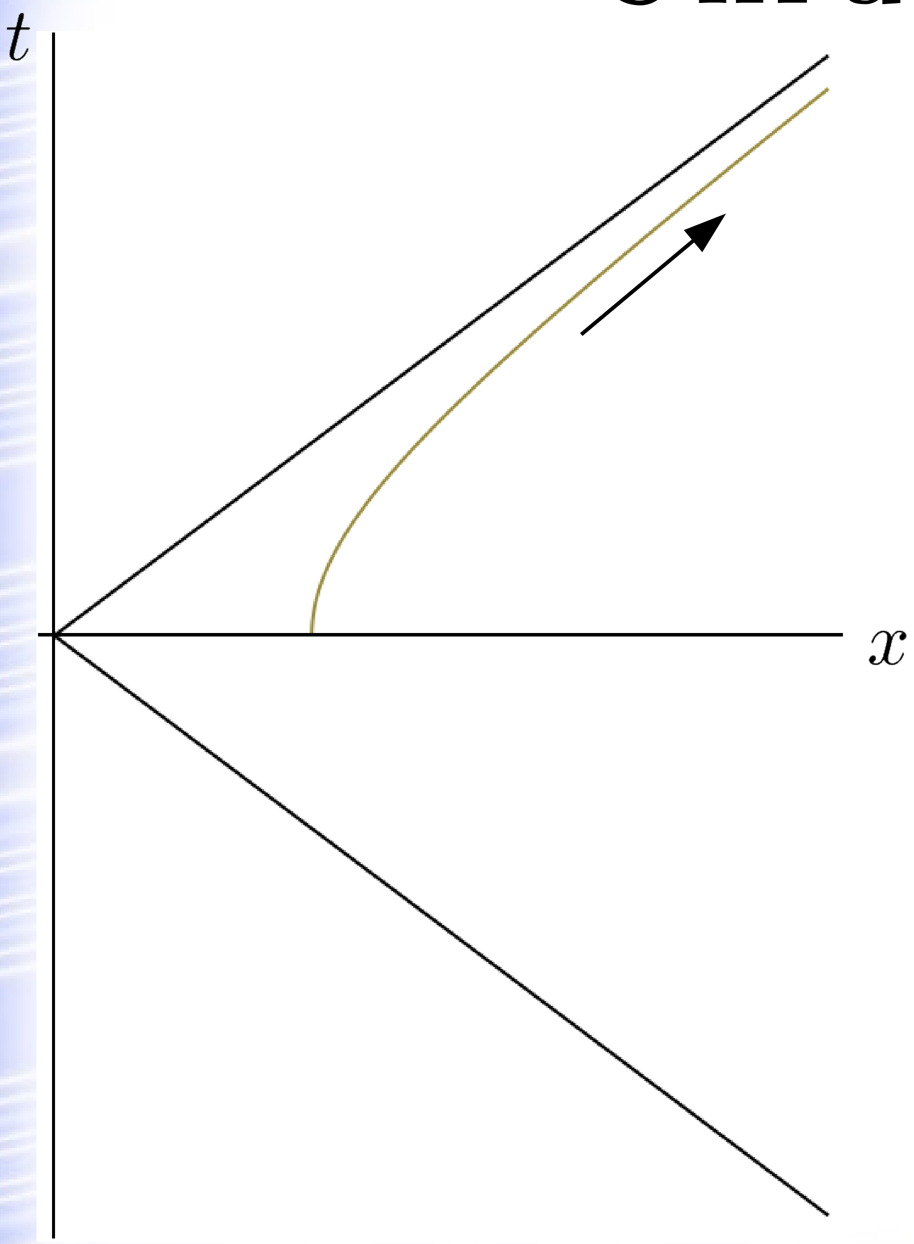
Unruh effect



Rindler ST

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2)$$

Unruh effect



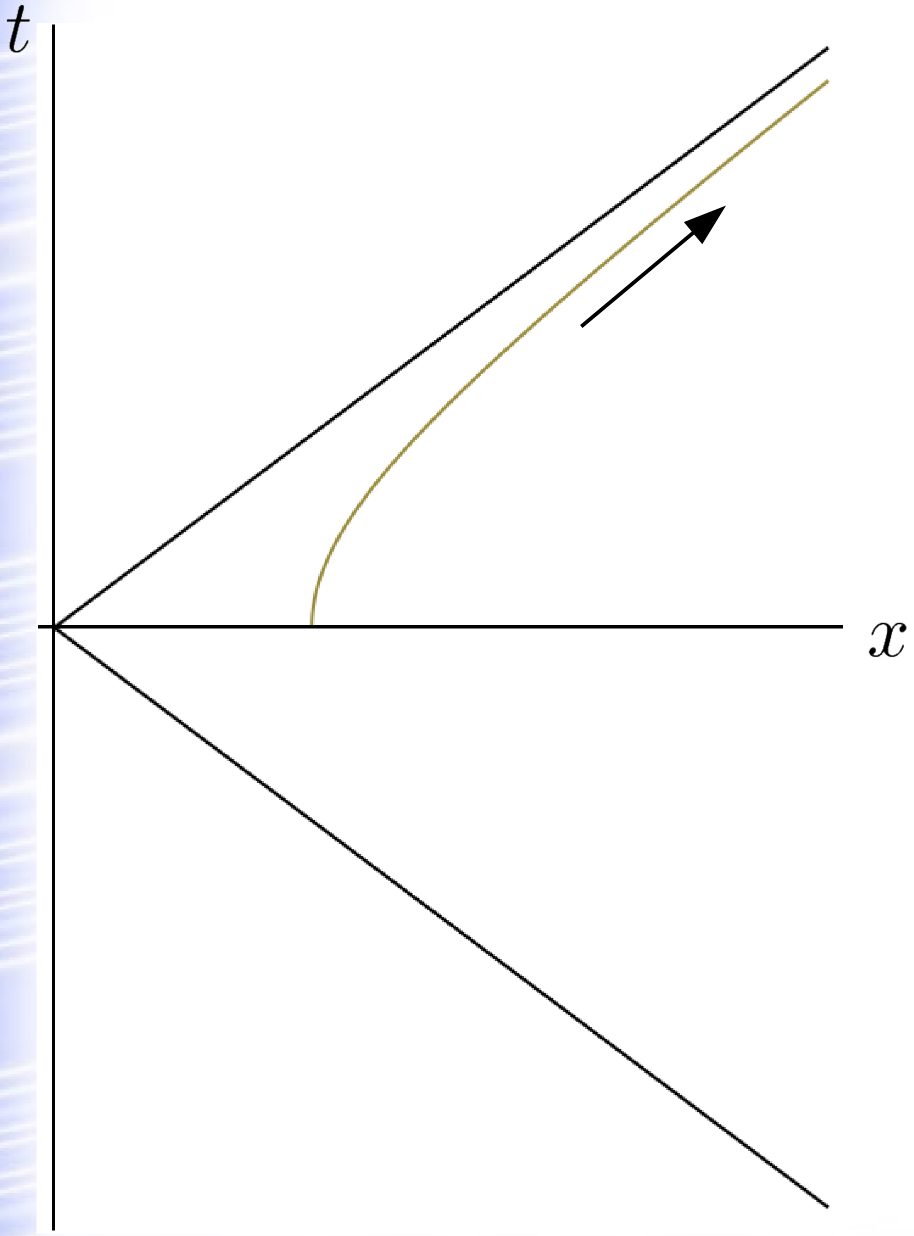
$$\Box\phi + m^2 e^{2a\xi}\phi = 0$$

$$\phi \sim e^{-i\omega\eta}\chi(\xi)$$

$$|vac\rangle_R$$

And of course
we already had $|vac\rangle_P$

Unruh effect

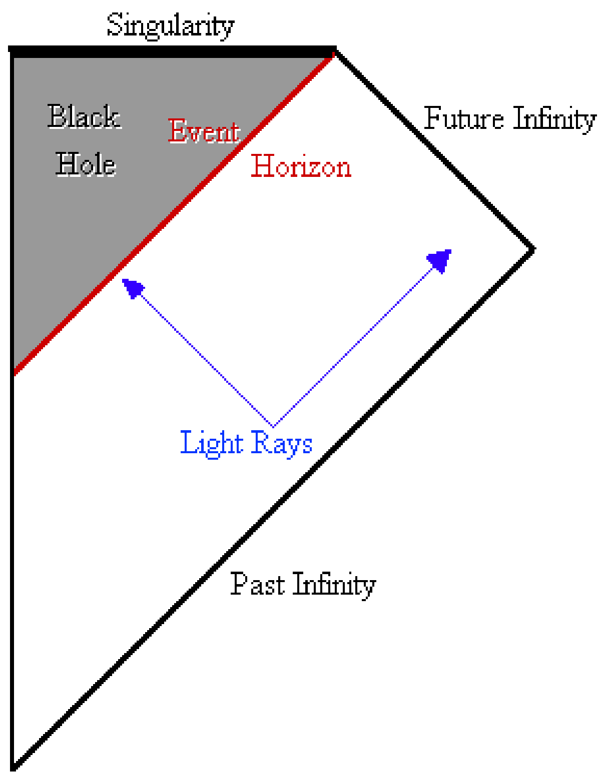


Final surprise!

$$ {}_P\langle vac|N_\omega^R|vac\rangle_P = \frac{1}{e^{\frac{2\pi\omega}{a}} - 1} $$

$$ T = \frac{a}{2\pi} $$

Hawking Radiation



Two kinds of observers/vacua

$$|vac\rangle_K \leftrightarrow |vac\rangle_T$$

or

$$|vac\rangle_{\rm in} \leftrightarrow |vac\rangle_{\rm out}$$

Anyway

$$T = \frac{1}{8\pi M}$$

when comparing $\langle N^{\rm out}\rangle_{\rm in}$ or $\langle N^T\rangle_K$

Further reading

- Birrel and Davies, *Quantum Fields in Curved Space*
- V. Mukhanov, *Introduction to Quantum Effects in Gravity*
- R. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*
- S.A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time*