A Model Experiment for Active Noise Cancellation in Opto-Mechanical Experiments

Aaron Buikema*

Mentors: Ludovico Carbone, Frank Brueckner, Andreas Freise

School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, UK

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Abstract

We discuss the design, construction, and characterization of a simple model system used to demonstrate feedforward active noise cancellation. This system consists of a set of piezoelectric actuators and accelerometer pairs, with the signal from one accelerometer used to send a signal to another piezo, canceling out the other accelerometer signal. The ideal feedforward transfer function from the accelerometer to piezo is derived and a rational function is fit to this function to produce the coefficients needed for time-domain filtering. Unfortunately, the fitting routine has very large numerical errors and produces unreasonable filter coefficients. Once a better fitting routine is found, the digital filter can be implemented.

1 Introduction

Active noise cancellation is the use of constructive interference to attenuate unwanted noise. This principle is used in noise-cancelling headphones to produce an enjoyable listening experience, but it can also be used to reduce mechanical noise in sensitive measurements.

Some sources of seismic noise in gravitation wave detectors cannot be removed by mechanical isolation alone. Gravity gradient noise, or Newtonian noise, which is due to coupling between the test mass and density fluctuations in the surrounding ground, cannot be removed with traditional mechanical isolation. Instead, this type of noise can be removed by measuring ground motion with accelerometers and seismometers and estimate how this will manifest itself as gravity gradient noise.

This project had two primary goals: first, using components already present in the lab, design and build a simple mechanical setup that could be used to demonstrate feedforward noise cancellation. Second, using this arrangement, develop and test a

*Haverford College, Haverford, PA, USA. Contact: aaronbuikema7@gmail.com

digital filtering system that would produce the needed signal to cancel the ground motion.

To perform this noise cancellation, we use a feedforward control loop. That is, rather than look at the output and adjust the system to produce a desired output (feedback), the system will be monitored for disturbances by an accelerometer and this information will be filtered and fed forward to the system to anticipate and compensate the effect on the system.

The following steps were required to demonstrate active noise cancellation:

1. Build mechanical setup.
2. Understand system in terms of transfer functions (TF).
3. Derive desired filter and analysis transfer function from complete transfer function schematic.
4. Measure all necessary transfer functions to determine ideal transfer function.
5. Fit rational function to ideal transfer function to find coefficients for time-domain filtering.
6. Implement and test filter.

To that end, the setup shown in figure 1 was devised. It consists of two pairs of an accelerometer and piezoelectric actuator separated by a post. The bottom piezo, which is driven by an external signal, usually white noise, acts as a simulation of ground noise and moves the whole setup. The bottom accelerometer, also called the witness sensor, measures this ground signal and sends the data to a digital filter. This filter manipulates the incoming signal in such a way that when sent to the top piezo, the top accelerometer outputs zero signal.

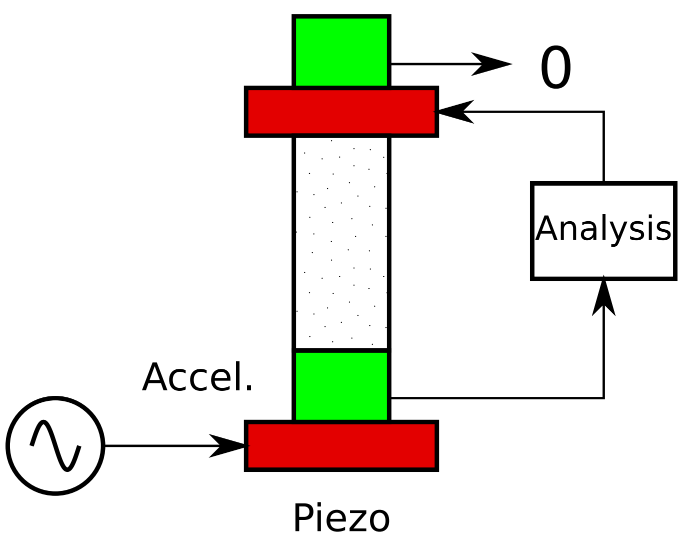

Figure 1: Schematic of noise cancellation setup. The goal is to use the signal from the witness accelerometer to cancel out the signal at the top accelerometer.

2 Mechanical Setup

Figure 2 is an image of the final setup used. The final tower was just under 20 cm tall. The piezos used were Piezomechanik HPSt 1000/25-15/5 (-200 V through +1000 V, max stroke 12/7 μm) and were driven by a Piezomechanik SVR 500/3 (-100 V through +500 V). The bottom actuator provides a known driving force, while the bottom accelerometer (MMF KS943B.100 triaxial accelerometer) acts as a witness sensor and detects this motion. This signal is analyzed and reshaped in such a way that when sent to the top piezo, the top accelerometer (MMF KS94B.100 single-axis accelerometer) will have zero signal.

Because this was a preliminary setup, only existing laboratory components were used to construct the tower. Thus, the primary building components were optical posts and post holders. This tower was constructed in such a way as to minimize off-axis motion, remove resonances, and limit the weight to reduce stress on the piezoelectric actuators. The setup was built in sections, with various intermediate transfer functions taken to ensure the proper performance. While the use of these optical components allowed easy adjustment, they had the tendency to relax quickly, changing the amplitude of motion and resonant frequencies of the system slightly. We do not expect these small changes to affect the performance of the system significantly.

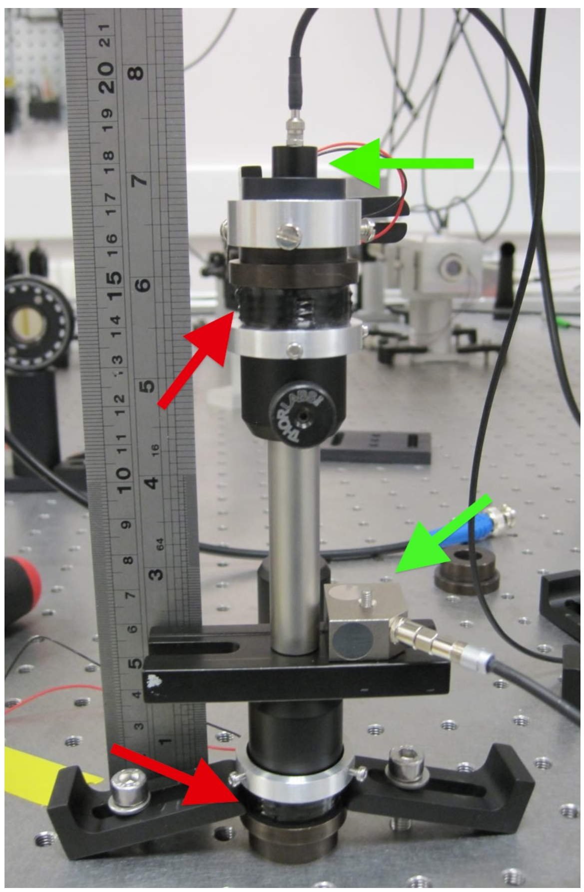

Figure 2: The final arrangement. Accelerometers are indicated with green arrows and piezoelectric actuators are indicated with red arrows.

3 Determining Ideal Feedforward Filter Transfer Function

Mathematically, the transfer function of a linear, time-invariant system is a complex function in frequency space given by

$$H(f) = \frac{Y(f)}{X(f)} \tag{1}$$

where $X(f)$ and $Y(f)$ are the Fourier transforms of the input and output signals, respectively. In frequency space, the response of two components in series is merely the product of the respective responses in frequency space. That is, for components with frequency responses $A(f)$ and $B(f)$, assuming a linear, time-independent system, the response of the two components in series is $A(f)B(f)$. Thus, by measuring a few critical transfer functions of the system and setting the output of the top accelerometer to zero, the desired filter transfer function can be determined for an arbitrary input signal.

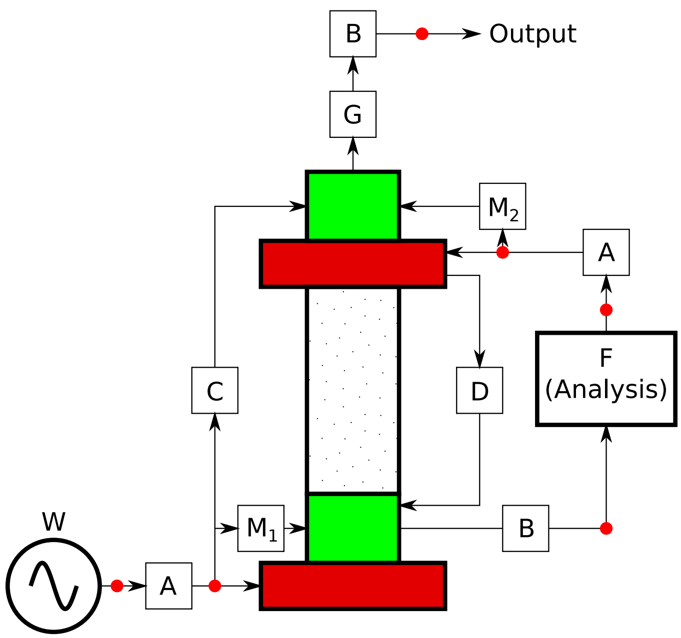

Figure 3: Schematic of setup following figure 1 with all transfer functions of interest added. The various labeled transfer functions are given in the text. The red dots are points where a signal can be directly measured.

The schematic shown in figure 1 is deceptively simple, as the final setup contained many more components that affected the performance of the system. For example, the piezos each required a high-voltage amplifier to drive them, and the accelerometers required control boxes that added electronic noise and other filtering effects. In fact, the piezos behave as capacitors, producing an intrinsic low-pass filter in our system. The transfer functions that need to be considered are shown in figure 3, and are as follows:

F = Ideal TF of digital filter (to solve)

(a) (b) (c)

Figure 4: A few controllers that complicate the ideal transfer function. (a) High-voltage amplifier for piezoelectric actuator control. Corresponds to A in figure 3. (b) Accelerometer controller. This box amplifies and filters the acceleromter signal. Corresponds to B in figure 3. (c) The FPGA used to send the filtered signal to the correcting piezo (see section 6). Corresponds to F in figure 3.

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A = TF of HV amp monitor from input

B = TF of blue box accelerometer controllers from accelerometer output

C = TF of top accelerometer from bottom HV amp monitor

D = TF of bottom accelerometer from top HV amp monitor

G = "Gain" (ratio of sensitivity of top accelerometer to bottom; this should be constant in the range of frequencies we're considering, and it is included only to make it unnecessary to draw extra TFs for each accelerometer)

M_1, M_2 = Mechanical TFs from piezo driver monitors to respective accelerometers

W = Whitenoise input (cancels out of all expressions)

Note that "monitor" in this context refers to the monitor output of the HV piezo driver, which is 1/1000 of the voltage sent to the piezo.

We want the signal of the top accelerometer to be zero, i.e., $a_2 = 0$. Then

$$0 = a_2 = BG \overbrace{[CAW + M_2AFB \underbrace{(M_1AW + DAFBM_1AW + \mathcal{O}((DAFB)^2)M_1AW]}_{\text{bottom accel input}}}^{\text{top accel input}} \tag{2}$$

where the addition terms come from the fact that the accelerometers will add signals. If we assume only first-order feedback from the correcting piezo to the bottom accelerometer is significant, i.e. $|DAFB|^2 \ll 1$, then (2) simplifies to a quadratic equation:

$$0 = aF^2 + bF + c$$

where

$$\begin{aligned} a &= M_2ABDABM_1GB \\ b &= M_2ABM_1GB \\ c &= GBC \end{aligned}$$

The solution for F is given by the quadratic formula, so we need only to produce the quantities a, b, and ac. After working out the various direct transfer functions above into measurable transfer functions, we find

$$\begin{aligned} a &= \left(\frac{a_2}{I_2}\right)\left(\frac{a_1}{I_2}\right)\left(\frac{a_1}{M_1}\right) \\ ac &= \left(\frac{a_2}{M_1}\right)\left(\frac{a_2}{I_2}\right)\left(\frac{a_1}{I_2}\right)\left(\frac{a_1}{M_1}\right) \\ b &= \left(\frac{a_2}{I_2}\right)\left(\frac{a_1}{M_1}\right) \end{aligned}$$

where a_i is the top (2) or bottom (1) accelerometer signal, M_1 is the monitor output of the bottom piezo, and I_2 is the source input of the top piezo driver.

However, for frequencies of interest, we have found that the coupling between the top piezo and bottom accelerometer is minimal in the frequencies of interest, so as a starting point we have performed this analysis assuming $D = 0$.

4 Transfer Function Measurements

Before taking transfer functions of the full system, we needed to take transfer functions of individual components to ensure that they were behaving as expected. The transfer function measurements were carried out by inputing white noise into a certain part of the system, usually the bottom piezo, and measuring how some part of the system reacts. This response in frequency space is then divided by the original signal.

All these measurements were carried out with a simple spectrum analyzer (Agilent 35670A). Most of these transfer function measurements required driving one piezo. This was done by inputing white noise from 0-3.2 kHz at 200 mV peak into the high-voltage amp. The accelerometer signal was sent to a signal conditioner (MMF M68D3), where it is passed through a high-pass filter at 0.1 Hz and a low-pass filter at 1 kHz and amplified by a factor of 100.

Note that while the accelerometers measure acceleration, the signal sent to the piezos is proportional to position. As such, we expect the magnitude of the transfer function of the accelerometer signal from the piezo signal to be proportional to ω^2 and exactly out of phase:

$$x = A\exp(i\omega t) \Longrightarrow \ddot{x} = -\omega^2 A \exp(i\omega t)$$

To demonstrate this, the accelerometer was placed almost directly onto the piezo, using only a 1/2-in lens post. Indeed, this behavior was observed for the region between 80-1000 Hz for both phase and magnitude (see figure 6). The deviation from this behavior at low frequencies was due to electronic noise overwhelming the signal, and at higher frequencies we reached a resonant frequency of the mechanical setup. This provided a starting region over which noise cancellation was attempted.

After these initial measurements, the necessary transfer functions noted in section 3 were taken. These measurements can be found in appendix A, and the calculated ideal filter transfer function can be seen in figure 7. The ideal transfer function appears as a low-pass filter in magnitude, but has very different behavior for phase. A simple analog filter cannot produce the desired output, so we will use a digital filter.

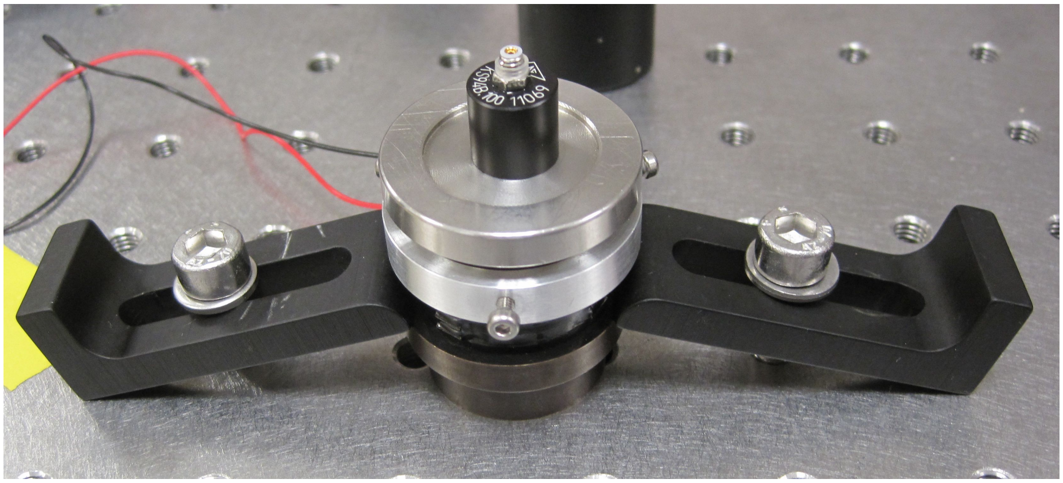


Figure 5: Measuring the direct transfer function between accelerometer and piezo input.

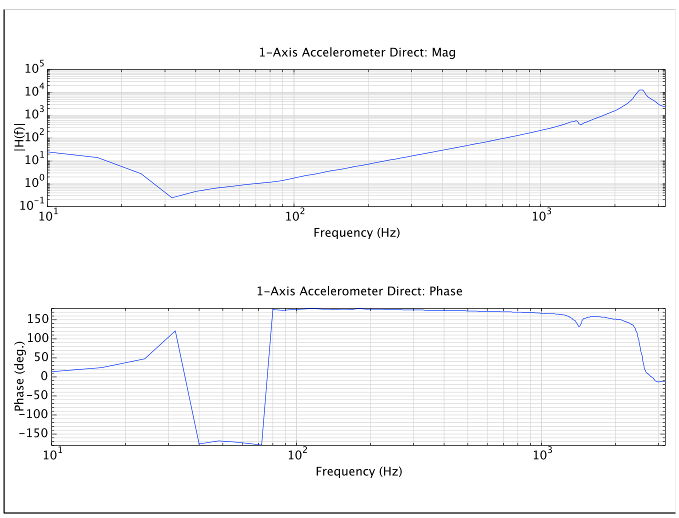


Figure 6: Transfer function of accelerometer signal from piezo signal. Note that the signal magnitude is proportional to ω^2 (linear on a log-log plot) and is 180° out of phase. The transfer functions displays non-ideal behavior at low frequencies because of noise and at high frequencies because of resonances of the mechanical setup.

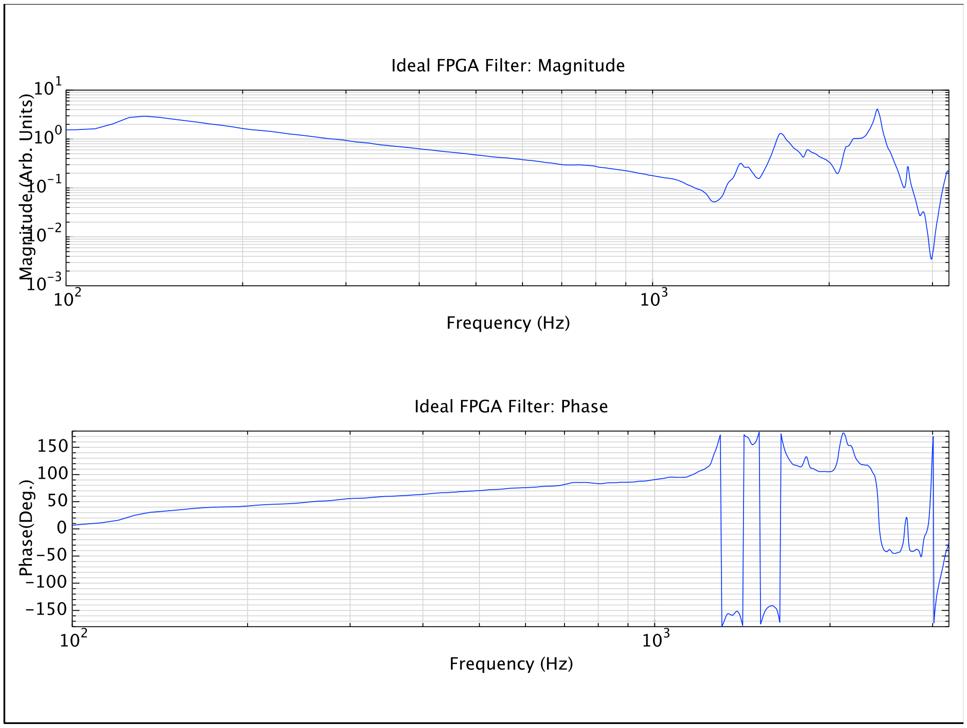


Figure 7: Calculated ideal transfer function of filter assuming no feedback from correcting piezo to witness accelerometer.

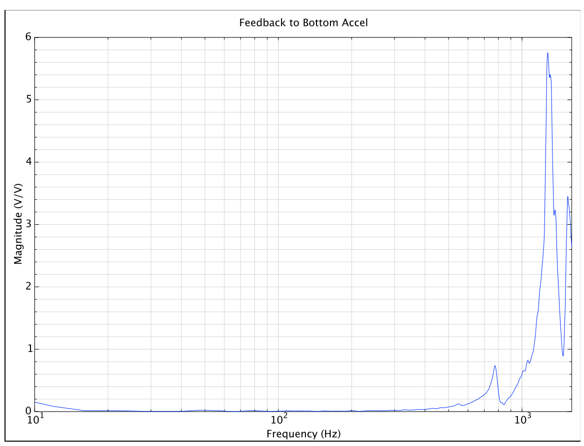


Figure 8: Transfer function magnitude from compensating piezo to witness accelerometer, demonstrating the undesired feedback. Note that the feedback is negligible for frequencies less than 600 Hz.

In the process of taking this measurements, we noted a few problems with this setup that will preclude total noise cancellation. Most problematic, at certain frequencies there is non-negligible feedback from the correcting piezo to the witness accelerometer, creating a positive-feedback loop. This is illustrated in figure 8. As a temporary solution, the accelerometer output will be low-pass filtered and only noise cancellation at frequencies below 1 kHz will be considered.

5 Fitting

The actual filtering of the signal occurs in the time domain, but we have a desired transfer function in frequency space. Moving from one domain to the other is not trivial, but this can be achieved by fitting a rational function to the desired transfer function.

We can write the output of a digital filter as a weighted sum of inputs $x[i]$ and outputs $y[i]$:

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k] \tag{3}$$

Taking the z-transform and rearranging, we are left with the following transfer function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \tag{4}$$

where $z = e^{sT}$, T is the sampling frequency, and $s = i\omega$. Thus, by finding a rational fit to the desired transfer function, we can find the coefficients a_i and b_i, which are used for time-domain filtering. Note that we can rewrite equation 4 in the following form:

$$H(z) = A\frac{(1-q_1z^{-1})(1-q_2z^{-1})\cdots(1-q_Mz^{-1})}{(1-p_1z^{-1})(1-p_2z^{-1})\cdots(1-p_Nz^{-1})} \tag{5}$$

The q_is and p_is are called the *zeros* and *poles* of the transfer function, respectively.

To perform this filtering, the MATLAB script VECTFIT is used [1, 2, 3]. This script uses a technique known as *vector fitting* to find a rational fit to a given function. The rational function is given in the following form:

$$f(s) = \sum_{m=1}^{N} \frac{r_m}{s - a_m} + d + sh \tag{6}$$

The script returns the poles (a_m), residues (r_m), and optional terms d and h. With these in hand, the zeros of the transfer function can also be calculated (after converting from the s coordinate plane to the z coordinate plane). The zeros and poles are merely the roots of the polynomials of equation 4, so the coefficients can easily be calculated.

Unfortunately, this routine has not worked optimally for our ideal transfer function. First, because we have chosen such a simple function to fit, the poles and zeros were ambiguous. Second, the `poly()` function in MATLAB, which returns the coefficients of a polynomial when passed the roots of the polynomial, comes with a warning that it is not very accurate. These two factors combine to produce very large coefficients (on the order of 10^{100} under some fitting parameters) that produce rational fits that are not accurate enough. As such, a more accurate fitting method needs to be used before noise cancellation tests can be carried out.

6 Field-Programmable Gate Array

6.1 Introduction

As already discussed, we cannot use a simple analog filter to produce the desired output. With enough computing power, though, digital filters can produce nearly any desired transfer function. Further, the behavior of digital filters can be changed by typing in a few lines of code, whereas for analog filters the circuit often needs to be rewired completely. For this reason, we chose to use a digital filtering system for the feedforward system.

However, one of the biggest disadvantages of digital filters is speed. To get around this problem, a field-programmable gate array (FPGA) is used to filter and analyze the signal. FPGAs consist of programmable logic components, which can be made into nearly any logic gate desired. This allows for parallel processing and much better performance than from a standard desktop PC. The FPGA used was part of a National Instruments device that included an ADC and DAC (National Instruments NI PXI-7852R). The FPGA is programmed with LabVIEW and the LabVIEW FPGA module.

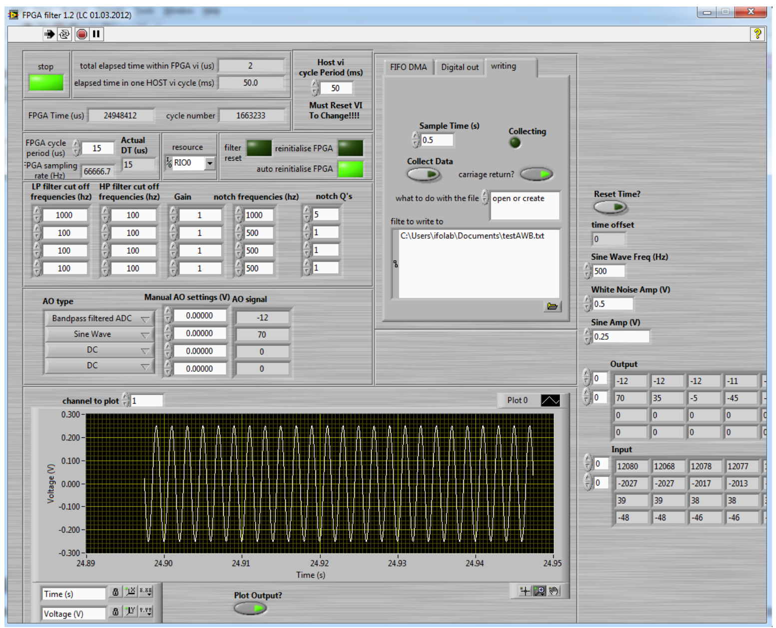


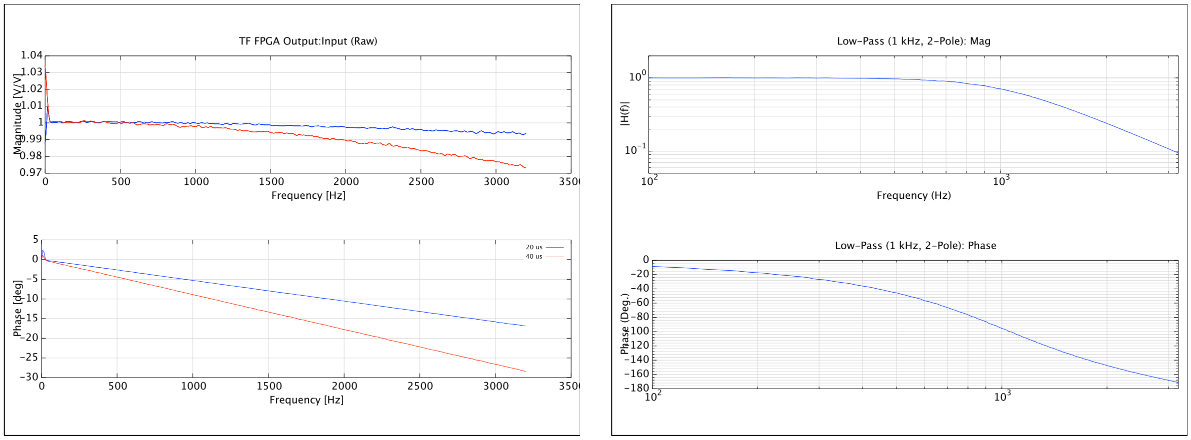
Figure 9: Front panel of FPGA LabVIEW interface showing sine wave generation.

6.2 LabVIEW Interface

A set of LabVIEW routines had already been developed by the Birmingham group; I was responsible for optimizing and debugging these routines and adding functionality to the program to allow us to perform customized real-time filtering. The current LabVIEW program is able to take an input, filter it, and output the resulting signal. It has the following capabilities:

- Adjustable sampling frequency and manual output time delay
- Standard Filters: low-pass, high-pass, band-pass, notch (with tunable cutoff frequencies)
- Custom Filters (when filter coefficient are given, see section 5)
- Function Generator: white noise, sine wave at user-defined amplitude and frequency

It is hoped that these features will allow us to produce any possible needed filter.



(a) (b)

Figure 10: FPGA performance. (a) Transfer function of the output of the FPGA when set to output exactly the input. Note that the phase is linear in frequency, and the slope of this line increases in magnitude as the cycle time is increased. (b) Transfer function of the two-pole low-pass filter.

6.3 Performance

The FPGA was used to send identical but inverted sine waves into the bottom and top piezos, respectively, to test that noise cancellation could be performed with this setup. Indeed, with these input signals, the output of the top accelerometer was within the noise level of the accelerometers.

Because the FPGA cannot filter a signal infinitely fast, there will always be some inherent delay t_0. To accurately determine this delay, the FPGA was set to output exactly the input at the highest sampling frequency. The transfer function of this operation was then measured. If the input signal is a sinusoid, then

$$\begin{aligned} \text{Input:} \quad & A\cos(\omega t) \\ \text{Output:} \quad & A\cos(\omega(t-t_0)) = A\cos(\omega t + \phi) \end{aligned}$$

Thus, the phase delay will be $\phi = -\omega t_0 = -2\pi f t_0$, and the magnitude of the slope of the line in the transfer function phase is $2\pi t_0$. The measured intrinsic delay was on the order of 15 μs, corresponding to a maximum sampling frequency of more than 65 kHz, more than enough to perform real-time filtering for this system.

Finally, all filters work as expected. Figure 10(b) demonstrates the behavior of a low-pass filter.

7 Conclusions

We have demonstrated initial steps in setting up a simple model for active noise cancellation. The mechanical setup was constructed and analyzed, the ideal filtering transfer function was calculated, and the FPGA was programmed to implement arbitrary filters. The final step is the use of a more reliable fitting routine to determine coefficients for time-domain filtering.

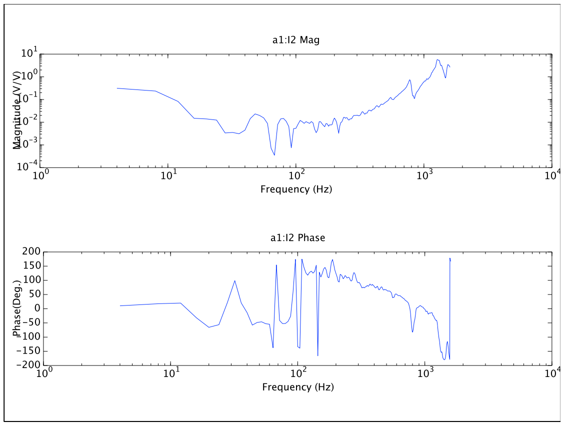
Future mechanical setups will need to use more stable components with behavior that is time independent. Further, it must be set up in such a way so as to minimize feedback from the correcting piezo to the witness sensor.

Further steps include trying this same experiment using more sensitive accelerometers to attempt to cancel out noise in the frequency range in which it will actually be canceled in second- and third-generation gravitational wave detectors ($\lesssim 10$ Hz).

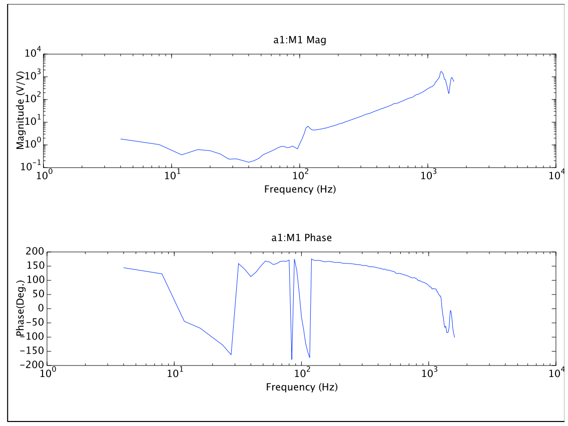
8 Acknowledgements

I would like to thank Ludovico Carbone, Frank Brueckner, and Andreas Freise for mentoring me this summer and answer my many questions. I'd also like to thank the NSF and the University of Florida IREU program for funding my research this summer. Lastly, I want like to thank the entire gravitational wave group at the University of Birmingham for welcoming me into their group for the past few months.

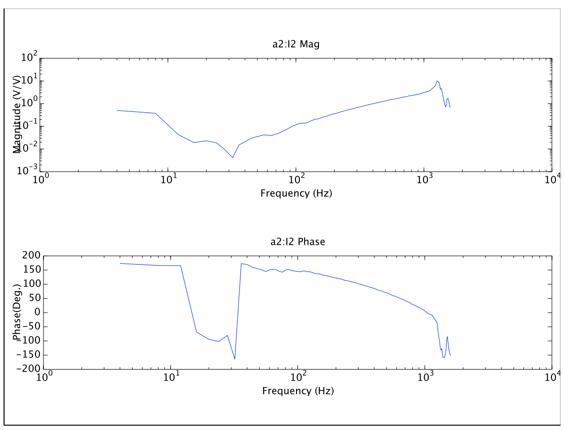
A Measured Transfer Functions



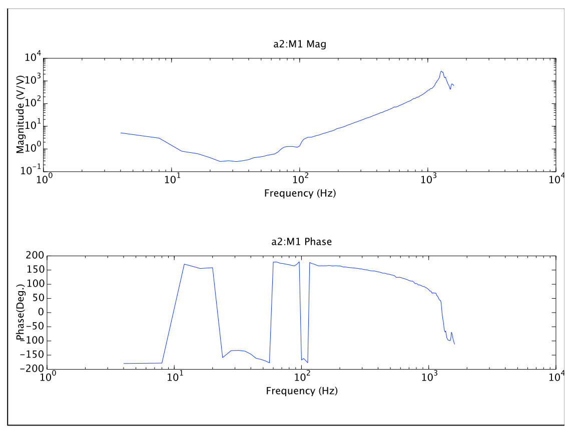
(a) TF of bottom accelerometer from top piezo driver input. Note that it is far smaller in magnitude than the other plots and the slope increases with frequency, indicating some mechanical coupling.



(b) TF of bottom accelerometer from bottom piezo driver monitor.



(c) TF of top accelerometer from top piezo driver input.



(d) TF of top accelerometer from bottom piezo driver monitor.

Figure 11: Various transfer functions used to calculate the ideal filter transfer function. Note that most of these plots exhibit the expected linear behavior over much of their range. They all illustrate some sort of mechanical resonance around 1 kHz. Further, they are all very noisy below 100 Hz, indicating the electronic noise is overwhelming the accelerometer signal. The TF of the bottom accelerometer from the top piezo driver input (11(a)) is the only one exhibiting very strange behavior, and we hope to minimize this response in future experimental setups.

References

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